C.E.M. TUITION

Student Name : ________________________________

Review Topic : Integration

(HSC - PAPER 4)

Year 12 - 2 Unit

19. **Calculate the volume of revolution formed when the curve $x^2 + 2y^2 = 8$ is rotated about the y axis.**

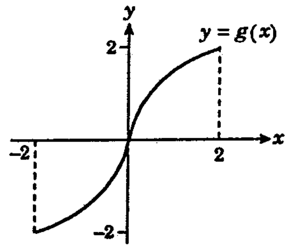
20. Consider the parabola $x^2 = 16y$.

(a) **(i)** Write down the coordinates of the focus and sketch the parabola.

(ii) A line is drawn through the focus parallel to the x axis. Calculate the area contained within the parabola below this line.

(b) That section of the parabola between $x = 0$ and $x = 4$ is rotated about the x axis. Calculate the volume so formed.

21.

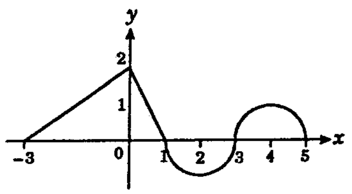


(a) If $y = g(x)$ is an odd function, write down the value of

$$\int_{-2}^{2} g(x)\, dx.$$

(b) If that part of the curve $y = g(x)$ is a quadrant of a circle, calculate the area between the curve $y = g(x)$ and the x axis for $-2 \le x \le 2$.

22.



The figure illustrated represents the function $y = h(x)$ and consists of two straight lines and two semi-circles of radius 1 unit.

(a) Calculate the value of $\int_{-3}^{5} h(x)\, dx$.

(b) Find the area contained between $y = h(x)$ and the x axis.

23. For even function $\phi(x)$ it has been noted that $\int_0^3 \phi(x)\, dx = 8 \cdot 5$.

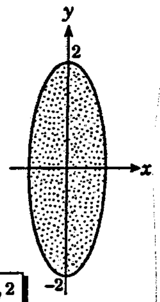
(a) Evaluate $\int_{-3}^{3} \phi(x)\, dx$.

(b) If $\phi(x)$ is completely above the x axis, calculate the area between the curve and the x axis for $-3 \le x \le 3$.

19. $x^2+2y^2=8$

When $x=0$,
$2y^2=8$
$y^2=4$
$y=\pm 2.$

Curve cuts y axis at -2 and 2.



Also, $x^2=8-2y^2$

Now $V=\pi\int_{-2}^{2} x^2\,dy$

$=\pi\int_{-2}^{2}(8-2y^2)\,dy$

$=2\pi\int_{0}^{2}(8-2y^2)\,dy$

Even function

$=2\pi\left[8y-\frac{2}{3}y^3\right]_0^2$

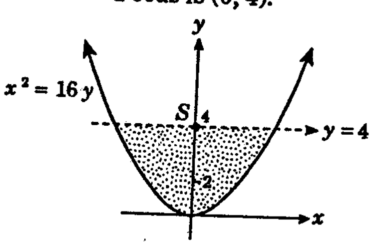
$=2\pi\left[\left(16-\frac{16}{3}\right)-0\right]$

$=\frac{64}{3}\pi.$

Volume is $\frac{64}{3}\pi$ units^3.

20. (a) (i) $x^2=16y$

Compare with $x^2=4ay$ where a is focal length,
$x^2=16y \Rightarrow x^2=4(4)y$
i.e., $a=4$.
Focus is (0, 4).



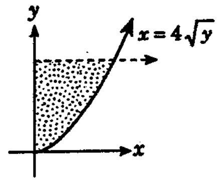
(ii) Required area is shaded on diagram. S is the focus at (0, 4).

Area $=2\int_0^4 x\,dy$

$=2\int_0^4 4\sqrt{y}\,dy$

$=8\int_0^4 y^{\frac{1}{2}}\,dy$

Area between curve and y axis. $x=\pm 4\sqrt{y}$



This gives half the area, i.e. $x=4\sqrt{y}$.

$=8\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^4$

$y^{\frac{3}{2}}=y\cdot y^{\frac{1}{2}}=y\sqrt{y}$

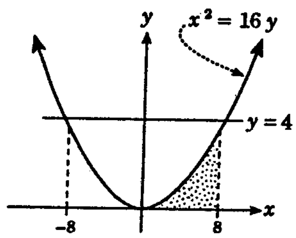
$=\frac{16}{3}\left[4\sqrt{4}-0\right]$

$=\frac{16}{3}(8)$

$=\frac{128}{3}.$

Area is $\frac{128}{3}$ units^2.

Alternatively, area could be found by finding area under $x^2=16y$ and subtracting it from area under $y=4$.



