

# TUTORIAL PROBLEMS (2) – WEEK 1

For the exercises below, do only every second question (the even numbered questions). The answers to all these questions are at the end of this handout – so if you are having difficulty getting correct answers for a particular exercise, go on and do all the questions in that exercise.

## 1. Basic Algebra

### Algebraic Factors

1. Write down the expansion of:

- |                         |                         |                   |
|-------------------------|-------------------------|-------------------|
| (a) $(x+5)(x+1)$        | (b) $(x-2)(x-3)$        | (c) $(a-3)(a+4)$  |
| (d) $(p-7)(p-3)$        | (e) $(2x+3)(x+5)$       | (f) $(3x-4)(x-2)$ |
| (g) $(3y-2)(4y+3)$      | (h) $(x-14)(6x-1)$      | (i) $(3x+4)^2$    |
| (j) $(4p-5)^2$          | (k) $(3x^2-5x+2)(2x-4)$ | (l) $(2a+b)^3$    |
| (m) $(x^2+5)(x^2-2x-3)$ | (n) $(x-2)(x+2)(x+2)$   | (o) $(5m-2n)^3$   |

2. Factorise

- |                  |                        |                     |
|------------------|------------------------|---------------------|
| (a) $bx+by$      | (b) $3n+3m$            | (c) $6y+12$         |
| (d) $18a-6b$     | (e) $y^2-4y$           | (f) $a^2-ax$        |
| (g) $12xy+16yz$  | (h) $17a^2x^5+2a^4x^3$ | (i) $bx+by-bz$      |
| (j) $3x^3-x^2+x$ | (k) $am^2+2am+3m$      | (l) $3xy-3yz+21y$   |
| (m) $a(x+y)-az$  | (n) $10a^2+a(c+b)$     | (o) $a(x+y)+b(x+y)$ |

3. Factorise

- |                         |                         |                       |
|-------------------------|-------------------------|-----------------------|
| (a) $xy+xz+yw+zw$       | (b) $5a+5b+xa+xb$       | (c) $3a+12c+ab+4bc$   |
| (d) $2xy-2xz+7y-7z$     | (e) $a^2+ab+ac+bc$      | (f) $5a+ab+5b+b^2$    |
| (g) $3ax-bx-3ay+by$     | (h) $15ac-10ad+3bc-2bd$ | (i) $ab^2+ac-b^2d-cd$ |
| (j) $a(x-y)+b(y-x)$     | (k) $2a-2b+ax-xb$       | (l) $x^4+x^3+2x+2$    |
| (m) $x^2(x+2y)-y(x+2y)$ | (n) $xz+z^2-x-z$        | (o) $2x^3-x^2-12x+6$  |

4. Factorise

- |                       |                    |                      |
|-----------------------|--------------------|----------------------|
| (a) $x^2-y^2$         | (b) $a^2-36$       | (c) $x^2-48$         |
| (d) $25-n^2$          | (e) $x^2-1$        | (f) $9-4a^2$         |
| (g) $x^2-16y^2$       | (h) $49a^2-121b^2$ | (i) $25a^2b^2-4c^2$  |
| (j) $a^2x^2-16b^2y^2$ | (k) $1-(a-b)^2$    | (l) $49x^2-64y^2z^2$ |
| (m) $100-x^4$         | (n) $a^4-y^4$      | (o) $(x+y)^2-x^2$    |

5. Factorise

- |                  |                   |                  |
|------------------|-------------------|------------------|
| (a) $x^2+3x+2$   | (b) $x^2+7x+12$   | (c) $x^2+13x+40$ |
| (d) $x^2-7x+12$  | (e) $x^2-13x+36$  | (f) $x^2-18x+56$ |
| (g) $x^2-2x-3$   | (h) $x^2-10x-11$  | (i) $x^2+15x+36$ |
| (j) $x^2-10x+25$ | (k) $x^2+6x+9$    | (l) $x^2+2x+1$   |
| (m) $x^2+6x+5$   | (n) $x^2-22x+120$ | (o) $x^2-x-42$   |

6. Factorise

(a)  $3x^2 + 7x + 4$   
 (d)  $4 - 3x - x^2$   
 (g)  $18x^2 - 9x - 2$

(b)  $4x^2 - 11x + 6$   
 (e)  $12x^2 - 7xy + y^2$   
 (h)  $3x^2 - 13x + 14$

(c)  $9x^2 + 24x + 16$   
 (f)  $2x^2 - 9x - 18$   
 (i)  $2 + x - 10x^2$

7. Factorise

(a)  $x^3 - y^3$   
 (d)  $b^3 - 1$   
 (g)  $4a^3 + 32$   
 (j)  $x^6 - y^6$   
 (m)  $5y^3 - 5000$

