C.E.M.TUITION

Student Name :________________________________

Review Topic : Double Angle Formulae

(Preliminary - Paper 1)

Year 12 - 3 Unit

1. Prove that $\dfrac{1-\cos 2x}{1+\cos 2x}=\tan^2 x$ and hence show that the exact value of $\tan\left(\frac{\pi}{8}\right)$ is $\sqrt{2}-1$.

2. Express $\sin x$ and $\cos x$ in terms of $t = \tan\left(\frac{x}{2}\right)$. Hence prove

that: $\frac{1+\sin x - \cos x}{1+\sin x + \cos x} = \tan\left(\frac{x}{2}\right)$

3. If $\cos x = \frac{7}{9}$ and $\sin y = \frac{1}{3}$, where angles x and y are acute,

(a) Show that $x = 2y$

(b) Find the exact value of $\tan(x + y)$

4. Given $\sin x = \frac{1}{\sqrt{3}}$, $\sin y = \frac{1}{\sqrt{2}}$, and $0 < x,\ y < \frac{\pi}{2}$, find the exact value of $\sin(x + y)$.

5. Given that $\dfrac{\cos(A-B)}{\cos(A+B)}=\frac{7}{3}$, show that $5\tan A = 2\cot B$.

Further, given that $\tan B = 2$, and A is acute, find in exact terms:

(a) $\tan(A+B)$ (b) $\sin A$ (c) $\cos 2A$

1. $\dfrac{1-\cos 2x}{1+\cos 2x}$

$= \dfrac{1-\left(1-2\sin^2 x\right)}{1+2\cos^2 x - 1}$

$= \sin^2 x \div \cos^2 x$

$= \tan^2 x$

Put $x = \frac{\pi}{8}$, then :

$\tan^2\left(\frac{\pi}{8}\right)$

$= \left(1-\frac{1}{\sqrt{2}}\right) \div \left(1+\frac{1}{\sqrt{2}}\right)$

$= \left(\sqrt{2}-1\right) \div \left(\sqrt{2}+1\right)$

$= \dfrac{\left(\sqrt{2}-1\right)^2}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}$

$\therefore\ \tan\left(\frac{\pi}{8}\right) = \sqrt{2}-1$

2. $t = \tan\left(\dfrac{x}{2}\right),\ \sin x = \dfrac{2t}{1+t^2}$

$\cos x = \dfrac{1-t^2}{1+t^2}$

$\dfrac{1+\sin x - \cos x}{1+\sin x + \cos x}$

$= \dfrac{1+\dfrac{2t}{1+t^2} - \dfrac{1-t^2}{1+t^2}}{1+\dfrac{2t}{1+t^2} + \dfrac{1-t^2}{1+t^2}}$

$= \dfrac{2t(1+t)}{2(1+t)}$

$= t$, i.e. $\tan\left(\dfrac{x}{2}\right)$

3. $\cos x = \frac{7}{9},\ \sin x = \frac{4\sqrt{2}}{9}$

$\sin y = \frac{1}{3},\ \cos y = \frac{2\sqrt{2}}{3}$

(a) $\sin 2y = 2\sin y \cos y$

$= \frac{4\sqrt{2}}{9}$

$\therefore \sin 2y = \sin x$

$x = 2y$ (x is acute)

(b) $\tan x = \frac{4\sqrt{2}}{7}$,

$\tan y = \frac{1}{2\sqrt{2}}$

$\tan(x+y) = \dfrac{\frac{4\sqrt{2}}{7} + \frac{1}{2\sqrt{2}}}{1-\frac{4\sqrt{2}}{14\sqrt{2}}}$

$\therefore\ \tan(x+y) = \frac{23}{2\sqrt{2}}$

4. x, y acute angles.

$\sin x = \frac{1}{\sqrt{3}},\ \cos x = \frac{\sqrt{2}}{\sqrt{3}}$

$\sin y = \frac{1}{\sqrt{2}},\ \cos y = \frac{1}{\sqrt{2}}$

$\sin(x+y) = \sin x \cos y$

$+ \cos x \sin y$

$= \dfrac{1+\sqrt{2}}{\sqrt{6}}$

5. $\dfrac{\cos(A-B)}{\cos(A+B)} = \frac{7}{3}$

$\dfrac{\cos A\cos B + \sin A \sin B}{\cos A \cos B - \sin A \sin B} = \frac{7}{3}$

Dividing both numerator and denominator by $\cos A \cos B$

$\dfrac{1+\tan A \tan B}{1-\tan A \tan B} = \frac{7}{3}$

$3 + 3\tan A \tan B$

$= 7 - 7\tan A \tan B$

$5\tan A \tan B = 2$

$\therefore\ 5\tan A = 2\cot B$

Given $\tan B = 2,\ \cot B = \frac{1}{2}$

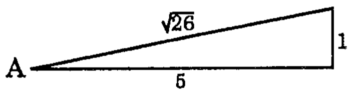
then $\tan A = \frac{1}{5}$

(a) $\tan(A+B)$

$= \left(\frac{1}{5}+2\right) \div \left(1-\frac{2}{5}\right)$

$= \frac{11}{3}$

(b) $\sin A = \frac{1}{\sqrt{26}}$



(c) $\cos 2A = 1 - 2\sin^2 A$

$= 1 - \frac{2}{26}$ or $\frac{12}{13}$.