CSSA
CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NSW

Centre Number

Student Number

2014
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Thursday 31 July 2014

General Instructions

- Reading time – 5 mins
- Working time – 3 hours
- Write using blue or black pen
  Black pen is preferred
- Use Multiple Choice Answer Sheet provided
- Board-approved calculators may be used
- A table of standard integrals is provided on a SEPARATE sheet
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks – 100

Section I Pages 2-6

10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II Pages 7- 17

90 marks
- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section

6200-1

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$\int x^n\,dx$	$=\frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\ \text{if } n<0$
$\int \frac{1}{x}\,dx$	$=\ln x,\quad x>0$
$\int e^{ax}\,dx$	$=\frac{1}{a}e^{ax},\quad a\neq 0$
$\int \cos ax\,dx$	$=\frac{1}{a}\sin ax,\quad a\neq 0$
$\int \sin ax\,dx$	$=-\frac{1}{a}\cos ax,\quad a\neq 0$
$\int \sec^2 ax\,dx$	$=\frac{1}{a}\tan ax,\quad a\neq 0$
$\int \sec ax\,\tan ax\,dx$	$=\frac{1}{a}\sec ax,\quad a\neq 0$
$\int \frac{1}{a^2+x^2}\,dx$	$=\frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$
$\int \frac{1}{\sqrt{a^2-x^2}}\,dx$	$=\sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$
$\int \frac{1}{\sqrt{x^2-a^2}}\,dx$	$=\ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$
$\int \frac{1}{\sqrt{x^2+a^2}}\,dx$	$=\ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE: $\ln x=\log_e x,\quad x>0$

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 Calculate the midpoint of $P(3,-4)$ and $Q(-1,2)$.

(A) $(-1,1)$
(B) $(1,-1)$
(C) $(1,1)$
(D) $(-1,-1)$

2 What is the derivative of $e^x(x^2+2x)$?

(A) $(2x+2)$
(B) $e^x(2x+2)$
(C) $e^x(x^2-2)$
(D) $e^x(x^2+4x+2)$

3 In a class of 30 girls, 13 are dancers and 23 are gymnasts. If 7 girls do both dance and gymnastics, what is the probability that a girl chosen at random does neither dance nor gymnastics?

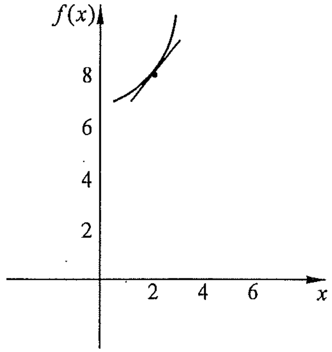
(A) $\frac{1}{5}$
(B) $\frac{7}{30}$
(C) $\frac{1}{30}$
(D) $\frac{8}{15}$

4 A function has the following properties:

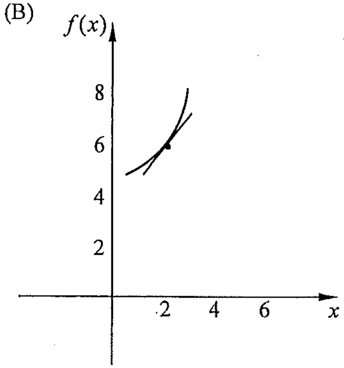
$f(2)=8$ $\qquad$ $f'(2)=6$ $\qquad$ $f''(2)=2$

Which sketch matches the graph of the function near $x=2$?

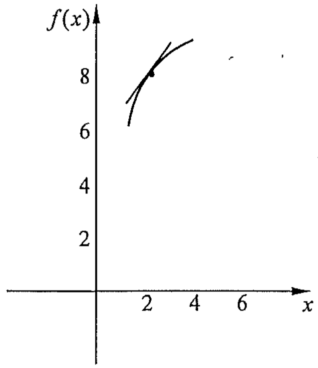
(A)



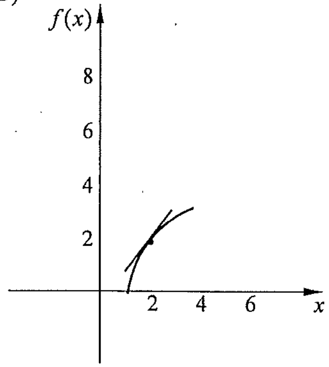
(B)



(C)



(D)



5 The quadratic equation $3x^2 - x - 4 = 0$ has roots α and β.

What is the value of $\alpha + \beta$?

(A) $-\frac{4}{3}$

(B) $-\frac{1}{3}$

(C) $\frac{1}{3}$

(D) $\frac{4}{3}$

6 Which expression is the correct simplification of $\frac{25^{2x}}{5^x}$?

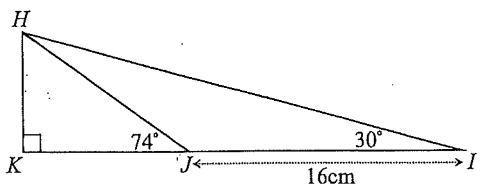
(A) 5^2

(B) 5^x

(C) 5^{2x}

(D) 5^{3x}

7



In the diagram, find the length of HK correct to two decimal places.

