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CENTRE OF EXCELLENCE IN MATHS TUITION

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MATHEMATICS SPECIMEN PAPER 1

EXPONENTIAL GROWTH & DECAY

1. The number n, of insects in a colony, is given by

$$n = 3500e^{0.08t}$$

Where t is the number of days after observations commence.

(a) Sketch the graph of n against t. [2]

(b) Find the population of the colony after 50 days. [2]

(c) How long does it take the population to double from when observations commenced? [2]

2. After police arrive at the scene of a murder the pathologist has determined that the body is cooling according to the law

$$T - T_0 = A_0 e^{-0\,0503t}$$

where T is the temperature of the body, T_0 is the temperature of the surroundings (the ambient temperature), t is the time in hours since the time of death and A is a constant to be found.

If the temperature of the body at 9.30 p.m. is 35.3°C, estimate the time the murder occurred, given that the surrounding temperature is 15°C and the normal body temperature is 37°C. [4]

3. When Emily did a parachute jump for charity the parachute opened shortly after she left the aircraft. Her velocity at time t seconds from when the parachute opened is v ms^{-1} where

$$v = 9 + 29e^{-0.063t}$$

(a) Sketch the graph of v against t. [2]

(b) What is Emily's speed at the instant the parachute opened? [1]

(c) What is the lowest speed she can possibly attain if she fell from a very great height? [1]

(d) If she actually landed after 45 seconds what was her speed on landing? [2]

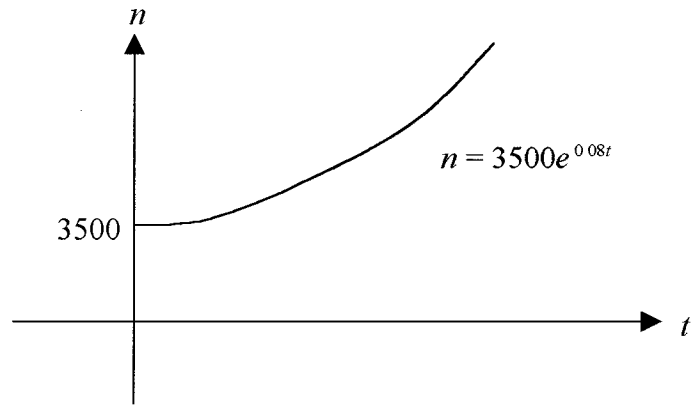
(e) How long did it take her to reach half the speed she had when the parachute opened? [3]

SOLUTIONS:

1. (a) When $t = 0$, $n = 3500e^0$

$= 3500$

The number of insects increases exponentially



(b) When $t = 50$

$n = 3500\, e^{0.08 \times 50}$

$= 3500e^4$

$n = 191094$ to the nearest insect

(c) When $n = 2 \times 3500 = 7000$

$7000 = 3500\, e^{0.08t}$

$2 = e^{0.08t}$

Take logarithms of both sides

$\ln 2 = \ln e^{0.08t}$

$\ln 2 = 0.08\, t$

$t = \dfrac{\ln 2}{0.08}$

$= 8.66$ to 2 decimal places

It takes 8.66 days for the population to double

2.

$T - T_0 = Ae^{-0.0503t}$

When $T = 0$, $T_0 = 15$

$37 - 15 = A$

$\Rightarrow \quad A = 22$

$T - 15 = 22e^{-0.0503t}$

When $T = 35.3$

$35.3 - 15 = 22e^{-0.0503t}$

$20.3 = 22e^{-0.0503t}$

$\div 22 \quad 0.923 = e^{-0.0503t}$

$-0.0503t = \ln 0.923$

$-0.0503t = -0.0804216$

$t = 1.599$ hours

The murder occurred at about 1.64 hours before 9.30 pm
$\therefore$ at approximately 7.54 pm

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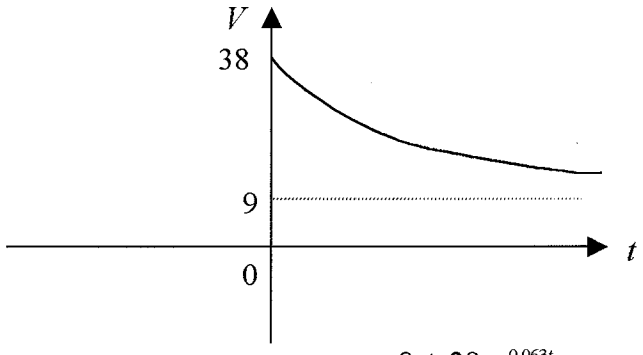
3. (a)



(b) $v = 9 + 29e^{-0.063t}$

When $t = 0$ $v = 9 + 29(1)$

$= 38$

$\Rightarrow$ Speed when parachute opened = 38 ms^{-1}

(c) $v = 9 + 29e^{-0.063t}$

As $t \to \infty$, $e^{-0.063t} \to 0$

$\Rightarrow$ $v \to 9$

$\Rightarrow$ Lowest speed she can attain = 9 ms^{-1}

(d) When $t = 45$

$v = 9 + 29e^{(-0.063 \times 45)}$

$= 10.7$

$\Rightarrow$ She landed at 10.7 ms^{-1}

(e) Speed when parachute opened = 38 ms^{-1}

Time to reach half this speed is given by

$19 = 9 + 29e^{-0.063t}$

$\div 29$ $e^{-0.063t} = \frac{10}{29}$

$\Rightarrow$ $-0.063t = \ln\left(\frac{10}{29}\right)$

$-0.063t = -1.0647$

$t = \frac{-1.0647}{-0.063}$

$= 16.9$

$\Rightarrow$Time taken to reach half the speed when parachute opened
= 16.9 seconds
