C.E.M. TUITION

Student Name :____________________________________

Review Topic : Circle Geometry

(Preliminary - Paper 1)

Year 12 - 3 Unit

1.

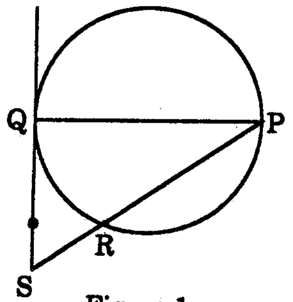


Figure 1.

PQ is a diameter of the circle. A tangent through Q meets the chord PR produced at S. Prove that

$RQ^2 = SR \cdot RP$ (Fig. 1)

2.

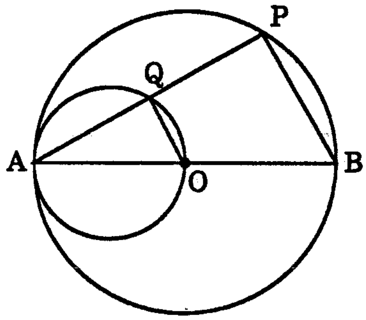


Figure 2.

A diameter AB is drawn in a circle, centre O. Another circle is drawn with OA as diameter. A chord AP of the larger circle cuts the smaller circle in Q. Prove that:

(i) **BP || OQ**

(ii) **AQ = QP** (Fig. 2)

3.

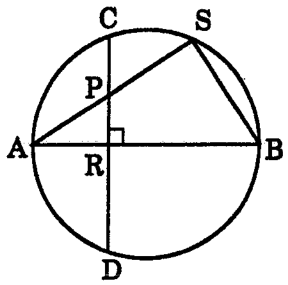


Figure 3.

AB is a diameter of a circle. CD ⊥ AB and CD intersects AB in R, and AS at P. Prove that:

(i) PRBS is a cyclic quadrilateral

(ii) $AP \cdot AS = AR \cdot AB$ (Fig.3)

4.

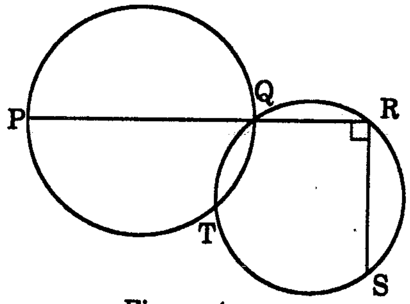


Figure 4.

In the Fig. 4 PQ is a diameter of the larger circle and PQR is a straight line. PR ⊥ RS. Prove that P, T and S are collinear.

5\.

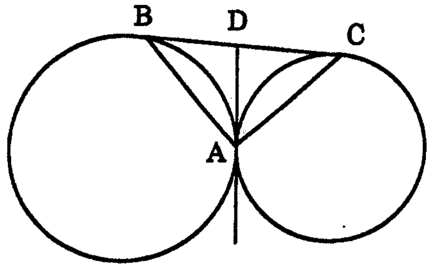


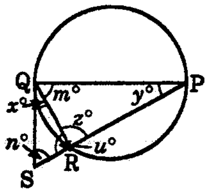
Figure 5.

In Fig. 5 BC and DA are common tangents to both the circles. Prove that:

(i) BD = DC

(ii) $\angle BAC = 90°$

1.



Given: SQ is a tangent, PQ is a diameter

Prove: $RQ^2 = SR \cdot RP$

Proof: Join RQ

In ΔPQR and ΔSQR:

$x = y$ ($\angle$ between tangent and chord = $\angle$ in the alt. segment)

$z = 90$ (PQ, diameter)

$u + z = 180$ (SRP, a st. $\angle$)

$\therefore$ $z = u$

$\therefore$ $m = n$ ($\angle$ sum of Δ = 180°, so 3rd respective $\angle$s are equal)

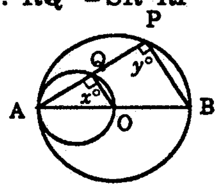
$\therefore$ $\Delta PQR \,|||\, \Delta SQR$

$\therefore$ sides are in the same ratio

$\therefore \dfrac{RQ}{SR} = \dfrac{RP}{RQ}$

$\therefore RQ^2 = SR \cdot RP$

2.



Given: AB and AO are the diameters. O is the centre of larger circle.

Prove: (i) OQ || BP
(ii) Q is the mid-point of AP

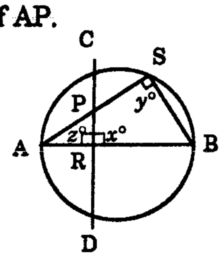
Proof:

(i) $x = 90$ (AB is diameter)
$y = 90$ (AO is diameter)
$\therefore$ $x = y$, but these are alt. $\angle$s.
$\therefore$ OQ || BP

(ii) In ΔAOQ and ΔABP:
$x = y$ (proved above)
$\angle A$ is common
Since the $\angle$ sum of any Δ is 180°, the 3rd respective $\angle AOQ = \angle ABP$
$\therefore$ $\Delta AOQ \,|||\, \Delta ABP$
$\therefore$ sides are proportional
$\therefore \dfrac{AO}{AB} = \dfrac{AQ}{AP}$

But $AB = 2AO$ (O is the centre)
$\therefore$ $AQ = \frac{1}{2}AP$
$\therefore$ Q is the mid-point of AP.

3.



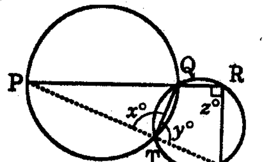
Given: AB is diameter, $CD \perp AB$

Prove: (i) PRBS is a cyclic quadrilateral
(ii) $AP \cdot AS = AR \cdot AB$

(i) Proof: Join BS
$x = 90$ ($CD \perp AB$)
$y = 90$ (AB, diameter)
$\therefore$ $x + y = 180$
$\therefore$ PRBS is a cyclic quadrilateral

(ii) In ΔARP and ΔABS:
$\angle A$ is common
$z = x = 90$ ($CD \perp AB$)
$\therefore$ The 3rd respective $\angle APR = \angle ABS$ ($\angle$ sum of Δ = 180°)
$\therefore$ $\Delta ARP \,|||\, \Delta ABS$
$\therefore$ sides are proportional
$\therefore \dfrac{AR}{AS} = \dfrac{AP}{AB}$
$\therefore$ $AP \cdot AS = AR \cdot AB$

4.



Given: PQ is the diameter of the larger circle, $PR \perp RS$.

Prove: P, T, S are collinear

Proof: Join PT, TS, QT.

$x = 90$ (PQ is a diameter)

$z = 90$ ($PR \perp RS$)

$z + y = 180$ (opp. $\angle$s of cyc. quad. TSRQ)

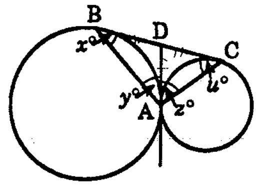
$\therefore$ $y = 90$

$\therefore$ $x + y = 180$

$\therefore$ PTS is a st. $\angle$

$\therefore$ P, T, S are collinear.

5.



Given: BC and DA are two common tangents.

Prove: BD = DC and $\angle BAC = 90°$.

Proof:

(i) DA = DC (Tangents from a point are equal in lengths)
DA = DB
$\therefore$ DB = DC

(ii) $x = y$ (DB = DA, ΔBDA isosceles)
$z = u$ (DA = DC, ΔDCA isosceles)
Now $(y + z) + x + u = 180$, being the $\angle$ sum of ΔABC
$\therefore$ $2(y + z) = 180$
$y + z = 90$
$\therefore$ $\angle BAC = 90$.