NAME :

Centre of Excellence in Mathematics
S201 / 414 GARDENERS RD. ROSEBERY 2018
www.cemtuition.com.au

MOBILE 0412 880 475

PHONE 96663331

REVIEW TOPICS:

FURTHER TRIG I & II & AUXILIARY ANGLES & GENERAL SOLUTIONS

CEM – Yr 12 – 3U Further Trig I & II, Auxiliary Angles, General Solutions – Review Booklet – Paper 1

1. (i) Express $\sqrt{3}\cos 2t - \sin 2t$ in the form $A\cos(2t + \alpha)$, with $A > 0$ and $0 < \alpha < \frac{\pi}{2}$.

   (ii) Find, in exact form, the general solutions to $\sqrt{3}\cos 2t - \sin 2t = 1$.

2. Evaluate $\sin\left[\cos^{-1}\left(\frac{2}{3}\right) + \tan^{-1}\left(-\frac{3}{4}\right)\right]$.

3. Find all values of θ in the range $0 \le \theta \le 2\pi$ for which $\sqrt{3}\cos\theta - \sin\theta = 1$

4. (i) Express $2\sin x + \sqrt{12}\cos x$ in the form $R\sin(x+\theta)$ where $R > 0$ and θ is a subsidiary angle in the range $0 \le \theta \le \frac{\pi}{2}$

(ii) Hence, give the general solution to the equation

$$2\sin x + \sqrt{12}\cos x = 2\sqrt{3}$$

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5. (i) Show that $\sin x - \cos 2x = 2\sin^2 x + \sin x - 1$

   (ii) Hence, or otherwise, solve $\sin x - \cos 2x = 0$ for $0 \le x \le 2\pi$

6. (i) Show that $\sqrt{3}\sin x - \cos x$ can be rewritten as $2\sin\left(x - \frac{\pi}{6}\right)$

   (ii) Hence or otherwise solve $\sqrt{3}\sin x - \cos x = 1$ for $0 \le x \le 2\pi$

7. (i) Show that the equation $\sqrt{3}\sec^2\theta - 2\tan\theta - 2\sqrt{3} = 0$ can be rewritten as $\left(\sqrt{3}\tan\theta + 1\right)\left(\tan\theta - \sqrt{3}\right) = 0$ [2]

(ii) Hence or otherwise find the general solutions to the equation

$$\sqrt{3}\sec^2\theta - 2\tan\theta - 2\sqrt{3} = 0$$ [2]

8. Prove that $\dfrac{\sin 8A}{1+\cos 8A} = \tan 4A$

9. Show that $\dfrac{2\cos A}{\operatorname{cosec} A - 2\sin A} = \tan 2A$

10. Prove that $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta}$ is independent of θ.

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Answers

1. b) i) $A\cos(2t+\alpha) = A\cos 2t\cos\alpha - A\sin 2t\sin\alpha$

$\Rightarrow \sqrt{3} = A\cos\alpha \;\&\; 1 = A\sin\alpha$

So, $A = 2$

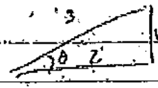
$\cos\alpha = \dfrac{\sqrt{3}}{2}, \sin\alpha = \dfrac{1}{2} \Rightarrow \alpha = \dfrac{\pi}{6}$

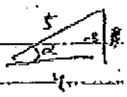
$\sqrt{3}\cos 2t - \sin 2t = 2\cos(2t+\alpha)$

ii) $\cos(2t+\frac{\pi}{6}) = \frac{1}{2} \Rightarrow 2t + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$

$t = n\pi - \dfrac{\pi}{4}, n\pi + \dfrac{\pi}{12}$

2. $\sin\left(\cos^{-1}\frac{2}{3} + \tan^{-1}\left(-\frac{3}{4}\right)\right)$





Let $\theta = \cos^{-1}\frac{2}{3}$, $\alpha = \tan^{-1}(-\frac{3}{4})$ (With the assumption that the range is $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$)

$\sin[\theta + \alpha]$

$= \sin\theta\cos\alpha + \cos\theta\sin\alpha$

$= \frac{\sqrt{5}}{3} \cdot \frac{4}{5} + \frac{2}{3} \times \frac{-3}{5}$ ✓

$= \frac{4\sqrt{5}}{15} + \frac{-6}{15} = \frac{4\sqrt{5} - 6}{15}$

3. $\sqrt{3}\cos\theta - \sin\theta = R\cos(\theta + \alpha)$

$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$R\cos\alpha = \sqrt{3}$

$R\sin\alpha = 1$

$R = \sqrt{(\sqrt{3})^2 + 1^2}$

$= 2$

$\tan\alpha = \dfrac{1}{\sqrt{3}}$

$\alpha = \dfrac{\pi}{6}$

$2\cos\left(\theta + \dfrac{\pi}{6}\right) = 1$

$\cos\left(\theta + \dfrac{\pi}{6}\right) = \dfrac{1}{2}$

$\theta + \dfrac{\pi}{6} = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

$\theta = \dfrac{\pi}{6}, \dfrac{3\pi}{2}$

4. i) $2\sin x + \sqrt{12}\cos x$ $\quad R\sin(x+\theta)$, $R > 0$

$R\sin(x+\theta) = R\sin x\cos\theta + R\cos x\sin\theta$

$2\sin x + \sqrt{12}\cos x = R\sin x\cos\theta + R\cos x\sin\theta$

using coefficients of $\sin x$: $R\cos\theta = 2$ ①

using coefficients of $\cos x$: $R\sin\theta = \sqrt{12}$ ②

squaring and adding ① and ②:

$R^2\cos^2\theta + R^2\sin^2\theta = 4 + 12$

$R^2(\sin^2\theta + \cos^2\theta) = 16$

$\therefore R^2 = 16$

$R = \pm\sqrt{16}$ but $R > 0$

so $R = 4$

From ①: $4\cos\theta = 2$

$\cos\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}$

From ②: $R\sin\theta = \sqrt{12}$

$4\sin\theta = \sqrt{4}\sqrt{3}$

$4\sin\theta = 2\sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\therefore \theta = \frac{\pi}{3}$

So θ is in the first quadrant with related angle $\frac{\pi}{3}$

so $2\sin x + \sqrt{12}\cos x = 4\sin\left(x + \frac{\pi}{3}\right)$.

ii) $2\sin x + \sqrt{12}\cos x = 2\sqrt{3}$

then $4\sin(x + \frac{\pi}{3}) = 2\sqrt{3}$

$\sin(x + \frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

$\sin(x + \frac{\pi}{3}) = \sin\frac{\pi}{3}$

∴ $x + \frac{\pi}{3} = 2n\pi + \frac{\pi}{3}$

$x = 2n\pi$

or $x = (2n+1)\pi - \frac{\pi}{3}$

$x = 2n\pi + \pi - \frac{\pi}{3}$

$x = 2n\pi + \frac{2\pi}{3}$

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5. (i) Show that $\sin x - \cos 2x = 2\sin^2 x + \sin x - 1$

LHS

We know $\cos 2x = 1 - 2\sin^2 x$

so $\sin x - \cos 2x$

$= \sin x - (1 - 2\sin^2 x)$

$= \sin x - 1 + 2\sin^2 x$

$= 2\sin^2 x + \sin x - 1$

$=$ RHS.

(ii) Solve $\sin x - \cos 2x = 0$ for $0 \le x \le 2\pi$

let $m = \sin x$

$\therefore 2\sin^2 x + \sin x - 1 = 0$

is $2m^2 + m - 1 = 0$

$2m^2 + 2m - m - 1 = 0$

$2m(m+1) - (m+1) = 0$

$(2m-1)(m+1) = 0$

$\therefore m = \frac{1}{2}$ or $m = -1$

Replace for m

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$\therefore x = \frac{\pi}{6}$ or $\pi - \frac{\pi}{6}$ or

$x = \frac{\pi}{6}$, $\frac{5\pi}{6}$ or $\frac{3\pi}{2}$

6.

$2\sin\left(x - \frac{\pi}{6}\right)$

$= 2\left(\sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6}\right)$ 1

$= 2\left(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right)$

$= \sqrt{3}\sin x - \cos x$ 1

$\therefore \sqrt{3}\sin x - \cos x = 2\sin\left(x - \frac{\pi}{6}\right)$

$2\sin\left(x - \frac{\pi}{6}\right) = 1$

$\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

$x - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}$

$0 \le x \le 2\pi$

$\therefore x = \frac{\pi}{3}, \pi$ for $0 \le x \le 2\pi$ 1

or

$r = \sqrt{\sqrt{3}^2 + 1^2}$

$= 2$

$\sqrt{3}\sin x - \cos x$

$= 2(\sin x \cos\alpha - \cos x \sin\alpha)$

$\therefore \sqrt{3} = 2\cos\alpha$

$1 = 2\sin\alpha$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$\frac{\pi}{6}$ is 1M

7.

$\sqrt{3}\sec^2\theta - 2\tan\theta - 2\sqrt{3} = 0$

$(\sqrt{3}\tan\theta + 1)(\tan\theta - \sqrt{3})$

$= \sqrt{3}\tan^2\theta - 3\tan\theta + \tan\theta - \sqrt{3}$ 1

$= \sqrt{3}(\sec^2\theta - 1) - 2\tan\theta - \sqrt{3}$ 1

$= \sqrt{3}\sec^2\theta - 2\tan\theta - 2\sqrt{3}$

$\therefore \sqrt{3}\sec^2\theta - 2\tan\theta - 2\sqrt{3} = 0$

can be rewritten as

$(\sqrt{3}\tan\theta + 1)(\tan\theta - \sqrt{3}) = 0$

$\tan\theta = -\frac{1}{\sqrt{3}}$ or $\tan\theta = \sqrt{3}$ 1

$\theta = \pi n - \frac{\pi}{6}$ or $\theta = \pi n + \frac{\pi}{3}$ 1

Alternatively

$\sqrt{3}(1 + \tan^2\theta) - 2\tan\theta - 2\sqrt{3} = 0$

$\sqrt{3}\tan^2\theta - 2\tan\theta - \sqrt{3} = 0$

on factorising

$(\sqrt{3}\tan\theta + 1)(\tan\theta - \sqrt{3}) = 0$

8. $\dfrac{\sin 8A}{1+\cos 8A} = \tan 4A$

$LHS = \dfrac{2\sin 4A \cos 4A}{1 + 2\cos^2 4A - 1}$

$= \tan 4A$

$= RHS$

$\therefore$ true

9. $\dfrac{}{\operatorname{cosec} A - 2\sin A} = \tan 2A$

$LHS = \dfrac{2\cos A}{\frac{1}{\sin A} - 2\sin A}$

$= \dfrac{2\cos A}{\frac{1-2\sin^2 A}{\sin A}}$

$= \dfrac{2\sin A\cos A}{\cos 2A}$

$= \dfrac{\sin 2A}{\cos 2A} = \tan 2A = RHS$

10. $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta}$

$= \dfrac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin(3\theta - \theta)}{\frac{1}{2}\sin 2\theta}$

$\dfrac{\sin 2\theta}{\frac{1}{2}\sin 2\theta}$