Exercises – Simultaneous Equations in 3 variables

Solve the following systems of equations using row reduction of the augmented matrix to echelon form:-

(1)
$$x + y + z = 2$$
$$3x - y - z = 10$$
$$x - 2y - 4z = 9$$

(2)
$$x - 5y - z = 13$$
$$2x + y + z = 0$$
$$-3x + y + 2z = 11$$

(3)
$$4x + y - 8z = 0$$
$$2x + 3y + z = 5$$
$$-x - y - 2z = 1$$

(4)
$$2x + y - z = 4$$
$$x - 4y + z = 11$$
$$5x + y - 2z = 13$$

(5)
$$2x + y + z = -2$$
$$-x + y - 2z = 4$$
$$5x + 2y - 3z = -12$$

(6)
$$x + 2y - z = 7$$
$$2x + 3y - 4z = 9$$
$$x - y - 3z = -6$$

(7)
$$x + y + z = 2$$
$$2x - 3y + 5z = 1$$
$$-2x + 5y - 7z = -3$$

(8)
$$x - y - z = 4$$
$$2x + y - 4z = 3$$
$$-x + 2y + 3z = -7$$

(9)
$$x - 4y - 5z = -6$$
$$2x - y - z = 1$$
$$x + 3y + 4z = 10$$

(10)
$$x + 2y + 4z = 10$$
$$-x + y + 5z = 5$$
$$-2x - y + z = -5$$

(11)
$$x + y + z = 6$$
$$2x + 3y + z = 13$$
$$x + 2y - z = 5$$

(12)
$$a - b + c = 10$$
$$4a + 2b - 3c = 8$$
$$3a - 5b + 2c = 34$$

$$(13) \quad \begin{aligned} x + y - z &= 4 \\ 2x - y + 3z &= 5 \\ x + 4y - 2z &= 19 \end{aligned}$$

$$(14) \quad \begin{aligned} a + 2b - c &= -5 \\ 2a - 3b + 4c &= 28 \\ 4a + 5b - 3c &= -10 \end{aligned}$$

Answers:

(1) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$
(2) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix}$
(3) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix}$
(4) No unique solution
Consistent equations
(solution is a line in 3-D)

(5) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$
(6) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -0.5 \\ 4 \\ 0.5 \end{pmatrix}$
(7) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix}$
(8) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.5 \\ -2 \\ -0.5 \end{pmatrix}$

(9) No unique solution.
Inconsistent equations

(10) No unique solution.
Consistent equations
(solution is a line in 3-D)

(11) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$
(12) $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$
(13) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix}$
(14) $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$

1. CONSIDER THE SIMULTANEOUS EQUATIONS.

$2x - y + 3z = 5$ --- (1), $x + y - z = 4$ --- (2), $x + 4y - 2z = 19$ ---(3).

(**i**) Form a new equation from:

  (**a**) Equations (1) and (2) by eliminating z; call this equation (A).

  (**b**) Equations (2) and (3) by eliminating z; call this equation (B).

(**ii**) Solve equations (A), (B) simultaneously to find x, y.

(**iii**) Substitute these values of x, y in equation (1) to find z.

(**iv**) Check your solutions of the three given simultaneous equations.

2. SOLVE THE FOLLOWING EQUATIONS AND VERIFY YOUR RESULTS BY DIRECT SUBSTITUTION in the original equations.

(**a**) $x + y + z = 6$ (**b**) $a - b + c = 10$ (**c**) $6x + 4y - 2z = 0$

  $2x + 3y + z = 13$ $4a + 2b - 3c = 8$ $3x - 2y + 4z = 3$

  $x + 2y - z = 5$ $3a - 5b + 2c = 34$ $5x - 2y + 6z = 3$

(**d**) $2a + b - c = 9$ (**e**) $3p - q - 2r = -15$ (**f**) $x - 2y \quad\quad = 3$

  $5a \quad\quad + 2c = -3$ $2p - 3q - 5r = -15$ $4y - 3z = 4$

  $7a - 2b \quad = 1$ $5p + 2q + 3r = -16$ $x \quad\quad + 3z = 2$

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1. (**i**) (**a**) $5x+2y=17$ (**b**) $x-2y=-11$ (**ii**) $x=1, y=6$ (**iii**) $z=3$

2. (**a**) $x=1, y=3, z=2$ (**b**) $a=5, b=-3, c=2$ (**c**) $x=1, y=-2, z=-1$ (**d**) $a=1, b=3, c=-4$
  (**e**) $p=-4, q=-1, r=2$ (**f**) $x=4, y=\frac{1}{2}, z=-2/3$