CHRISTIAN BROTHERS' HIGH SCHOOL, LEWISHAM

MATHEMATICS - 2/3 UNIT COMMON

Year 11/12 — November 2009

Instructions:

- Show all working.
- Marks may be deducted for careless or poorly arranged work.
- HSC approved calculators may be used.
- Diagrams not to scale.
- Time - 50 minutes

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1. For the parabola $x^2 = 12y$ find: [6 marks]

(i) the focal length
(ii) the co-ordinates of the vertex
(iii) the co-ordinates of the focus
(iv) the equation of the axis
(v) the equation of the directrix
(vi) the length of the latus rectum

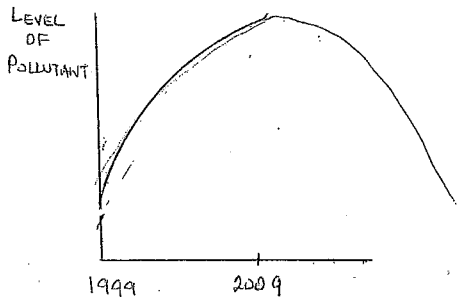
2. Find the equation of the circle with centre $(1,-5)$ and radius $3\sqrt{3}$ units. [2 marks]

3. If $f(x) = x^3 + 12x^2 + 45x$, find the values of x for which the function is:

(i) increasing [3 marks]

(ii) concave up [2 marks]

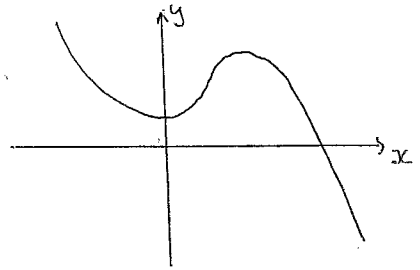
4. (i) The graph below shows the level of pollutant in the atmosphere over the past 20 years. Describe briefly how the level of this pollutant has changed over this period of time i.e. what does the graph imply about the 1st and 2nd derivatives? [2 marks]



Question 4 continued on page 2...

Question 4 continued:

(ii) The diagram shows the graph of a certain function $f(x)$. [2 marks]



(a) Copy this graph on to your answer sheet.
(b) On the same set of axes draw a sketch of the derivative $f'(x)$.

5. (i) A parabola has vertex $(-3,-1)$ and focus $(-3,-2)$. Determine its equation. [3 marks]

(ii) $P(4,2)$ lies on the parabola $x^2 = 8y$

(a) Find the equation of the tangent to the parabola at P. [2 marks]

(b) Find the equation of the normal to the parabola at P. [2 marks]

(c) If the tangent and the normal meet the x axis at A and B respectively, find the area of the triangle PAB. [3 marks]

6. Consider the parabola $4y = x^2 - 2x + 5$. Find

(i) the co-ordinates of the vertex. [2 marks]

(ii) the co-ordinates of the focus. [2 marks]

7. A function is defined by $f(x) = x^3 - 3x^2 - 9x + 22$

(i) Find the co-ordinates of the turning points of the graph $y = f(x)$ and determine their nature. [3 marks]

(ii) Find the co-ordinates of the point of inflexion. [2 marks]

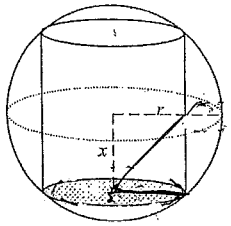
(iii) Hence sketch the graph of $y = f(x)$, showing the turning points, the point of inflexion and where the curve crosses the y axis. [2 marks]

(iv) For what values of x is the graph of $y = f(x)$ concave down. [1 mark]

(v) What is the maximum value of the curve if the domain is restricted to $-6 \le x \le 6$? [1 mark]

8. The curve $y = ax^3 + bx^2 + cx + 5$ has a stationary point at $x = 0$ and a point of inflexion at $(2,-11)$. Find a, b and c. [4 marks]

9. A cylinder is to be made to fit inside a sphere of radius r cm as shown.



Let x be the distance of the base of the cylinder from the centre of the sphere as shown.

(i) Find an expression for the radius of the base of the cylinder in terms of r and x. [1 mark]

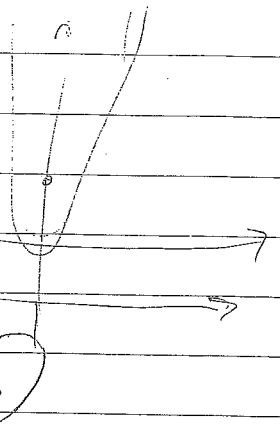
(ii) Show that the volume, V, of the cylinder is given by [2 marks]

$$V = 2\pi x\left(r^2 - x^2\right)$$

(iii) Find, in terms of r, the maximum volume of the cylinder. Give your answer in exact form. [3 marks]

Start here for Question Number:

1. i. 3 ✓
ii. (2,0) ✓
iii. (0,3) ✓
iv. $x=0$ ✓
v. $y=-3$ ✓
vi. 12 ✓



(6)

2. $(x-1)^2+(y+5)^2=27$ ✓ (2)

3. i. $f'(x)=3x^2+24x+45$

$3x^2+24x+45>0$.