(b)  $a^3 + 27$   
 (e)  $x^3 + 125$   
 (h)  $5m^3 - 5$   
 (k)  $8x^3 + 27$   
 (n)  $a^3 + b^3 + a + b$

(c)  $x^3 - 64$   
 (f)  $a^3 - 1000$   
 (i)  $64 + y^3$   
 (l)  $x^7y^4 - x^4y^7$   
 (o)  $(x+5)^3 + (x-2)^3$

**Algebraic Fractions**

1. Simplify the following fractions.

(a) $\frac{2ab}{3ac}$	(b) $\frac{5a^2}{15ab}$	(c) $\frac{2x+4}{3x+6}$	(d) $\frac{a^2 - a}{a^2 + a}$
(e) $\frac{x^3 + 8}{x^2 - 2x + 4}$	(f) $\frac{x^2 + 2x + 1}{x^2 + 3x + 2}$	(g) $\frac{3x^2y - 6xy}{2x^2y - 4xy^2}$	(h) $\frac{a^2 - 5a}{a^2 - 4a - 5}$
(i) $\frac{3x^2 - 12}{10 - 5x}$	(j) $\frac{6x^2 - 150}{3x + 15}$	(k) $\frac{8a^2 - 8}{4a^2 + 8a + 4}$	(l) $\frac{a^3 - 27}{a^2 - 9}$
(m) $\frac{(a+b)^2 - c^2}{3a + 3b - 3c}$	(n) $\frac{1 - \frac{y^2}{x^2}}{1 + \frac{y^2}{x^2}}$	(o) $\frac{\frac{1}{x} - \frac{2}{x^2} - \frac{3}{x^3}}{\frac{9}{x} - x}$	(p) $\frac{2x^2 - x - 6}{\frac{4}{x^2} - 1}$

2. Simplify the following fractions.

(a) $\frac{5}{x} \times \frac{15}{y}$	(b) $\frac{3x}{5} \times \frac{1}{6x}$	(c) $\frac{2m}{3n} \times \frac{6}{m^2}$
(d) $\frac{18}{7x} \div \frac{9}{28x}$	(e) $\frac{3ab}{4a} \times \frac{8a^2}{12b}$	(f) $\frac{10x-15}{6} \times \frac{1}{8x-12}$
(g) $\frac{3m-6}{4} \times \frac{8m}{m^2-2m}$	(h) $\frac{4a-6}{5} \div \frac{6a-9}{15}$	(i) $\frac{x-y}{4} \times \frac{8}{x^2-y^2}$
(j) $\frac{m^2-mn}{n^2-n} \times \frac{n-1}{m-n}$	(k) $\frac{1-a}{b+b^2} \div \frac{1-a^2}{1-b^2}$	(l) $\frac{x^2-4}{2x-4} \times \frac{2}{x+2}$
(m) $\frac{a^2+5a+6}{a^2-25} \div \frac{a+3}{a-5}$	(n) $\frac{x-3y}{x^3y} \div \frac{3y-x}{xy^3}$	(o) $\frac{x^2-y^2}{x^2-2xy+y^2} \times \frac{xy-y^2}{xy+y^2}$

3. Express each one of the following as single fraction.

(a) $\frac{x}{3} + \frac{x}{5}$	(b) $\frac{m}{2} + \frac{2m}{3}$
(c) $\frac{x+7}{3} + \frac{2x-1}{2}$	(d) $\frac{3x-1}{3} - \frac{x+2}{6}$
(e) $\frac{1}{x} + \frac{2}{x+y}$	(f) $\frac{2}{a+1} + \frac{1}{a-3}$
(g) $\frac{a}{b-c} - \frac{3a}{5b-5c}$	(h) $\frac{5c}{a^2+ab} - \frac{c}{a+b}$