(A) 8.00cm

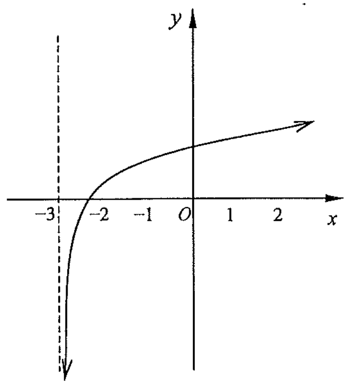
(B) 11.07cm

(C) 11.52cm

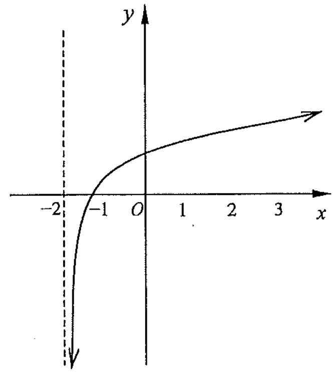
(D) 55.80cm

8 Which diagram shows the graph of $y = 2\log_e(x+3)$?

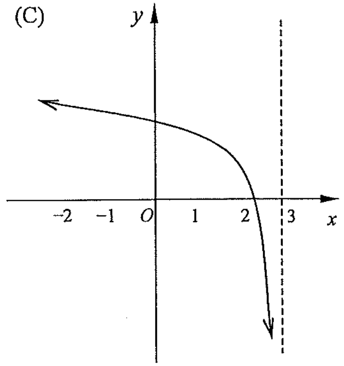
(A)



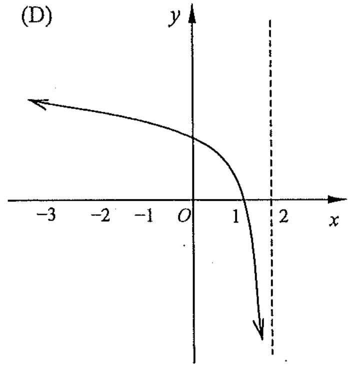
(B)



(C)

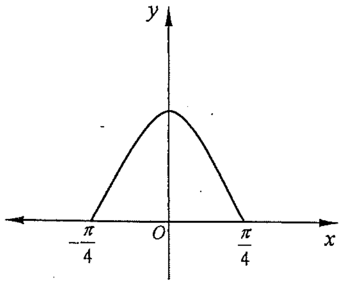


(D)



9 The diagram below shows the region bounded by the curve $y=\sqrt{5}\cos 2x$ and the x-axis for $-\frac{\pi}{4}\le x\le\frac{\pi}{4}$. The region is rotated about the x-axis to form a solid.

Which of the following gives the volume of the solid?



(A) $V=5\pi\int_0^{\frac{\pi}{4}}\cos 2x\ dx$

(B) $V=10\pi\int_0^{\frac{\pi}{4}}\cos 2x\ dx$

(C) $V=10\pi\int_0^{\frac{\pi}{4}}\cos 4x\ dx$

(D) $V=25\pi\int_0^{\frac{\pi}{4}}\cos 2x\ dx$

10 What is the limiting sum of the geometric series

$x^2-2x^3+4x^4-8x^5+\ldots$ where $|x|<1$?

(A) $\frac{x^2}{1+2x}$

(B) $\frac{x^2}{1-2x}$

(C) $\frac{1}{1+2x}$

(D) $\frac{x^2\left(1-(-2x)^n\right)}{1+2x}$

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Find the value of r correct to 2 significant figures if $\frac{4}{3}\pi r^3=200$. **2**

(b) Factorise m^2-m-42. **1**

(c) Find $\int \frac{1}{x^2}\,dx$. **2**

(d) Differentiate $y=\frac{\sin x}{x+1}$. **2**

(e) Evaluate $\lim_{x\to 1}\frac{2x^2}{5x^2+1}$ **1**

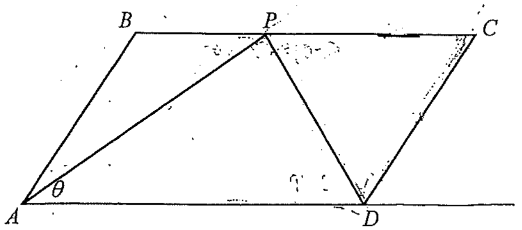
Question 11 continues on page 8

Question 11 (continued)

(f) Show that $\log_e \frac{1}{\sqrt{e}} = -\frac{1}{2}$. **2**

(g) Find the value of k given $\int_0^2 4x + k \, dx = 18$. **2**

(h)



$ABCD$ is a parallelogram. P is a point chosen on side BC such that AP bisects $\angle DAB$ and $\angle APD = 90°$.

