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YEAR 11 – EXT.1 MATHS

REVIEW TOPIC : AUXILIARY ANGLE METHOD – BOOK 2

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Question 6:

Find all values of θ between 0° and 360° satisfying the equation

$$2\sin\theta + 4\sin(\theta + 60) = 1 .$$

Give your answer correct to the nearest degree. [7]

$\theta = 128^0, 330^0$ (to the nearest deg)

Question 7:

(a) Show that $15 \sin \theta + 8 \cos \theta$ may be written in the form $R \sin(\theta + \alpha)$ where R and α are constants to be found such that $R > 0$ and $0° < \alpha < 90°$. [3]

$\boxed{17\sin\left(\theta + 28.1^0\right)}$

(b) Hence find the maximum and minimum points of the expression $15 \sin \theta + 8 \cos \theta$ in the range $0° \leq \theta \leq 360°$. What are the values of θ that gives the maximum and minimum point? [5]

$\boxed{\text{Max } = 17 \text{ when } \theta = 61.9^c ; \text{Min } = -17 \text{ when } \theta = 241.9^c}$

(c) Sketch the graph of $y = 15 \sin \theta + 8 \cos \theta$ for $0° \leq \theta \leq 360°$. [2]

Question 8:

Given that $4\sin\theta - 3\cos\theta \equiv R\sin(\theta - \alpha)$

Find the value of R and the value of α where $R > 0$ and $0° < \alpha < 90°$. [3]

$\boxed{5\sin\left(\theta - 36.87^0\right)}$

Hence find all values of θ between 0° and 360° satisfying the equations

(a) $4\sin\theta - 3\cos\theta = 2$ [4]

$\boxed{\theta = 60.5^0, 193.3^0}$

(b) $4\sin2\theta - 3\cos2\theta = 2$ [4]

$\boxed{\theta = 30.2^0, 96.6^0, 210.2^0, 276.6^0}$

Question 9:

Given that $\sin\theta + 2\cos\theta = R\sin(\theta + \alpha)$

find the value of R and the value of α where $R > 0$ and $0° < \alpha < 90°$. [3]

$\boxed{\sqrt{5}\sin(\theta + 63.4°)}$

Hence find the greatest and least values of the expression

$$\frac{6}{\sin\theta + 2\cos\theta + 4}$$

and give the corresponding values of θ between $-180°$ and $180°$. [6]

$\boxed{\text{Max value} = \frac{6}{11}\left(4+\sqrt{5}\right) \text{ when } \theta = -153.4^0, \text{Min value} = \frac{6}{11}\left(4-\sqrt{5}\right) \text{ when } \theta = 26.6^0}$

Question 10:

(a) Express $\cos\theta - \sqrt{3}\sin\theta$ in the form $R\cos(\theta + \alpha)$

where $r > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. [3]

(b) Find the maximum and minimum values of

$$\frac{1}{\cos\theta - \sqrt{3}\sin\theta + 4}$$

stating the values of θ for which they occur in the range $-\pi < \theta < \pi$. [4]

(c) Solve the equation

$$\cos\theta - \sqrt{3}\sin\theta = \sqrt{3}$$

for values of θ between 0 and 2π inclusive. [4]

SOLUTIONS TO Q6 TO 10:

Question 6:

$2\sin\theta + 4\sin(\theta + 60) = 1$

Expand $\sin(\theta + 60)$ as follows:

$$\sin(\theta + 60) = \sin\theta\cos 60 + \sin 60\cos\theta$$
$$= \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$$

The equation becomes

$$2\sin\theta + 4\left(\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right) = 1$$
$$2\sin\theta + 2\sin\theta + 2\sqrt{3}\cos\theta = 1$$
$$4\sin\theta + 2\sqrt{3}\cos\theta = 1$$
$$2\sqrt{3}\cos\theta + 4\sin\theta \equiv R\cos(\theta - \alpha)$$

Where $R > 0$ and $0 \le \alpha \le 90°$

$$2\sqrt{3}\cos\theta + 4\sin\theta \equiv R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$$
$$= R\cos\alpha\cos\theta + R\sin\alpha\sin\theta)$$