$3(x^2+8x+15)>0$

$3(x+5)(x+3)>0$

$x<-5,\ x>-3$ ✓

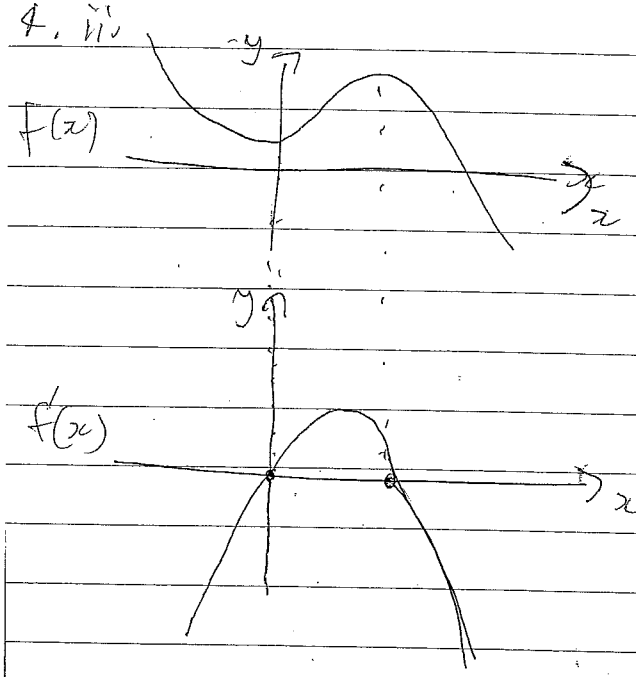
ii. $f''(x)=6x+24$

$6x+24>0$ ✓

$x>-4$ (5)

4. i. 1st derivative >0. ✓
2nd derivative <0. ✓ (2)

4. ii.



(2)

5. i. $-(x+3)^2=4(y+1)$ ✓ (3)

ii. a. $y'=\frac{x}{4}$

$m_1=1$.

$x-4=y-2$.

$y=x-2$ ①

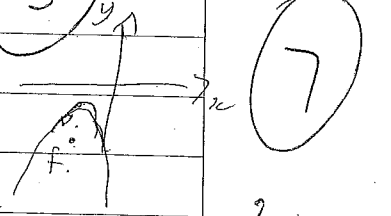
at $x=0$, $y=-2$ ∴ (0,−2)

b. $m_2=-1$

$-x+4=y-2$ ✓

$y=-x+6$. ② at $x=0$, $y=6$.

∴ (0,6)



(7)

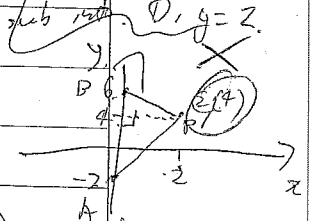
C. ① = ②

$2x-8=-x+6$

$3x=14$

$x=4\frac{2}{3}$ sub into ①, $y=2\frac{2}{3}$

$P=(4\frac{2}{3}, 2\frac{2}{3})$



$\frac{1}{2}\times 8\times 2$

$=8$ units^2

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6. $4y = (x-1)^2 + 5 - 1$

$4y - 4 = (x-1)^2$

$4(y-1) = (x-1)^2$

i. $(1, 1)$. ✓

ii. $(1, 2)$. ✓ (4)

7. i. $f'(x) = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$= 3(x-3)(x+1)$

$x = 3, -1$

Stationary points, $(-1, 27)$, $(3, -5)$. ✓

$f''(x) = 6x - 6$.