Let $\angle PAD = \theta$.

(i) Prove that $\angle CPD = (90 - \theta)$. **1**

(ii) Prove that $PC = DC$. **2**

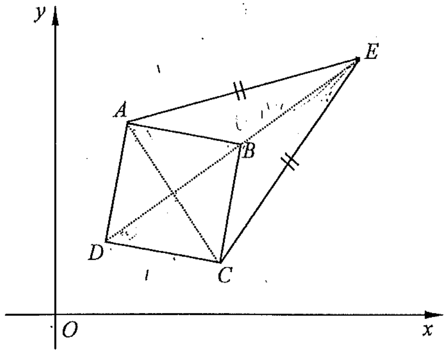
Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Sketch the graph of $y = x^3 - 1$. **1**

(ii) Find the equation of the normal at the point where $x = -1$. **2**

(iii) At what point does the normal to the curve $y = x^3 - 1$ cross the y-axis? **1**

(b) The points $A(4, 11)$, $B(12, 10)$, $C(11, 2)$ and $D(3, 3)$ form a square as shown below.
E is a point on DB extended such that $AE = CE$ and $DB = BE$.



(i) Find the length of AC. Leave your answer in exact form. **1**

(ii) Find the coordinates of E. **2**

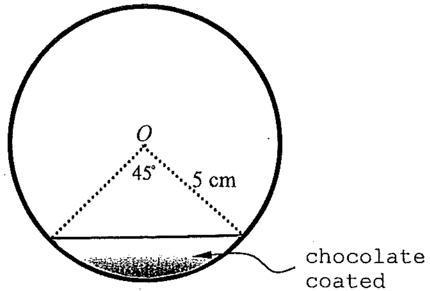
(iii) Explain why $ADCE$ is a kite. **1**

(iv) Hence, find the area of $ADCE$. **2**

Question 12 continues on page 10

Question 12 (continued)

(c) A circular wafer slice, to be used for a dessert, is to have a portion of its front coated in chocolate as shown in the diagram. The radius of the wafer slice is 5cm. **3**

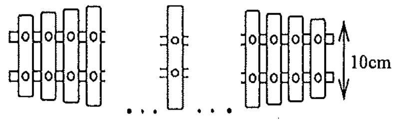


What area of the wafer will remain uncoated?
Give your answer correct to the nearest cm^2.

(d) Evaluate $\sum_{n=1}^{4} \frac{n^2}{3n-1}$ **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a)



A percussionist is experimenting with designs for a xylophone. It is to be **symmetrical** in shape as shown in the diagram above.

The shortest wooden bar is to be 10cm long and the consecutive bars will differ in length by b cm. The total length of all the wooden bars is S cm.

Let the number of wooden bars be $2n+1$.

(i) Show that $S = bn^2 + 20n + 10$. **2**

(ii) Given that $S = 360$ cm and $b = 1.5$ cm find the number of wooden bars.

(b) Julian's house is infested with termites. The population P of termites grows exponentially according to the equation $P = Ae^{kt}$ where A and k are constants, and t is the time in days.

When Julian called the exterminator there were 80 thousand termites in his house. Three days later, when the exterminator arrived to assess the house, the termite population had increased to 310 thousand

(i) Show that $P = Ae^{kt}$ satisfies $\frac{dP}{dt} = kP$. **1**

(ii) Find the value of k correct to 4 significant figures. **2**

(iii) When the termite population exceeds 2 million, the damage they cause will deem the house unsafe. After the initial assessment, the exterminator cannot return for another week to treat the house. **2**

Will there be enough time to save Julian's house?

Question 13 continues on page 12

(c) Consider the parabola $(x-2)^2 = 12y + 6$.

(i) Find the coordinates of the vertex. **1**

(ii) Find the coordinates of the focus. **1**

(d) (i) Draw neat sketches of the curve $f(x) = 3\cos x$ and the curve $g(x) = \sin 2x$ in the domain $0 \le x \le \pi$. **2**

(ii) Find the area of the region bounded by the curve $f(x) = 3\cos x$ and the curve $g(x) = \sin 2x$ in the given domain. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the function $f(x) = x^4 - x^3 - 2x^2 + 3x$.

(i) Show that $f'(x) = (4x-3)(x-1)(x+1)$. **2**

(ii) One of the stationary points of $y = f(x)$ is approximately $(0.75, 1.02)$. Find the other two stationary points. **2**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the graph of $y = f(x)$, showing the stationary points and the y-intercept. **2**

(b) Use the Trapezoidal rule to find an approximation for $\int_3^{11} f(x)\,dx$, using the table of values given below. Give your answer correct to two decimal places. **2**

x	3	5	7	9	11
$f(x)$	0	$\ln 3$	$\ln 5$	$\ln 7$	$\ln 9$

(c) Xavier is playing a variation of Chess called Makruk on-line. Each game is graded and Xavier begins with a 0.6 probability of winning and a 0.3 probability of losing.

