C.E.M. Maths Ext. 2 - Exercises with partial fractions:

1. Express $\dfrac{x^2-4x+7}{(x-2)(x-1)^2}$

in terms of partial fractions. [5]

2. Given that

$$f(x) = \frac{2x^2-12x-2}{(1+x^2)(3-x)} \equiv \frac{A+Bx}{1+x^2} + \frac{C}{3-x}$$

Show that $A = 0$. Also, find the values of B and C. [4]

3. Express $\dfrac{8x^2-5x+22}{(x-1)(x^2+4)}$

in partial fractions. [4]

4. Express $\dfrac{x^2-7x+4}{x(x-2)^2}$

in partial fractions. [4]

5. Express $\dfrac{9+x-x^2}{(x-1)(x+2)^2}$

in partial fractions [4]

6. Calculate the values of the constants A, B and C for which

$$\frac{11x^2-17x+50}{(x-3)(x^2+5)} \equiv \frac{A}{(x-3)} + \frac{Bx+C}{(x^2+5)}.$$ [4]

C.E.M. Maths Ext. 2 – Solutions to exercises with partial fractions:

3101S

1. Question

Express $\dfrac{x^2-4x+7}{(x-2)(x-1)^2}$

in terms of partial fractions. [5]

Solution

$$\frac{x^2-4x+7}{(x-2)(x-1)^2} \equiv \frac{A}{(x-2)}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}$$

Multiply both sides by $(x-2)(x-1)^2$

$$x^2-4x+7 \equiv A(x-1)^2+B(x-2)(x-1)+C(x-2)$$

Let $x=1$

$$(1)^2-4(1)+7 = C(1-2)$$
$$4 = -C$$
$$\Rightarrow \quad C = -4$$

Let $x=2$

$$(2)^2-4(2)+7 = A(2-1)^2$$
$$3 = A$$
$$A = 3$$

Compare coefficients of x^2 terms

$$1 = A+B$$
$$1 = 3+B$$
$$B = -2$$

$$\frac{x^2-4x+7}{(x-2)(x-1)^2} \equiv \frac{3}{(x-2)}-\frac{2}{(x-1)}-\frac{4}{(x-1)^2}$$

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2. Question

Given that

$$f(x) = \frac{2x^2-12x-2}{(1+x^2)(3-x)} \equiv \frac{A+Bx}{1+x^2}+\frac{C}{3-x}$$

Show that $A = 0$. Also, find the values of B and C. [4]

Solution

$$\frac{2x^2-12x-2}{(1+x^2)(3-x)} \equiv \frac{(A+Bx)}{(1+x^2)}+\frac{C}{(3-x)}$$

Multiply both sides by $(1+x^2)(3-x)$

$$2x^2 - 12x - 2 \equiv (A + Bx)(3 - x) + C(1 + x^2)$$

Let $x = 3$

$$\begin{aligned} 2(3)^2 - 12(3) - 2 &\equiv C(1 + 3^2) \\ -20 &= 10C \\ C &= -2 \end{aligned}$$

Compare coefficients of x^2

$$\begin{aligned} 2 &= -B + C \\ 2 &= -B - 2 \\ B &= -4 \end{aligned}$$

Compare constant terms

$$\begin{aligned} -2 &= 3A + C \\ -2 &= 3A - 2 \\ A &= 0 \end{aligned}$$

$$f(x) = -\frac{4x}{1+x^2}-\frac{2}{3-x}$$

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3. Question

Express $\dfrac{8x^2-5x+22}{(x-1)(x^2+4)}$

in partial fractions. [4]

Solution

$$\frac{8x^2-5x+22}{(x-1)(x^2+4)} \equiv \frac{A}{(x-1)}+\frac{(Bx+C)}{(x^2+4)}$$

Multiply both sides by $(x-1)(x^2+4)$

$$8x^2-5x+22 \equiv A(x^2+4)+(Bx+C)(x-1)$$

Let $x = 1$

$$\begin{aligned} 8(1)^2-5(1)+22 &\equiv A(1^2+4) \\ 25 &= 5A \\ A &= 5 \end{aligned}$$

