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MATHEMATICS
SPECIMEN PAPER 1

SUM TO INFINITY - GP

1. The second term of a geometric series is 60 and the fourth term is 21.6.

   (a) Find possible values of the common ratio and the corresponding first term of the series. [4]

   (b) Find the sum to infinity of the series, taking the positive value of the common ratio. [2]

2. The third term of a geometric series is 90 and the sixth term is $3\frac{1}{3}$.

(a) Find the common ratio and the first term of the geometric series. [4]

(b) Find the sum to infinity of the series. [2]

3. The sum of the first three terms of a geometric series is 27.1, and the sum to infinity is 100.

(a) Write down two equations involving a and r, the first term and common ratio respectively. [2]

(b) Hence find the values of a and r. [2]

(c) Write down an expression for the sum of n terms of this series. [1]

(d) How many terms would be required for the sum to be greater than 95? [3]

4. Given the infinite geometric series $1 - 2x + 4x^2 - 8x^3 + \ldots$

(a) Find the common ratio. [1]

(b) Write down an expression for the nth term. [1]

(c) Find the range of values of x for which the series has a sum to infinity and find this sum. [1]

(d) Given $x = 0.3$ find the sum of the first 10 terms. [1]

(e) Find the difference between the sum of the first 10 terms and the sum to infinity. Express this answer in standard form. [1]

5. A rubber ball is dropped from a height of 2m. It rises to a height of 1.6m and falls again. The heights to which it rises after each bounce are in geometric sequence.

(a) Find the height to which the ball rises after the seventh bounce. [3]

(b) Find the total distance travelled by the ball. [3]

6. The fifth term of a geometric series is 144 and the eighth term is -18.

(a) Find the common ratio. [3]

(b) Find the sum to infinity. [2]

SOLUTIONS:

1. (a)

$$ar = 60 \qquad -\{1\}$$
$$ar^3 = 21.6 \qquad -\{2\}$$

$\frac{\{2\}}{\{1\}}$

$$\frac{ar^3}{ar} = \frac{21.6}{60}$$
$$r^2 = 0.36$$
$$r = \pm 0.6$$

Substitute $r = 0.6$ in $\{1\}$

$$a \times 0.6 = 60$$
$$a = 100$$

Substitute $r = -0.6$ in $\{1\}$

$$a = -100$$

(b) The sum to infinity $S\infty = \frac{a}{1-r}$ if $|r| < 1$

Taking $r = 0.6$

$$S\infty = \frac{100}{1-0.6}$$
$$S\infty = 250$$

2. (a)

$$ar^2 = 90 \qquad -\{1\}$$
$$ar^5 = 3\frac{1}{3} \qquad -\{2\}$$

$\frac{\{2\}}{\{1\}}$

$$\frac{ar^5}{ar^2} = \frac{3\frac{1}{3}}{90}$$
$$r^3 = \frac{10}{3} \times \frac{1}{90}$$
$$r^3 = \frac{1}{27}$$
$$r = \sqrt[3]{\frac{1}{27}}$$
$$r = \frac{1}{3}$$

Substitute $r = \frac{1}{3}$ in $\{1\}$

$$a\left(\frac{1}{3}\right)^2 = 90$$

$$a \times \frac{1}{9} = 90$$

$\times 9$ $$a = 810$$

(b) The sum to infinity $S\infty = \dfrac{a}{1-r}$ if $|r| < 1$

$$= \frac{810}{\left(1-\frac{1}{3}\right)}$$

$$S\infty = 1{,}215$$

3\. (a)

$$a + ar + ar^2 = 27.1 \qquad -\{1\}$$

$$\frac{a}{(1-r)} = 100 \qquad -\{2\}$$

(b) From {2} $a = 100(1 - r)$

Substitute in {1}

$100(1 - r) + 100r(1 - r) + 100r^2(1 - r) = 27.1$

$\Rightarrow$

$$100 - 100r^3 = 27.1$$
$$100(1 - r^3) = 27.1$$
$$(1 - r^3) = 0.271$$
$$r^3 = 1 - 0.271$$
$$r^3 = 0.729$$
$$r = \sqrt[3]{0.729}$$
$$r = 0.9$$