$$(i) \frac{x^2}{x^2+3x+2} - \frac{2x}{x+2}$$

$$(k) \frac{x-1}{2} + \frac{x+3}{5} + \frac{x+7}{10}$$

$$(m) \frac{1}{x+2} + \frac{1}{x+3}$$

$$(o) \frac{1}{2x^2-x-1} - \frac{3}{6x^2-x-2}$$

$$(j) \frac{m}{m^2+mn} + \frac{n}{n^2+mn}$$

$$(l) \frac{2a^2-b^2}{a^2} - \frac{b^2-c^2}{b^2} - \frac{c^2-a^2}{c^2}$$

$$(n) \frac{1}{x^2-9x+20} + \frac{1}{x^2-11x+30}$$

$$(p) \frac{x-3}{x+2} - \frac{x-2}{x+3} + \frac{1}{x-1}$$

## Surds

1. Express each of the surds in the simplest form.

$$(a) \sqrt{8}$$

$$(b) \sqrt{12}$$

$$(c) \sqrt{32}$$

$$(d) 3\sqrt{63}$$

$$(e) \sqrt{72}$$

$$(f) \sqrt{96}$$

$$(g) \sqrt{288}$$

$$(h) \frac{1}{6}\sqrt{18}$$

$$(i) \frac{3}{2}\sqrt{192}$$

$$(j) \sqrt{320}$$

$$(k) \sqrt{\frac{10}{25}}$$

$$(l) \sqrt{\frac{72}{25}}$$

2. Simplify each of the following surds.

$$(a) 4\sqrt{3} + 3\sqrt{3} - 2\sqrt{3}$$

$$(b) 3\sqrt{5} + 5\sqrt{5} - \sqrt{5}$$

$$(c) 3\sqrt{45} - \sqrt{20} + 7\sqrt{5}$$

$$(d) \sqrt{12} + \sqrt{27}$$

$$(e) \sqrt{18} + \sqrt{50}$$

$$(f) 4\sqrt{48} - 5\sqrt{27}$$

$$(g) \sqrt{128} + 3\sqrt{18} - \sqrt{162}$$

$$(h) 3\sqrt{32} + 2\sqrt{75} - 5\sqrt{162}$$

$$(i) 2\sqrt{363} - 5\sqrt{320} - \sqrt{192}$$

$$(j) -2\sqrt{a} - 4\sqrt{a} + 3\sqrt{a}$$

$$(k) 5\sqrt{y} - \sqrt{x} - 2\sqrt{y}$$

$$(l) 2\sqrt{x} + 2\sqrt{y} - 3\sqrt{x}$$

3. Express each of the surds in the simplest form.

$$(a) \sqrt{3} \times \sqrt{3}$$

$$(b) 4\sqrt{5} \times 2\sqrt{2}$$

$$(c) (\sqrt{5})^2$$

$$(d) (2\sqrt{7})^2$$

$$(e) \sqrt{8} \times \sqrt{5} \times \sqrt{125}$$

$$(f) \sqrt{2}(\sqrt{3} + 2\sqrt{2})$$

$$(g) (2\sqrt{7} - 1)^2$$

$$(h) 2\sqrt{5}(3\sqrt{3} - 4\sqrt{2})$$

$$(i) \sqrt{2}(\sqrt{18} + \sqrt{8} - 4\sqrt{2})$$

$$(j) (\sqrt{3} + 4)(\sqrt{3} - 2)$$

$$(k) (2\sqrt{5} - 3)(2\sqrt{5} + 3)$$

$$(l) (3\sqrt{5} - 2\sqrt{2})(2\sqrt{5} + 3\sqrt{2})$$

$$(m) (2\sqrt{x} + \sqrt{y})(\sqrt{x} - 3\sqrt{y})$$

$$(n) (3\sqrt{a} - \sqrt{b})^2$$

$$(o) \sqrt{x}(6\sqrt{y} - 3\sqrt{x})$$

4. Multiply each surd by its conjugate and write down the answer.

$$(a) \sqrt{2} - 1$$

$$(b) \sqrt{11} - \sqrt{5}$$

$$(c) 2\sqrt{5} + \sqrt{3}$$

$$(d) 2\sqrt{11} - 3\sqrt{5}$$

5. Express each of the surds in the simplest form with a rational denominator.

$$(a) \frac{1}{\sqrt{7}}$$

$$(b) \frac{3}{\sqrt{5}}$$

$$(c) \frac{4}{3\sqrt{7}}$$

$$(d) \frac{3\sqrt{2}}{\sqrt{8}}$$

$$(e) \frac{\sqrt{12}}{5\sqrt{3}}$$

$$(f) \frac{4\sqrt{2}}{3\sqrt{8}}$$