At the end of each game, players receive points.

If Xavier wins he receives 5 points. If he loses he receives 2 points and a draw will result in Xavier receiving 3 points.

(i) What is the probability that Xavier's first game ends in a draw? **1**

In the next game, after grading has occurred, Xavier now has a 0.4 probability of winning and a 0.4 probability of losing.

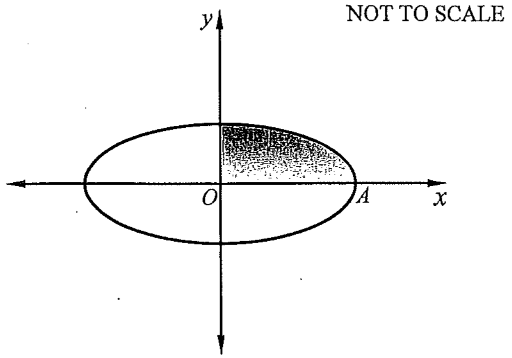
Find the probability that after two games:

(ii) Xavier receives ten points. **2**

(iii) Xavier receives five points or less. **2**

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram below shows an ellipse with equation $4x^2 + 9y^2 = 36$.



(i) Show that the coordinates of A are $(3,0)$. **1**

(ii) The area shaded in the diagram is rotated about the x-axis. Find the volume generated. Leave your answer in terms of π. **3**

(b) A particle is moving in a straight line with velocity $v = 3e^t - 12e^{-2t}$. Initially the particle is at the origin, t is measured in seconds and v in metres per second.

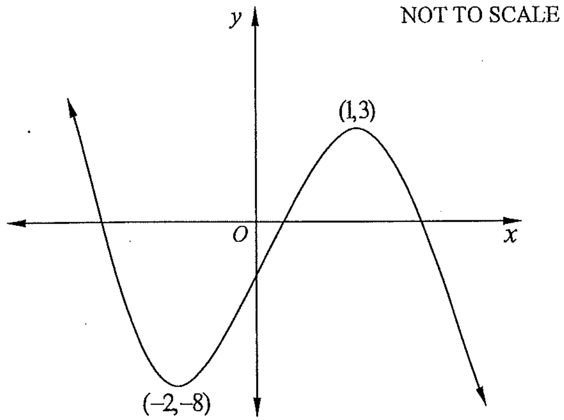
(i) Find the initial velocity of the particle. **1**

(ii) Is the particle ever at rest? Support your answer with suitable calculations. **2**

(iii) Find the displacement of the particle at $t = 4$ seconds. Write your answer correct to one decimal place. **2**

Question 15 continues on page 15

(c) Consider the graph of the function $y = f(x)$, with the coordinates of its turning points shown.



(i) On a separate diagram sketch the graph of $y = f'(x)$. **2**

(ii) Find the area of the region bounded by $y = f'(x)$ and the x-axis. **2**

(d) Given that $x = \log_a 3$ and $y = \log_a 5$, **2**

show that $\log_a\left(\frac{a^2}{75}\right) = 2 - 2y - x$.

Question 16 (15 marks) Use a SEPARATE writing booklet.

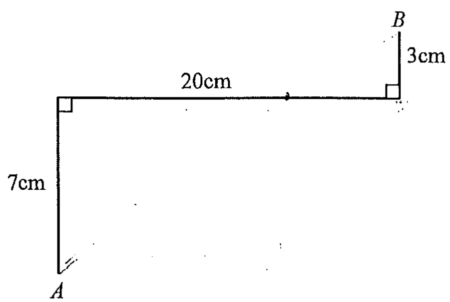
(a) Monica invests \$250 into an account at the Bank of Newton. She invests the money at the beginning of each month for n years. Interest is to be paid at a rate of 6%p.a. compounded monthly.

(i) Show that the total value of her investment (A_n) at the end of n years is given by $A_n = \$250\left(1.005 + 1.005^2 + ... + 1.005^{12n}\right)$. **2**

(ii) Find the value of the investment at the end of 7 years. **2**

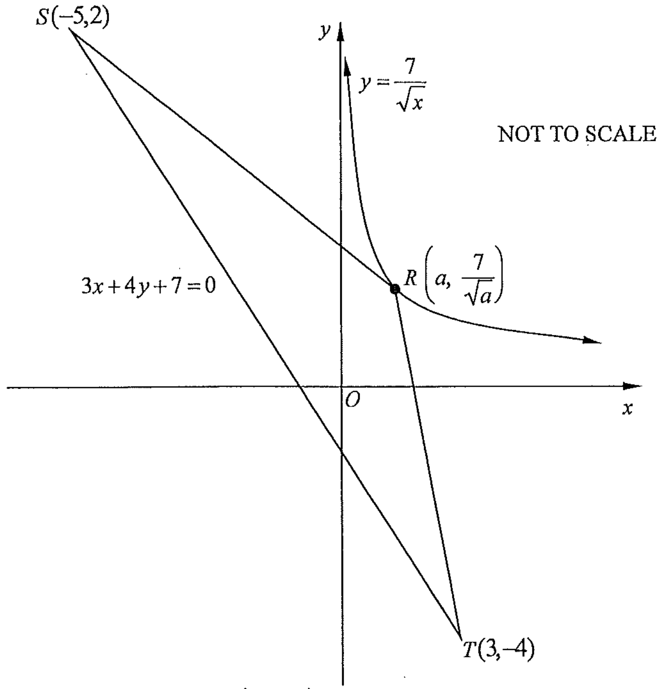
(iii) What single investment at the beginning of the 7 years would yield the same final value for Monica? You may assume interest is compounded monthly. **2**