at $x = -1$, $f''(x) < 0$ ∴ Maximum turning point. ✓

at $x = 3$, $f''(x) > 0$ ∴ Minimum turning point ✓

ii. $f''(x) = 0$

$6x - 6 = 0$

$x = 1$

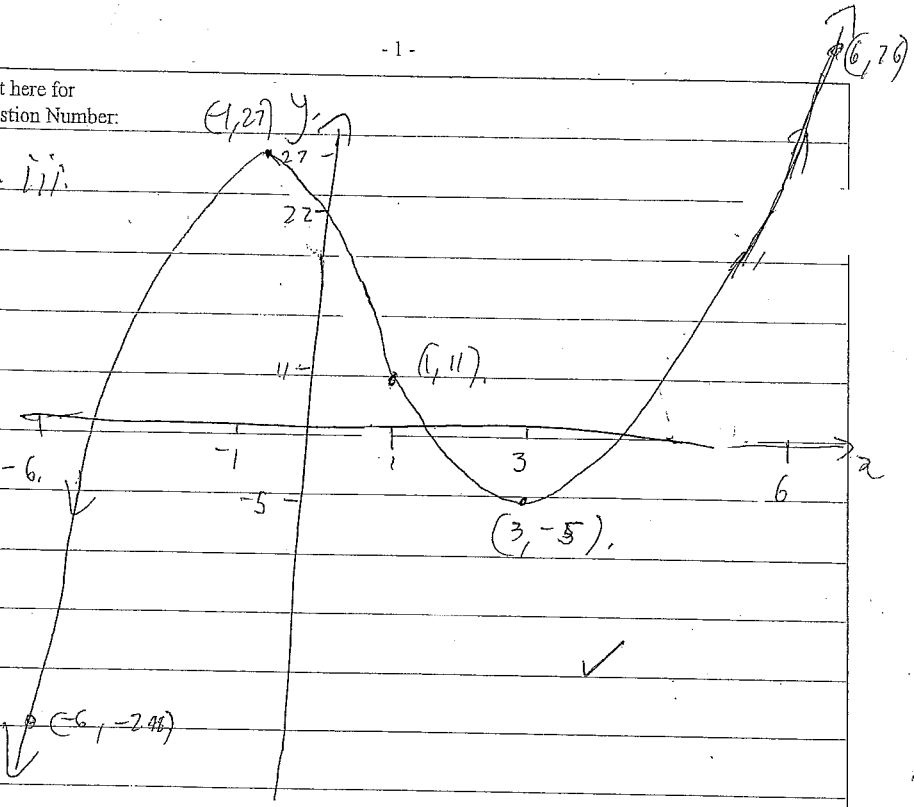
x	0.5	1	1.5
$f''(x)$	−	0	+

∴ Point of inflexion. ✓

Possible point of inflexion. $(1, 11)$.

Start here for Question Number:

7. iii.



iv. $x < 1$ ✓

v. at $x = -6$ $y = -248$.

$x = 6$ $y = 76$

∴ from diagram, maximum value is 76. ✓

(9)

8. $-11 = 8a + 4b + 2c + 5$

$-16 = 8a + 4b + 2c$

$-8 = 4a + 2b + c$ ①.

$f'(x) = 3ax^2 + 2bx + c$

$f'(0) = 0.$

$0 = c$ ②. ✓

$f''(x) = 6ax + 2b.$

$f''(2) = 0.$

$0 = 12a + 2b$ ③.

$0 = 6a + b.$ ✓

$x^3 - 6x^2 + 5.$

$3x^2 - 12x$

$6x - 12$

Sub ② into ①

$-8 = 4a + 2b.$ ④.

③ − ②.

$8 = 8a$

$a = 1$

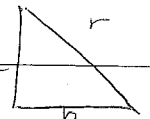
Sub $a = 1$ into ③, $b = -6.$ ✓

✓

$\therefore a = 1, b = -6, c = 0.$ (4)

9. ii) $r^2 = x^2 + R^2$

$R = \sqrt{r^2 - x^2}$



✓ 1

iii) $V = \pi R^2 \cdot 2x$ $(V = \pi r^2 h)$

$= 2x\pi(r^2 - x^2).$ ✓ 2

iii) $V' = 2\pi x \cdot -2x + (r^2 - x^2) \cdot 2\pi$

$= 2\pi(-2x^2 + r^2 - x^2)$

$= 2\pi(r^2 - 3x^2)$

$= 2\pi(r + \sqrt{3}x)(r - \sqrt{3}x).$

When $V' = 0$, $x = \pm \frac{r}{\sqrt{3}}$

$= \pm \frac{r\sqrt{3}}{3}$

But x is a distance, hence $x = \frac{r\sqrt{3}}{3}$.

$V'' = 2\pi \cdot -6x$ positive values of

$= -12\pi x < 0$ for all x.

hence $x = \frac{r\sqrt{3}}{3}$ is the x coordinate of a maximum turning point.

∴ maximum volume $= 2\pi\left(\frac{r\sqrt{3}}{3}\right)\left[r^2 - \left(\frac{r\sqrt{3}}{3}\right)^2\right]$

$= \frac{2\pi r\sqrt{3}}{3}\left(r^2 - \frac{3r^2}{9}\right)$

$= \frac{2\pi r\sqrt{3}}{3}\left(\frac{2r^2}{3}\right)$

$= \frac{4r^3\pi\sqrt{3}}{9}$ units^3 (2).

✓ (6)