$$(g) \frac{\sqrt{2}+3}{\sqrt{5}}$$

$$(h) \frac{1-\sqrt{3}}{\sqrt{3}}$$

$$(i) \frac{\sqrt{5}-\sqrt{2}}{\sqrt{3}}$$

$$(j) \frac{4\sqrt{3}+2\sqrt{2}}{3\sqrt{2}}$$

$$(k) \frac{\sqrt{x}}{\sqrt{y}}$$

$$(l) \frac{2\sqrt{a}}{\sqrt{ab}}$$

6. Simplify the following surds by rationalizing the denominator.

$$(a) \frac{1}{\sqrt{3}+1}$$

$$(b) \frac{1}{\sqrt{3}+\sqrt{7}}$$

$$(c) \frac{1}{5+3\sqrt{2}}$$

$$(d) \frac{2}{2\sqrt{2}-1}$$

$$(e) \frac{3}{\sqrt{5}-2}$$

$$(f) \frac{\sqrt{3}}{4\sqrt{3}+5}$$

$$(g) \frac{6}{7+2\sqrt{5}}$$

$$(h) \frac{\sqrt{5}}{\sqrt{5}-1}$$

$$(i) \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$$

$$(j) \frac{2\sqrt{3}-1}{2\sqrt{3}+1}$$

$$(k) \frac{\sqrt{5}+\sqrt{3}}{2\sqrt{10}-\sqrt{6}}$$

$$(l) \frac{5\sqrt{2}-1}{5\sqrt{2}+1}$$

## Answers to Exercises

### 1. Basic Algebra

#### Algebraic Factors

- |                                    |                              |  |
|------------------------------------|------------------------------|--|
| (a) $x^2 + 6x + 5$                 | (b) $x^2 - 5x + 6$           | (c) $a^2 + a - 12$                     |
| (d) $p^2 - 10p + 21$               | (e) $2x^2 + 13x + 15$        | (f) $3x^2 - 10x + 8$                   |
| (g) $12y^2 + y - 6$                | (h) $6x^2 - 85x + 14$        | (i) $9x^2 + 24x + 16$                  |
| (j) $16p^2 - 40p + 25$             | (k) $6x^3 - 22x^2 + 24x - 8$ | (l) $8a^3 + 12a^2b + 6ab^2 + b^3$      |
| (m) $x^4 - 2x^3 + 2x^2 - 10x - 15$ | (n) $x^3 + 2x^2 - 4x - 8$    | (o) $125m^3 - 150m^2n + 60mn^2 - 8n^3$ |
- |                       |                            |                      |
|-----------------------|----------------------------|----------------------|
| (a) $b(x + y)$        | (b) $3(n + m)$             | (c) $6(y + 2)$       |
| (d) $6(3a - b)$       | (e) $y(y - 4)$             | (f) $a(a - x)$       |
| (g) $4y(3x + 4z)$     | (h) $a^2x^3(17x^2 + 2a^2)$ | (i) $b(x + y - z)$   |
| (j) $x(3x^2 - x + 1)$ | (k) $m(am + 2a + 3)$       | (l) $3y(x - z + 7)$  |
| (m) $a(x + y - z)$    | (n) $a(10a + c + b)$       | (o) $(x + y)(a + b)$ |
- |                         |                         |                         |
|-------------------------|-------------------------|-------------------------|
| (a) $(y + z)(x + w)$    | (b) $(a + b)(5 + x)$    | (c) $(a + 4c)(3 + b)$   |
| (d) $(y - z)(2x + 7)$   | (e) $(a + b)(a + c)$    | (f) $(5 + b)(a + b)$    |