(b) Find the straight line distance from A to B. Leave your answer in exact form. **2**



Question 16 continues on page 17

(c)



In the diagram, point $R\left(a, \frac{7}{\sqrt{a}}\right)$ is a variable point on the curve $y = \frac{7}{\sqrt{x}}$.

The points $S(-5,2)$ and $T(3,-4)$ lie on the straight line $3x + 4y + 7 = 0$.

(i) Show that the area of the triangle RST is $A = 3a + \frac{28}{\sqrt{a}} + 7$. **3**

(ii) Find the value of a which will produce the triangle of minimum area. Give your answer correct to one decimal place. **3**

(iii) Hence find the minimum area of triangle RST. **1**

End of paper

CSSA

CATHOLIC SECONDARY SCHOOLS ASSOCIATION
2014 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS – MARKING GUIDELINES

Section I

Questions (1 mark each)

Question	Answer	Outcomes Assessed	Targeted Performance Bands
1	B	P3	2-3
2	D	H3	3-4
3	C	H5	4-5
4	A	H6	3-5
5	C	P5	2-3
6	D	P3	2-3
7	B	P3	4-5
8	A	H3	3-4
9	B	H8	4-5
10	A	H5	3-4

Section II

Question 11 (15 marks)

(a) (2 marks)

Sample answer

$\frac{4}{3}\pi r^3 = 200$

$r = \sqrt[3]{\frac{150}{\pi}}$

$r = 3.6$ to 2 significant figures

(b) (1 mark)

Sample answer

$m^2 - m - 42$

$= (m-7)(m+6)$

(c) (2 marks)

Sample answer

$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx$

$= \frac{x^{-1}}{-1} + C = \frac{-1}{x} + C$

(d) (2 marks)

Sample answer

$y = \frac{\sin x}{x+1}$

$y' = \frac{(x+1)(\cos x) - (1)(\sin x)}{(x+1)^2}$

$y' = \frac{x\cos x + \cos x - \sin x}{(x+1)^2}$

(11) (e) (1 mark)

Sample answer

$\lim_{x\to 1} \frac{2(1)^2}{5(1)^2+1}$

$= \frac{1}{3}$

(f) (2 marks)

Sample answer

$\ln\frac{1}{\sqrt{e}} = \ln 1 - \ln\sqrt{e} = 0 - \ln e^{1/2}$

$= -\frac{1}{2}\ln e$

$= -\frac{1}{2}$

(g) (2 marks)

Sample answer

$\int_0^2 4x + k\ dx = \left[2x^2 + kx\right]_0^2$

$\therefore\ 8 + 2k = 18$

$k = 5$

(h) (i) (1 mark)

Sample answer

$\angle PDA = (90 - \theta)$ (angle sum of ΔPAD)

$\angle CPD = (90 - \theta)$ (alternate angles, $DC \parallel AB$)

(h) (ii) (2 marks)

Sample answer

$\angle PCD = 2\theta$ (opposite angles of a parallelogram are equal)

$\angle CPD + \angle PCD + \angle PDC = 180°$ (angle sum of ΔPCD)

$90 - \theta + 2\theta + \angle PDC = 180°$

$\angle PDC = 180 - (90 - \theta + 2\theta)$

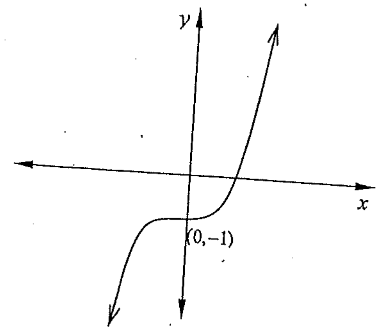
$= 90 - \theta$

so $PC = DC$ ($\angle PDC = \angle CPD$ is isosceles), as required.