Compare coefficients of $\cos\theta$

$$2\sqrt{3} = R\cos\alpha \qquad -\{1\}$$

Compare coefficients of $\sin\theta$

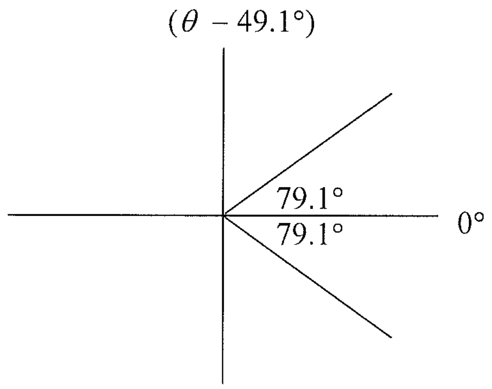
$$4 = R\sin\alpha \qquad -\{2\}$$

Squaring {1} and {2} and adding gives

$$\left(2\sqrt{3}\right)^2 + 4^2 = R^2(\cos^2\alpha + \sin^2\alpha)$$
$$\Rightarrow \quad R^2 = 28$$
$$R = \sqrt{28}$$
$$\frac{\{2\}}{\{1\}} = \quad \frac{R\sin\alpha}{R\cos\alpha} = \frac{4}{2\sqrt{3}}$$
$$\tan\alpha = \frac{2}{\sqrt{3}}$$
$$\alpha = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$
$$\alpha = 49.1°$$
$$\Rightarrow \quad 2\sqrt{3}\cos\theta + 4\sin\theta \equiv \sqrt{28}\cos(\theta - 49.1)$$

The equation becomes

$$\sqrt{28}\cos(\theta - 49.1) = 1$$
$$\cos(\theta - 49.1) = \frac{1}{\sqrt{28}}$$
$$\cos^{-1}\left(\frac{1}{\sqrt{28}}\right) = 79.1°$$



$\Rightarrow \quad \theta - 49.1 = 79.1 \quad \text{or} \quad \theta - 49.1 = 360 - 79.1$

$\theta = 128.2° \qquad \theta = 330.0°$

$\Rightarrow \quad \theta = 128°, 330°$ correct to the nearest degree

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Question 7:

(a)

$$15\sin\theta + 8\cos\theta \equiv R\sin(\theta + \alpha)$$
$$\equiv R(\sin\theta\cos\alpha + \sin\alpha\cos\theta)$$
$$\equiv R\cos\alpha\sin\theta + R\sin\alpha\cos\theta$$

Compare coefficients of $\sin\theta$

$$15 = R\cos\alpha \qquad -\{1\}$$

Compare coefficients of $\cos\theta$

$$8 = R\sin\alpha \qquad -\{2\}$$

Squaring {1} and {2} and adding gives

$$15^2 + 8^2 = R^2(\cos^2\alpha + \sin^2\alpha)$$
$$\Rightarrow \quad R^2 = 289$$
$$R = 17$$

$\frac{\{2\}}{\{1\}}$

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{8}{15}$$
$$\Rightarrow \quad \tan\alpha = \frac{8}{15}$$
$$\alpha = \tan^{-1}\left(\frac{8}{15}\right)$$
$$\alpha = 28.1°$$
$$\Rightarrow \quad 15\sin\theta + 8\cos\theta \equiv 17\sin(\theta + 28.1°)$$