| (g) $(3a - b)(x - y)$   | (h) $(3c - 2d)(5a + b)$ | (i) $(b^2 + c)(a - d)$  |
| (j) $(x - y)(a - b)$    | (k) $(a - b)(2 + x)$    | (l) $(x + 1)(x^3 + 2)$  |
| (m) $(x + 2y)(x^2 - y)$ | (n) $(x + z)(z - 1)$    | (o) $(2x - 1)(x^2 - 6)$ |
- |                            |                                 |                                      |
|----------------------------|---------------------------------|--------------------------------------|
| (a) $(x + y)(x - y)$       | (b) $(a + 6)(a - 6)$            | (c) $(x + \sqrt{48})(x - \sqrt{48})$ |
| (d) $(5 + n)(5 - n)$       | (e) $(x + 1)(x - 1)$            | (f) $(3 + 2a)(3 - 2a)$               |
| (g) $(x + 4y)(x - 4y)$     | (h) $(7a + 11b)(7a - 11b)$      | (i) $(5ab + 2c)(5ab - 2c)$           |
| (j) $(ax + 4by)(ax - 4by)$ | (k) $(1 + a - b)(1 - a + b)$    | (l) $(7x + 8yz)(7x - 8yz)$           |
| (m) $(10 + x^2)(10 - x^2)$ | (n) $(a^2 + y^2)(a + y)(a - y)$ | (o) $y(2x + y)$                      |
- |                      |                        |                       |
|----------------------|------------------------|-----------------------|
| (a) $(x + 1)(x + 2)$ | (b) $(x + 4)(x + 3)$   | (c) $(x + 8)(x + 5)$  |
| (d) $(x - 3)(x - 4)$ | (e) $(x - 4)(x - 9)$   | (f) $(x - 4)(x - 14)$ |
| (g) $(x + 1)(x - 3)$ | (h) $(x - 11)(x + 1)$  | (i) $(x + 12)(x + 3)$ |
| (j) $(x - 5)^2$      | (k) $(x + 3)^2$        | (l) $(x + 1)^2$       |
| (m) $(x + 5)(x + 1)$ | (n) $(x - 10)(x - 12)$ | (o) $(x + 6)(x - 7)$  |
- |                        |                        |                        |
|------------------------|------------------------|------------------------|
| (a) $(3x + 4)(x + 1)$  | (b) $(4x - 3)(x - 2)$  | (c) $(3x + 4)^2$       |
| (d) $(4 + x)(1 - x)$   | (e) $(3x - y)(4x - y)$ | (f) $(2x + 3)(x - 6)$  |
| (g) $(6x + 1)(3x - 2)$ | (h) $(3x - 7)(x - 2)$  | (i) $(2 + 5x)(1 - 2x)$ |
- |                               |                              |                                 |
|-------------------------------|------------------------------|---------------------------------|
| (a) $(x - y)(x^2 + xy + y^2)$ | (b) $(a + 3)(a^2 - 3a + 9)$  | (c) $(x - 4)(x^2 + 4x + 16)$    |
| (d) $(b - 1)(b^2 + b + 1)$    | (e) $(x + 5)(x^2 - 5x + 25)$ | (f) $(a - 10)(a^2 + 10a + 100)$ |
| (g) $4(a + 2)(a^2 - 2a + 4)$  | (h) $5(m - 1)(m^2 + m + 1)$  | (i) $(4 + y)(16 - 4y + y^2)$    |