Question 12 (15 marks)

(a) (i) (1 mark)

Sample answer



(a) (ii) (2 marks)

Sample answer

$y = x^3 - 1$

$\frac{dy}{dx} = 3x^2$

at $x = -1$, $m = 3$

$\therefore$ gradient of normal $= \frac{-1}{3}$, and point $(-1,-2)$

$\therefore$ Equation of normal: $y + 2 = \frac{-1}{3}(x+1)$

$x + 3y + 7 = 0$

(12) (a) (iii) (1 mark)

Sample answer

$0+3y+7=0$

$\therefore \left(0, -\tfrac{7}{3}\right)$

(b) (ii) (2 marks)

Sample answer

Let $E(x,y)$

$DB=BE$

$\frac{3+x}{2}=12$ and $\frac{3+y}{2}=10$

$\therefore E(21, 17)$

(b) (iv) (2 marks)

Sample answer

$DE=\sqrt{(21-3)^2+(17-3)^2}$

$=\sqrt{520}$ units

Area $=\frac{1}{2}\times\sqrt{130}\times\sqrt{520}$

$=130$ units^2

(d) (2 marks)

Sample answer

$$\sum_{n=1}^{4}\frac{n^2}{3n-1}=\frac{(1)^2}{3(1)-1}+\frac{2^2}{3(2)-1}+\frac{(3)^2}{3(3)-1}+\frac{(4)^2}{3(4)-1}$$

$$=3.8795=\frac{1707}{440}$$

Question 13 (15 marks)

(a) (i) (2 marks)

Sample answer

Each side of the xylophone has n wooden bars and there is the extra one in the middle. Treat this as two series:

$$S=\frac{n}{2}[20+(n-1)b]+\frac{n+1}{2}[20+nb]$$

$$=10n+\frac{bn^2}{2}-\frac{nb}{2}+10n+10+\frac{bn^2}{2}+\frac{nb}{2}$$

(b) (i) (1 mark)

Sample answer

$AC=\sqrt{(11-4)^2+(2-11)^2}$

$=\sqrt{130}$ units

(b) (iii) (1 mark)

Sample answer

Both pairs of adjacent sides of $ADCE$ are equal

(c) (3 marks)

Sample answer

Area of circular wafer $=\pi\times5^2=78.5398\ldots\text{cm}^2$

Area of chocolate segment $=\frac{1}{2}\times5^2\times\left(\frac{\pi}{4}-\sin\frac{\pi}{4}\right)$

$=0.9786\ldots\text{cm}^2$

$\therefore$ Area uncoated $=78.5398-0.9786=77.5611$

$\approx 78\ \text{cm}^2$

(13) (a)(ii) (2 marks)

Sample answer

$360=\frac{3}{2}n^2+20n+10$

$3n^2+40n-700=0 \quad \therefore \quad (3n+70)(n-10)=0$

$n=10,\ n>0$

Number of wooden bars $=2(10)+1=21$

(b)(ii) (2 marks)

Sample answer

$310\ 000=80\ 000e^{3k}$

$\frac{31}{8}=e^{3k} \quad \ln\left(\frac{31}{8}\right)=3k$

$\frac{1}{3}\ln\left(\frac{31}{8}\right)=k \qquad \therefore k\cong0.451515\ldots \qquad k\cong0.4515$ (4 sig. fig)

(b) (iii) (2 marks)

Sample answer

$t=3+7=10$ days

$P=80\ 000e^{0.4515\times10}$

$P=7\ 310\ 205$ termites

So Julian's house will not be safe from the termites.

(c) (i) (1 mark)

Sample answer

$(x-2)^2=12y+6 \quad \therefore (x-2)^2=12\left(y+\frac{1}{2}\right)$ So Vertex $=\left(2,-\frac{1}{2}\right)$

(c) (ii) (1 mark)

Sample answer

$4a=12$

$a=3$

So Focus $=\left(2,\ \frac{5}{2}\right)$

(b)(i) (1 mark)

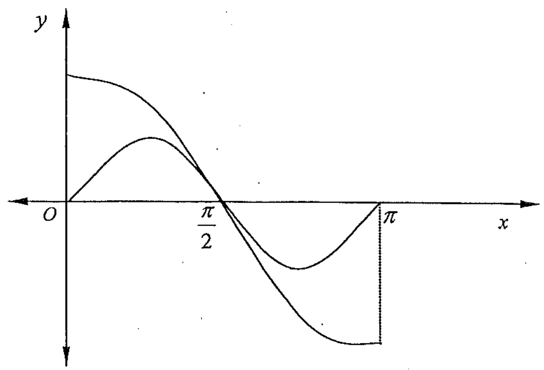
Sample answer

$P=Ae^{kt}$

$\frac{dP}{dt}=kAe^{kt}=kP$

(d) (i) (2 marks)

Sample answer



(13) (d) (ii) (2 marks)

Sample answer

$$2\int_0^{\frac{\pi}{2}} 3\cos x - \sin 2x \; dx = 2\left[3\sin x + \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$$

$$2\left[\left(3\sin\frac{\pi}{2} + \frac{1}{2}\cos 2\left(\frac{\pi}{2}\right)\right) - \left(3\sin 0 + \frac{1}{2}\cos 0\right)\right] = 4 \text{ units squared}$$

Question 14 (15 marks)
(a)(i) (2 marks)

Sample answer

$f(x) = x^4 - x^3 - 2x^2 + 3x$
$f'(x) = 4x^3 - 3x^2 - 4x + 3$
$= x^2(4x-3) - (4x-3)$
$= (x^2-1)(4x-3)$
$= (x-1)(x+1)(4x-3)$

(a)(ii) (2 marks)

Sample answer
Stationary when
$f'(x) = 0$
$(x-1)(x+1)(4x-3) = 0$

at $x = 1, y = 1$
at $x = -1, y = -3$
at $x = 0.75, y = 1.02$

so the stationary points are $(1,1), (-1,-3)$ and $(0.75, 1.02)$.