(b) The function $17\sin(\theta + 28.1°)$ has a minimum value when

$$\sin(\theta + 28.1) = -1$$

$\Rightarrow$ Minimum value of $15\sin\theta + 8\cos\theta = -17$

The function $17\sin(\theta + 28.1°)$ has a maximum value when

$$\sin(\theta + 28.1) = 1$$

$\Rightarrow$ Maximum value of $15\sin\theta + 8\cos\theta = 17$

Minimum value when

$$\sin(\theta + 28.1) = -1$$

$$\Rightarrow \quad \theta + 28.1 = 270$$

$$\theta = 241.9°$$

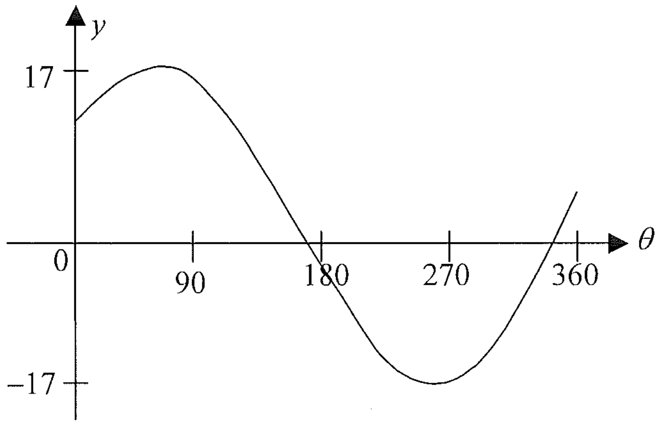
Maximum value when

$$\sin(\theta + 28.1) = 1$$

$$\theta + 28.1 = 90$$

$$\theta = 61.9°$$

(c)



The graph of $y = 17\sin(\theta + 28.1°)$ is obtained from the graph of $y = \sin\theta$ by a translation of $\begin{pmatrix} -28.1° \\ 0 \end{pmatrix}$ and a stretch of factor 17 along the y axis.

Question 8:

$$4\sin\theta - 3\cos\theta \equiv R\sin(\theta - \alpha)$$

$$\equiv R(\sin\theta\cos\alpha - \sin\alpha\cos\theta)$$

$$\equiv R\cos\alpha\sin\theta - R\sin\alpha\cos\theta$$

Compare coefficients of $\sin\theta$

$$4 = R\cos\alpha \qquad -\{1\}$$

Compare coefficients of $\cos\theta$

$$-3 = -R\sin\alpha$$

$$3 = R\sin\alpha \qquad -\{2\}$$

Squaring {1} and {2} and adding gives

$$4^2 + 3^2 = R^2(\cos^2\alpha + \sin^2\alpha)$$

$$\Rightarrow \quad R^2 = 25$$

$$R = 5$$

$$\frac{\{2\}}{\{1\}} \qquad \frac{R\sin\alpha}{R\cos\alpha} = \frac{3}{4}$$

$$\Rightarrow \quad \tan\alpha = 0.75$$

$$\alpha = \tan^{-1}0.75$$

$$\alpha = 36.87°$$

$$\Rightarrow \quad 4\sin\theta - 3\cos\theta \equiv 5\sin(\theta - 36.87°)$$

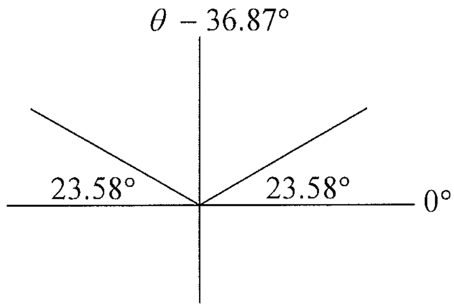
(a)

$$4\sin\theta - 3\cos\theta = 2$$

$$\Rightarrow \quad 5\sin(\theta - 36.87) = 2$$

$$\sin(\theta - 36.87) = \frac{2}{5}$$

$$\sin^{-1}0.4 = 23.58°$$



$$\Rightarrow \quad \theta - 36.87 = 23.58 \quad \text{or} \quad \theta - 36.87 = 180 - 23.58$$

$$\theta = 60.45 \quad \text{or} \quad \theta = 193.29$$

$\Rightarrow$ $\theta = 60.5°$, $193.3°$ correct to 1 decimal place

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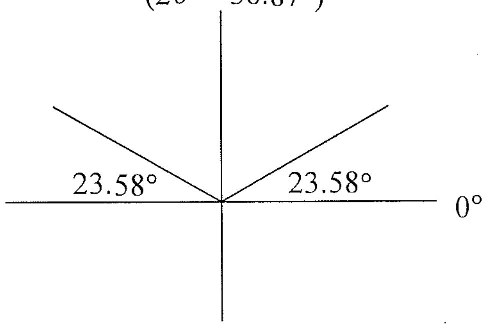
(b)

$$4\sin 2\theta - 3\cos 2\theta = 2$$

Replace θ with 2θ in the quadrant diagram as follows:



$$\Rightarrow \quad 2\theta - 36.87 = 23.58 \quad \text{or} \quad 2\theta - 36.87 = 180 - 23.58$$
$$2\theta = 60.45 \qquad 2\theta = 193.29$$
$$\theta = 30.235 \qquad \theta = 96.65$$

$$\text{or} \quad 2\theta - 36.87 = 360 + 23.58 \quad \text{or} \quad 2\theta - 36.87 = 540 - 23.58$$
$$2\theta = 420.45 \qquad 2\theta = 553.29$$
$$\theta = 210.23 \qquad \theta = 276.64$$