(j)  $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$

(l)  $x^4y^4(x-y)(x^2+xy+y^2)$

(n)  $(a+b)(a^2-ab+b^2+1)$

(k)  $(2x+3)(4x^2-6x+9)$

(m)  $5(y-10)(y^2+10y+100)$

(o)  $(2x-3)(x^2+3x+39)$

### Algebraic Fractions

1. (a)  $\frac{2b}{3c}$

(b)  $\frac{a}{3b}$

(c)  $\frac{2}{3}$

(d)  $\frac{a-1}{a+1}$

(e)  $x+2$

(f)  $\frac{x+1}{x+2}$

(g)  $\frac{3(x-2)}{2(x-2y)}$

(h)  $\frac{a}{a+1}$

(i)  $\frac{-3(x+2)}{5}$

(j)  $2(x-5)$

(k)  $\frac{2(a-1)}{a+1}$

(l)  $\frac{a^2+3a+9}{a+3}$

(m)  $\frac{a+b+c}{3}$

(n)  $\frac{x^2-y^2}{x^2+y^2}$

(o)  $-\frac{(x+1)}{x^2(3+x)}$

(p)  $-\frac{x^2(2x+3)}{2+x}$

2. (a)  $\frac{75}{xy}$

(b)  $\frac{1}{10}$

(c)  $\frac{4}{mn}$

(d) 8

(e)  $\frac{a^2}{2}$

(f)  $\frac{5}{24}$

(g) 6

(h) 2

(i)  $\frac{2}{x+y}$

(j)  $\frac{m}{n}$

(k)  $\frac{1-b}{b(1+a)}$

(l) 1

(m)  $\frac{a+2}{a+5}$

(n)  $-\frac{y^2}{x^2}$

(o) 1

3 (a)  $\frac{8x}{15}$

(b)  $\frac{7m}{6}$

(c)  $\frac{8x+11}{6}$

(d)  $\frac{5x-4}{6}$

(e)  $\frac{3x+y}{x(x+y)}$

(f)  $\frac{3a-5}{(a+1)(a-3)}$

(g)  $\frac{2a}{5(b-c)}$

(h)  $\frac{c(5-a)}{a(a+b)}$

(i)  $\frac{-x}{x+1}$

(j)  $\frac{2}{m+n}$

(k)  $\frac{4(x+1)}{5}$

(l)  $\frac{a^4b^2-b^4c^2+a^2c^4}{a^2b^2c^2}$

(m)  $\frac{2x+5}{(x+2)(x+3)}$

(n)  $\frac{2}{(x-4)(x-6)}$

(o)  $\frac{1}{(x-1)(2x+1)(3x-2)}$

(p)  $\frac{x^2+11}{(x-1)(x+2)(x+3)}$

## Surds - ANSWERS

1. (a)  $2\sqrt{2}$  (b)  $2\sqrt{3}$  (c)  $4\sqrt{2}$  (d)  $9\sqrt{7}$   
 (e)  $6\sqrt{2}$  (f)  $4\sqrt{6}$  (g)  $12\sqrt{2}$  (h)  $\frac{1}{2}\sqrt{2}$   
 (i)  $12\sqrt{3}$  (j)  $8\sqrt{5}$  (k)  $\frac{\sqrt{10}}{5}$  (l)  $\frac{6\sqrt{2}}{5}$

2. (a)  $5\sqrt{3}$  (b)  $7\sqrt{5}$  (c)  $14\sqrt{5}$   
 (d)  $5\sqrt{3}$  (e)  $8\sqrt{2}$  (f)  $\sqrt{3}$   
 (g)  $8\sqrt{2}$  (h)  $10\sqrt{3} - 33\sqrt{2}$  (i)  $14\sqrt{3} - 40\sqrt{5}$   
 (j)  $-3\sqrt{a}$  (k)  $3\sqrt{y} - \sqrt{x}$  (l)  $2\sqrt{y} - \sqrt{x}$

3. (a) 3 (b)  $8\sqrt{10}$  (c) 5  
 (d) 28 (e)  $50\sqrt{2}$  (f)  $\sqrt{6} + 4$   
 (g)  $29 - 4\sqrt{7}$  (h)  $6\sqrt{15} - 8\sqrt{10}$  (i) 2  
 (j)  $2\sqrt{3} - 5$  (k) 11 (l)  $18 + 5\sqrt{10}$   
 (m)  $2x - 5\sqrt{xy} - 3y$  (n)  $9a - 6\sqrt{ab} + b$  (o)  $6\sqrt{xy} - 3x$

4. (a) 1 (b) 6 (c) 17 (d) -1

5. (a)  $\frac{\sqrt{7}}{7}$  (b)  $\frac{3\sqrt{5}}{5}$  (c)  $\frac{4\sqrt{7}}{21}$  (d)  $\frac{3}{2}$   
 (e)  $\frac{2}{5}$  (f)  $\frac{2}{3}$  (g)  $\frac{\sqrt{10+3\sqrt{5}}}{5}$  (h)  $\frac{\sqrt{3}}{3} - 1$   
 (i)  $\frac{\sqrt{15} - \sqrt{6}}{3}$  (j)  $\frac{2\sqrt{6} + 2}{3}$  (k)  $\frac{\sqrt{xy}}{y}$  (l)  $\frac{2\sqrt{b}}{b}$

6. (a)  $\frac{\sqrt{3} - 1}{2}$  (b)  $\frac{\sqrt{7} - \sqrt{3}}{4}$  (c)  $\frac{5 - 3\sqrt{2}}{7}$  (d)  $\frac{4\sqrt{2} + 2}{7}$   
 (e)  $3(\sqrt{5} + 2)$  (f)  $\frac{12 - 5\sqrt{3}}{23}$  (g)  $\frac{42 - 12\sqrt{5}}{29}$  (h)  $\frac{5 + \sqrt{5}}{4}$   
 (i)  $\frac{5 - \sqrt{21}}{2}$  (j)  $\frac{13 - 4\sqrt{3}}{11}$  (k)  $\frac{13\sqrt{2} + 3\sqrt{30}}{34}$  (l)  $\frac{51 - 10\sqrt{2}}{49}$