(a)(iii) (2 marks)

Sample answer
$f''(x) = 12x^2 - 6x - 4$

x	-1	1	0.75
$f''(x)$	14	2	-1.75
concavity	up	up	down

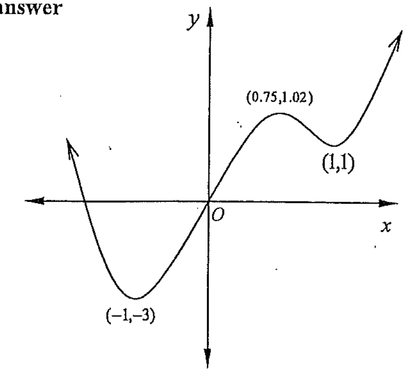
So (1, 1) is a MINIMUM turning point

(-1, -3) is a MINIMUM turning point and

$(0.75, 1.02)$ is a MAXIMUM turning point.

(14) (a)(iv) (2 marks)

Sample answer



(b) (2 marks)

Sample answer

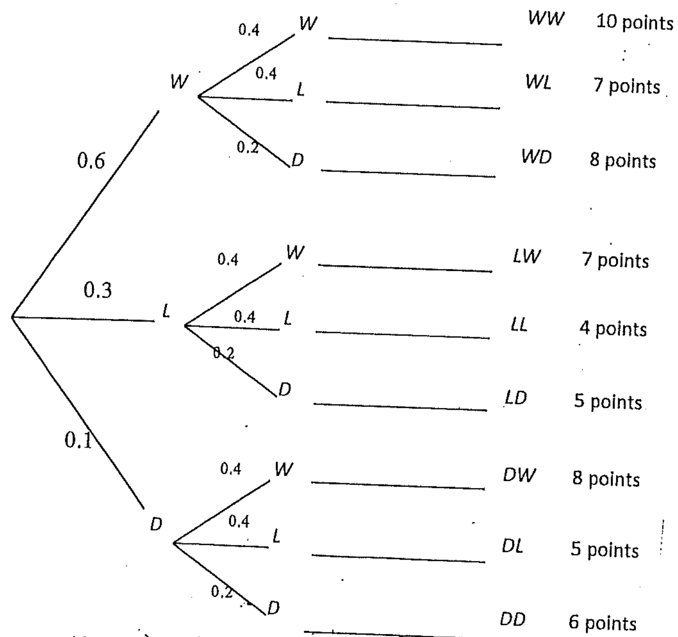
$$\int_3^{11} f(x)\,dx \cong \frac{2}{2}[0 + 2\ln 3 + 2\ln 5 + 2\ln 7 + \ln 9]$$
$\cong 11.50514...$
$\cong 11.51$ (2 d.p.)

(c) (i) (1 mark)

Sample answer
P(draw) = 0.1

(c) (ii) (2 marks)

Sample answer



$P(\text{Xavier scores 10 points}) = P(\text{win/win})$
$= 0.6 \times 0.4$
$= 0.24$

(14) (c) (iii) (2 marks)

Sample answer

$P(\text{Xavier scores 5 points or less}) = P(\text{lose/lose}) + P(\text{lose/draw}) + P(\text{draw/lose})$

$= 0.3 \times 0.4 + 0.3 \times 0.2 + 0.1 \times 0.4$

$= 0.22$

Question 15 (15 marks)

(a)(i) (1mark)

Sample answer:

$4x^2 + 9y^2 = 36$

Let $y = 0$ $\quad 4x^2 = 36 \quad x = 3 \quad \therefore A(3,0)$

(a)(ii) (3marks)

Sample answer:

$V = \pi \int_a^b y^2\,dx \quad V = \pi \int_0^3 4 - \frac{4}{9}x^2\,dx$

$= \pi \left[4x - \frac{4}{27}x^3 \right]_0^3 = \pi[(12-4)-(0)] = 8\pi$ units^3

(b)(i) (1mark)

Sample answer:

$v = 3e^t - 12e^{-2t}$

When $t = 0 \quad \therefore v = 3e^0 - 12e^0 = 3 - 12 = -9$ m/sec.