$\therefore$

$$a = 100(1 - 0.9)$$
$$a = 10$$

(c) Sum of n terms of a geometric series is

$$S_n = a\frac{(1-r^n)}{(1-r)}$$

$$= 10\frac{(1-0.9^n)}{(1-0.9)}$$

$$S_n = 100(1-0.9^n)$$

$$a\left(\frac{1}{3}\right)^2 = 90$$

$$a\times\frac{1}{9} = 90$$

$\times 9$ $$a = 810$$

(b) The sum to infinity $$S\infty = \frac{a}{1-r} \quad \text{if } |r|<1$$

$$= \frac{810}{\left(1-\frac{1}{3}\right)}$$

$$S\infty = 1{,}215$$

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3. (a)

$$a + ar + ar^2 = 27.1 \quad -\{1\}$$

$$\frac{a}{(1-r)} = 100 \quad -\{2\}$$

(b) From {2} $$a = 100(1-r)$$

Substitute in {1}

$$100(1-r) + 100r(1-r) + 100r^2(1-r) = 27.1$$

$$\Rightarrow \quad 100 - 100r^3 = 27.1$$

$$100(1-r^3) = 27.1$$

$$(1-r^3) = 0.271$$

$$r^3 = 1 - 0.271$$

$$r^3 = 0.729$$

$$r = \sqrt[3]{0.729}$$

$$r = 0.9$$

$\therefore$ $$a = 100(1-0.9)$$

$$a = 10$$

(c) Sum of n terms of a geometric series is

$$S_n = a\frac{(1-r^n)}{(1-r)}$$

$$= 10\frac{(1-0.9^n)}{(1-0.9)}$$

$$S_n = 100(1-0.9^n)$$

(d) If $S_n > 95$

$100(1 - 0.9^n) > 95$

$\div 100 \quad 1 - 0.9^n > 0.95$

$\Rightarrow \quad 0.9^n < 0.05$

Take ln of both sides

$\ln 0.9^n < \ln 0.05$

$n \ln 0.9 < \ln 0.05$

$-0.1054n < -2.9957$

$\div -0.1054 \quad n > \frac{-2.9957}{-0.1054}$ (Remember to reverse the inequality)

$n > 28.4$

$\Rightarrow$ 29 terms would be required for the sum to be greater than 95.

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4. (a) Common ratio $r = \frac{T_2}{T_1}$

$= -\frac{2x}{1}$

$r = -2x$

(b) nth term of a geometric series is

$T_n = ar^{n-1}$

$= 1(-2x)^{n-1}$

$T_n = (-1)^{n-1}(2x)^{n-1}$

(c) In order that the series has a sum to infinity then

$|r| < 1$

$|-2x| < 1$

$-1 < 2x < 1$

$\div 2 \quad -\frac{1}{2} < x < \frac{1}{2}$

The sum to infinity of a geometric series

$S\infty = \frac{a}{1-r}$

$= \frac{1}{1-(-2x)}$

$S\infty = \frac{1}{(1+2x)}$

(d) $x = 0.3$ $\therefore$ $r = -2 \times 0.3$

$= -0.6$

Sum of the first n terms of a geometric series is

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum of the first 10 terms $S_{10} = \dfrac{1(1-(-0.6)^{10})}{(1-(-0.6))}$

$S_{10} = 0.6212209$ to 7 decimal places.

(e) The sum to infinity, $S\infty = \dfrac{a}{1-r}$

$$= \frac{1}{1-(-0.6)}$$

$$= 0.625$$

$\therefore$ $S\infty - S_{10} = 0.625 - 0.6212209$

$= 0.003779$

$S\infty - S_{10} = 3.8 \times 10^{-3}$

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5. (a) $h_1 = 2$

$h_2 = 1.6$

As these heights are in geometric sequence

$$r = \frac{1.6}{2}$$

$$r = 0.8$$

The height to which the ball rises after the seventh bounce $= h_8$

$T_n = ar^{n-1}$

$h_8 = 2 \times 0.8^{8-1}$

$h_8 = 0.42$m to 2 d.p.

(b) The total distance travelled by the ball is the sum to infinity

$2 + 2 \times 1.6 + 2 \times 1.28 + \ldots\ldots\ldots$

After each bounce the ball rises and falls the same distance

$= 2 + 2[1.6 + 1.28 + \ldots..]$

$S\infty = 1.6 + 1.28 + \ldots\ldots.$

$$S\infty = \frac{1.6}{1-0.8}$$

$$= 8$$

$\therefore$ The distance travelled by the ball $= 2 + 2 \times 8$

The distance travelled by the ball $=$ 18m

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6. (a) $T_5 = ar^4 = 144 \quad \ldots .(1)$

$T_8 = ar^7 = -18 \quad \ldots .(2)$

(2) $\div$ (1) $\dfrac{T_8}{T_5} = \dfrac{ar^7}{ar^4} = \dfrac{-18}{144}$

$$r^3 = -\frac{1}{8}$$

Common ratio $r = -\dfrac{1}{2}$

First term is $a = \dfrac{144}{\left(-\dfrac{1}{2}\right)^4} = 2304$

(b) $S_\infty = \dfrac{a}{1-r} = \dfrac{2304}{1+\dfrac{1}{2}} = 1536$