$\Rightarrow \quad \theta = 30.2°,\ 96.6°,\ 210.2°,\ 276.6°$ correct to 1 decimal place

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Question 9:

$$\sin\theta + 2\cos\theta \equiv R\sin(\theta + \alpha)$$
$$\equiv R(\sin\theta\cos\alpha + \sin\alpha\cos\theta)$$
$$\equiv R\cos\alpha\sin\theta + R\sin\alpha\cos\theta$$

Compare coefficients of $\sin\theta$

$$1 = R\cos\alpha \qquad \text{-}\{1\}$$

Compare coefficients of $\cos\theta$

$$2 = R\sin\alpha \qquad \text{-}\{2\}$$

Squaring {1} and {2} and adding gives

$$1^2 + 2^2 = R^2(\cos^2\alpha + \sin^2\alpha)$$
$$\Rightarrow \quad R^2 = 5$$
$$R = \sqrt{5}$$

$$\frac{\{2\}}{\{1\}} \qquad \frac{R\sin\alpha}{R\cos\alpha} = \frac{2}{1}$$

$$\Rightarrow \quad \tan\alpha = 2$$
$$\alpha = \tan^{-1}2$$
$$\alpha = 63.4°$$
$$\Rightarrow \quad \sin\theta + 2\cos\theta \equiv \sqrt{5}\sin(\theta + 63.4°)$$

$$\frac{6}{\sin\theta + 2\cos\theta + 4} \equiv \frac{6}{\sqrt{5}\sin(\theta + 63.4°) + 4}$$

The minimum value of the expression will occur when $\sqrt{5}\sin(\theta + 63.4°) + 4$ is a maximum

$\Rightarrow \quad \sin(\theta + 63.4) = 1$

$\Rightarrow \quad \text{minimum value} = \dfrac{6}{\sqrt{5}(1) + 4}$

$$= \frac{6}{(\sqrt{5}+4)}\frac{(\sqrt{5}-4)}{(\sqrt{5}-4)}$$

$$= \frac{6(\sqrt{5}-4)}{5-16}$$

$$\text{minimum value} = \frac{6}{11}(4-\sqrt{5})$$

This will occur when

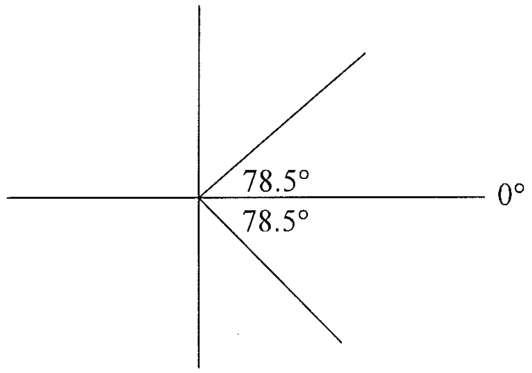
$$\theta + 63.4° = 90°$$
$$\theta = 26.6°$$

The maximum value of the expression will occur when $\sqrt{5}\sin(\theta + 63.4°) + 4$ is a minimum

$\Rightarrow \quad \sin(\theta + 63.4°) = -1$

$\text{maximum value} = \dfrac{6}{\sqrt{5}(-1) + 4}$

$\theta = 53.1°$



$$= \frac{6}{(4-\sqrt{5})}$$

$$= \frac{6}{(4-\sqrt{5})}\frac{(4+\sqrt{5})}{(4+\sqrt{5})}$$

$$= \frac{6\left(4+\sqrt{5}\right)}{16-5}$$

$$= \frac{6}{11}\left(4+\sqrt{5}\right)$$

This will occur when

$$\theta + 63.4° = -90°$$

$$\theta = -153.4°$$

Question 10:

(a)

$$\cos\theta - \sqrt{3}\sin\theta \equiv R\cos(\theta + \alpha)$$

$$\equiv R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$$

$$\equiv R\cos\alpha\cos\theta - R\sin\alpha\sin\theta$$

Compare coefficients of $\cos\theta$

$$1 = R\cos\alpha \qquad \text{-}\{1\}$$

Compare coefficients of $\sin\theta$

$$-\sqrt{3} = -R\sin\alpha$$

$$\sqrt{3} = R\sin\alpha \qquad \text{-}\{2\}$$

Squaring {1} and {2} and adding gives

$$1^2 + \left(\sqrt{3}\right)^2 = R^2(\cos^2\alpha + \sin^2\alpha)$$

$$R^2 = 4$$

$$R = 2$$

$\frac{\{2\}}{\{1\}}$:

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow \quad \tan\alpha = \sqrt{3}$$

$$\alpha = \tan^{-1}\sqrt{3}$$

$$\alpha = \frac{\pi}{3}$$

$$\Rightarrow \quad \cos\theta - \sqrt{3}\sin\theta \equiv 2\cos\left(\theta + \frac{\pi}{3}\right)$$

(b)

$$\frac{1}{\cos\theta - \sqrt{3}\sin\theta + 4} \equiv \frac{1}{2\cos\left(\theta + \frac{\pi}{3}\right) + 4}$$

Maximum value of the expression $\frac{1}{2\cos\left(\theta + \frac{\pi}{3}\right) + 4}$ occurs when

$\cos\left(\theta + \frac{\pi}{3}\right)$ is a minimum.

$$\Rightarrow \quad \cos\left(\theta+\frac{\pi}{3}\right) = -1$$

$$\text{Maximum value} = \frac{1}{2\cos\left(\theta+\frac{\pi}{3}\right)+4}$$

$$= \frac{1}{2(-1)+4}$$

$$\text{Maximum value} = \frac{1}{2}$$

When

$$\cos\left(\theta+\frac{\pi}{3}\right) = -1$$

$$\theta+\frac{\pi}{3} = \pi$$

$$\theta = \frac{2\pi}{3}$$

Minimum value of the expression $\dfrac{1}{2\cos\left(\theta+\frac{\pi}{3}\right)+4}$ occurs when

$\cos\left(\theta+\frac{\pi}{3}\right)$ is a maximum

$$\Rightarrow \quad \cos\left(\theta+\frac{\pi}{3}\right) = 1$$

$$\text{Minimum value} = \frac{1}{2\cos\left(\theta+\frac{\pi}{3}\right)+4}$$

$$= \frac{1}{2(1)+4}$$

$$\text{Minimum value} = \frac{1}{6}$$

When

$$\cos\left(\theta+\frac{\pi}{3}\right) = 1$$

$$\theta+\frac{\pi}{3} = 0$$

$$\theta = -\frac{\pi}{3}$$

Maximum $\left(\frac{2\pi}{3},\frac{1}{2}\right)$, Minimum $\left(\frac{1}{6},-\frac{\pi}{3}\right)$

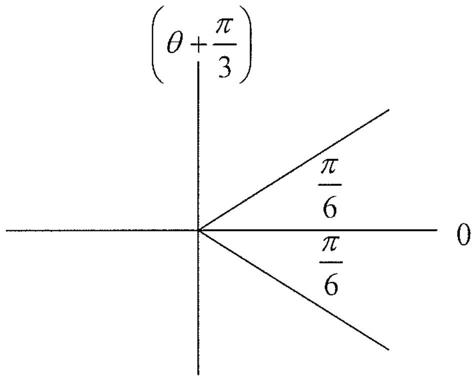
(c)

$$\cos\theta-\sqrt{3}\sin\theta = \sqrt{3}$$

$$\Rightarrow \quad 2\cos\left(\theta+\frac{\pi}{3}\right) = \sqrt{3}$$

$$\cos\left(\theta+\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$$



$$\theta+\frac{\pi}{3}=2\pi-\frac{\pi}{6} \quad \text{or} \quad \theta+\frac{\pi}{3}=2\pi+\frac{\pi}{6}$$

$$\theta=\frac{3\pi}{2} \qquad \theta=11\frac{\pi}{6}$$

$$\theta = \frac{3\pi}{2},\frac{11\pi}{6}$$