(b)(ii) (2marks)

Sample answer:

Let $v = 0 \quad 0 = 3e^t - 12e^{-2t} \quad 3e^t = \frac{12}{e^{2t}}$

$\therefore e^{3t} = 4 \quad \therefore 3t = \ln 4 \quad \therefore t = \frac{\ln 4}{3} = 0.462 \quad$ $\therefore$ Yes, the particle is at rest after $\frac{\ln 4}{3}$ seconds.

(b)(iii) (2marks)

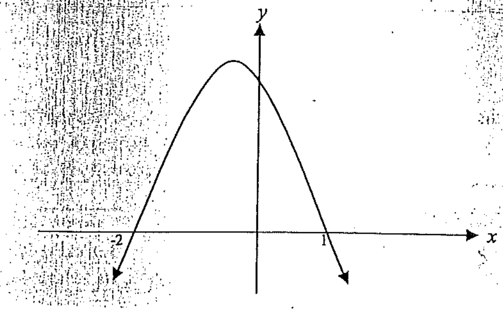
Sample answer:

$\frac{dx}{dt} = 3e^t - 12e^{-2t}$

$x = \int_0^4 3e^t - 12e^{-2t}\,dt = \left[3e^t + 6e^{-2t}\right]_0^4$

(15) (c)(i) (2marks)

Sample answer:



(c)(ii) (2marks)

Sample answer:

$A = \int_{-2}^{1} f'(x)\,dx = [f(x)]_{-2}^{1} = f(1) - f(-2) = 3 - (-8) = 11\ u^2$

(d) (2 marks)

Sample answer:

$LHS = \log_a\left(\frac{a^2}{75}\right)$

$= \log_a a^2 - \log_a 75$

$= 2\log_a a - \log_a(5^2 \times 3)$

$= 2\log_a a - \log_a(5^2 \times 3)$

$= 2 - 2\log_a 5 - \log_a 3$

$= 2 - 2y - x$

Question 16 (15 marks)

(a)(i) (2marks)

Sample answer:

$r = \frac{6}{12} = 0.5\%$ and n years $= 12n$ months

$A_1 = 250\left(1 + \frac{0.5}{100}\right) = 250(1.005)$

$A_2 = 250(1.005)^2 \quad , \ldots , \quad A_{12n} = 250(1.005)^{12n}$

$\therefore A = 250\left(1.005 + 1.005^2 + \ldots + 1.005^{12n}\right)$

(16) (a)(ii) (2marks)

Sample answer:

When $n = 7$ years $= 7 \times 12 = 84$ months

$$\therefore A = 250 \times \frac{1.005(1.005^{84} - 1)}{1.005 - 1} = \$26\ 148.57$$

(a)(iii) (2marks)

Sample answer:

$26\ 148.57 = P(1.005)^{84}$

$\therefore P = \$17\ 198.83$

(b) (2marks)

Sample answer:

$AB = \sqrt{20^2 + 10^2} = \sqrt{500} = 10\sqrt{5}$

(c)(i) (3marks)

Sample answer:

The perpendicular distance from $R\ (a, \frac{7}{\sqrt{a}})$ to the line $3x + 4y + 7 = 0$ is :

$$d = \frac{\left|3a + \frac{28}{\sqrt{a}} + 7\right|}{\sqrt{3^2 + 4^2}} = \frac{1}{5}\left(3a + \frac{28}{\sqrt{a}} + 7\right)$$

The distance $ST = \sqrt{(-5-3)^2 + (2+4)^2} = \sqrt{64+36} = 10$

$\therefore$ the area of the triangle RST is $A = \frac{1}{2} \times 10 \times \frac{1}{5}\left(3a + \frac{28}{\sqrt{a}} + 7\right)$

$$\therefore A = \left(3a + \frac{28}{\sqrt{a}} + 7\right)$$

(c)(ii) (3marks)

Sample answer:

$$A = \left(3a + \frac{28}{\sqrt{a}} + 7\right) = 3a + 28a^{-\frac{1}{2}} + 7$$

$$\therefore \frac{dA}{da} = 3 - 14a^{-\frac{3}{2}}$$

Let $\frac{dA}{da} = 0 \quad \therefore 0 = 3 - 14a^{-\frac{3}{2}} \quad \therefore \frac{14}{a^{\frac{3}{2}}} = 3$

$\therefore a^{\frac{3}{2}} = \frac{14}{3} \quad \therefore a = \left[\frac{14}{3}\right]^{\frac{2}{3}} = 2.79257$

$\therefore \frac{d^2A}{da^2} = 21a^{-\frac{5}{2}} > 0$ when $a = 2.79257$

$\therefore$ the area is minimum when $a = 2.8$ correct to 1 *d.p.*

(c)(iii) (1mark)

Sample answer:

$A = \left(3a + \frac{28}{\sqrt{a}} + 7\right)$ when $a = 2.8$

$A = 8.4 + \frac{28}{\sqrt{2.8}} + 7$

$\therefore A = 32.1$ units squared