Compare coefficients of x^2 terms

$$\begin{aligned} 8 &= A+B \\ 8 &= 5+B \\ B &= 3 \end{aligned}$$

Compare constant terms

$$\begin{aligned} 22 &= 4A-C \\ 22 &= 4(5)-C \\ C &= -2 \end{aligned}$$

$$\frac{8x^2-5x+22}{(x-1)(x^2+4)} \equiv \frac{5}{x-1}+\frac{3x-2}{x^2+4}$$

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4. Question

Express $\dfrac{x^2-7x+4}{x(x-2)^2}$

in partial fractions. [4]

Solution

$$\frac{x^2-7x+4}{x(x-2)^2} \equiv \frac{A}{x}+\frac{B}{(x-2)}+\frac{C}{(x-2)^2}$$

Multiply both sides by $x(x-2)^2$

$$x^2-7x+4 \equiv A(x-2)^2+Bx(x-2)+Cx$$

Let $x = 0$

$$\begin{aligned} 4 &= A(-2)^2 \\ 4 &= 4A \\ A &= 1 \end{aligned}$$

let $x = 2$

$$\begin{aligned} (2)^2-7(2)+4 &= C(2) \\ -6 &= 2C \\ C &= -3 \end{aligned}$$

Compare coefficients of x^2 terms

$$\begin{aligned} 1 &= A+B \\ 1 &= 1+B \\ B &= 0 \end{aligned}$$

$$\frac{x^2-7x+4}{x(x-2)^2} = \frac{1}{x}-\frac{3}{(x-2)^2}$$

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5. Question

Express $\dfrac{9+x-x^2}{(x-1)(x+2)^2}$

in partial fractions . [4]

Solution

$$\frac{9+x-x^2}{(x-1)(x+2)^2} \equiv \frac{A}{(x-1)}+\frac{B}{(x+2)}+\frac{C}{(x+2)^2}$$

Multiply both sides by $(x-1)(x+2)^2$

$$9+x-x^2 \equiv A(x+2)^2+B(x-1)(x+2)+C(x-1)$$

Let $x=1$

$$\begin{aligned} 9+(1)-(1)^2 &= A(1+2)^2 \\ 9 &= 9A \\ A &= 1 \end{aligned}$$

Let $x=-2$

$$\begin{aligned} 9+(-2)-(-2)^2 &= C(-2-1) \\ 3 &= -3C \\ C &= -1 \end{aligned}$$

Compare x^2 terms

$$\begin{aligned} -1 &= A+B \\ -1 &= 1+B \\ B &= -2 \end{aligned}$$

$$\frac{9+x-x^2}{(x-1)(x+2)^2} \equiv \frac{1}{(x-1)}-\frac{2}{(x+2)}-\frac{1}{(x+2)^2}$$

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6. Question

Calculate the values of the constants A, B and C for which

$$\frac{11x^2-17x+50}{(x-3)(x^2+5)} \equiv \frac{A}{(x-3)}+\frac{Bx+C}{(x^2+5)} \quad .$$ [4]

Solution

$$\frac{11x^2-17x+50}{(x-3)(x^2+5)} \equiv \frac{A}{(x-3)}+\frac{(Bx+C)}{(x^2+5)}$$

Multiply both sides by $(x-3)(x^2+5)$

$$11x^2-17x+50 \equiv A(x^2+5)+(Bx+C)(x-3)$$

let $x = 3$

$$\begin{aligned} 11(3)^2-17(3)+50 &= A(3^2+5) \\ 98 &= 14A \\ A &= 7 \end{aligned}$$

Compare coefficients of x^2 terms

$$\begin{aligned} 11 &= A+B \\ 11 &= 7+B \\ B &= 4 \end{aligned}$$

Compare constant terms

$$\begin{aligned} 50 &= 5A-3C \\ 50 &= 5(7)-3C \\ 3C &= -15 \\ C &= -5 \end{aligned}$$

$$\frac{11x^2-17x+50}{(x-3)(x^2+5)} \equiv \frac{7}{(x-3)}+\frac{4x-5}{(x^2+5)}$$