When $y=4$, $x^2=64$
$x=\pm 8$

$x^2=16y$

$y=\frac{1}{16}x^2$

Even function

$A=\pi\int_{-8}^{8} y\,dx$

$=\frac{1}{16}\int_{-8}^{8} x^2\,dx$

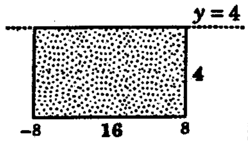
$=2\times\frac{1}{16}\int_0^8 x^2\,dx$

$=\frac{1}{8}\left[\frac{1}{3}x^3\right]_0^8$

$=\frac{1}{8}\cdot\frac{1}{3}\left[8^3-0\right]$

$=\frac{64}{3}.$

Now area under $y=4$ is a rectangle, length 16, width 4:
$A=4\times 16=64$

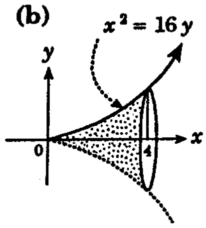


Area required

$=64-\frac{64}{3}$

$=\frac{128}{3}$ units^2.

(b)



$y=\frac{x^2}{16}$

$\therefore\ y^2=\frac{x^4}{256}$

$V=\pi\int_0^4 y^2\,dx$

$=\pi\int_0^4\frac{x^4}{256}\,dx$

$=\frac{\pi}{256}\int_0^4 x^4\,dx$

$=\frac{\pi}{256}\left[\frac{1}{5}x^5\right]_0^4$

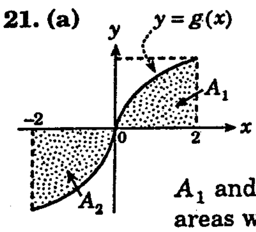
$=\frac{\pi}{256}\left[\frac{1}{5}(4)^5-0\right]$

$=\frac{\pi}{256}\cdot\frac{1}{5}\cdot 1024$

$=\frac{4\pi}{5}.$

Volume is $\frac{4\pi}{5}$ units^3.

21. (a)



A_1 and A_2 are equal areas with opposite signs.

Since $y = g(x)$ is an odd function, $A_2 = -A_1$, therefore

$$\pi\int_{-2}^{2} g(x)\,dx = A_2 + A_1 = -A_1 + A_1 = 0$$

i.e. $\pi\int_{-2}^{2} g(x)\,dx = 0$

OR (alternate solution), since $y = g(x)$ is odd, then

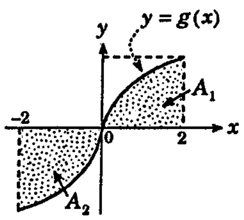
$$\int_{-2}^{0} g(x)\,dx = -\int_{0}^{-2} g(x)\,dx$$

i.e. $\int_{-2}^{2} g(x)\,dx$

$$= \int_{-2}^{0} g(x)\,dx + \int_{0}^{2} g(x)\,dx$$

$= 0.$

(b)



$$A_1 = \int_{0}^{2} g(x)\,dx$$

$$A_2 = \left|\int_{-2}^{0} g(x)\,dx\right|$$

> *Note* Absolute value since area is under the x axis.

but $A_1 = |A_2|$

$$\therefore A = 2\int_{0}^{2} g(x)\,dx$$

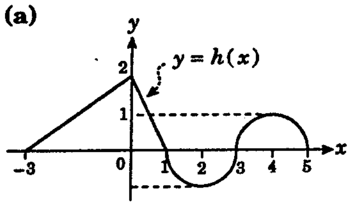
but $\int_{0}^{2} g(x)\,dx$ = area of the quadrant of a circle with radius 2

$$= \frac{1}{4}\pi r^2 = \frac{1}{4}\times\pi\times 2^2 = \pi.$$

Therefore, required area

$$= 2\times\int_{0}^{2} g(x)\,dx = 2\times\pi = 2\pi \text{ unit}^2.$$

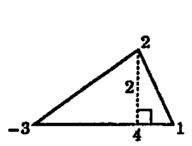
22. (a)



> *Note* The area of the two semi-circles cancel out, as one is positive and the other negative, and they have same size.

$$\therefore \int_{-3}^{5} h(x)\,dx = \text{area of triangle only}$$

$$= \frac{1}{2}\times 4\times 2 = 4.$$



base = 4, height = 2

> *Note* With definite *a* integral we add the various areas of different sections together, regardless of sign.

(b) Area between the curve $y = h(x)$ and the x axis $= 4 + 2\times$area of a semi-circle with radius 1 unit where 4 is the area of Δ,

i.e. $\int_{-3}^{1} h(x)\,dx$

$$= 4 + 2\times\frac{1}{2}\pi r^2 = 4 + \pi r^2 = 4 + \pi\times 1^2 \quad [\text{radius} = 1] = 4 + \pi.$$

Therefore, area between $y = h(x)$ and the x axis is $(4+\pi)$ unit^2.

> *Remember* For area we need a positive quantity.
>
> $$\int_{1}^{5} h(x)\,dx = \left|\int_{1}^{3} h(x)\,dx\right| + \int_{3}^{4} h(x)\,dx$$

23. (a) $\int_{-3}^{3} \phi(x)\,dx$

$= 2\times\int_{0}^{3}\phi(x)\,dx$ for even fn.

$= 2\times 8.5 = \underline{17 \text{ sq. units}}.$

(b) Same answer as (a)