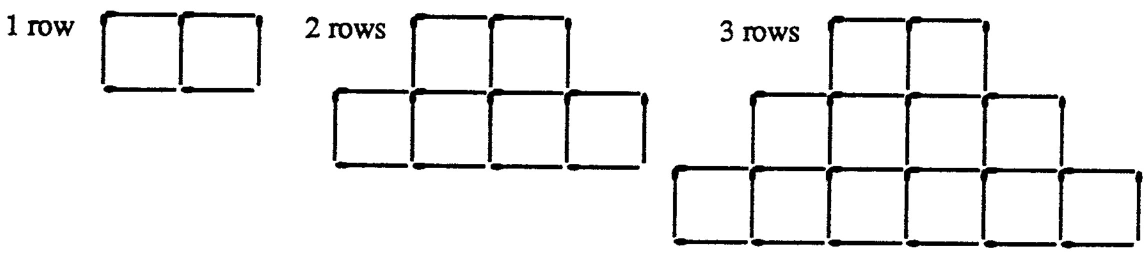
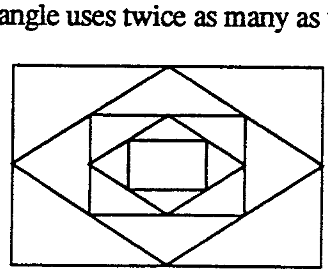
Exercises on Series and Applications

1. Find x if 2x, 7, and $9-x$ are the first three terms of an arithmetic series.
2. Find the thirteenth term of the arithmetic series $3+9+15+\dots$.
3. Find the sixteenth term of the series $57+49+41+\dots$.
4. If the first term of an arithmetic series is 4 and the nineteenth term is 31, find the thirtieth term.
5. Find the first term and common difference of the arithmetic series with ninth term 47 and seventeenth term 103.
6. Find the sum of the first ten terms of the arithmetic series $2+7+12+\dots$.
7. Find the first term of an arithmetic series if the common difference is -3 and the sum of the first twelve terms is 54.
8. The first term of an arithmetic series is 3 and the thirteenth term is 99. Find the sum of the first thirteen terms.
9. The skirt of a dancer's costume is to be decorated with 25 rows of sequins. The top row will contain 16 sequins, the next 18, the third 20 and so on. How many sequins will be used?
10. Jerry is employed under contract for 18 weeks. He will be paid \$270 for the first week, and each week will receive an additional \$35.
    (a) What will he be paid for the last week?
    (b) What will be his total earnings over the period of the contract?
11. Kim is using matches to make a pattern.



    (a) How many matches are used in the first row?
    (b) How many extra matches are used in (i) the second row, (ii) the third row?
    (c) How many extra matches will be required for the tenth row?
    (d) What is the total number of matches needed to complete 10 rows?
12. Find x if 12, 30, and x are the first three terms of a geometric series.
13. Find all possible values of a and b if $a+b+5+\dots$ is an arithmetic series and $a+b-4+\dots$ is a geometric series.
14. Find the eleventh term in the geometric series $3+12+48+\dots$.
15. The first term of a geometric series is 4 and the eighth term is 8748. Find the twelfth term.
16. Find the sum of the first twenty terms of the geometric series with $a=7$ and $r=-2$.
17. Find the sum of the first nine terms of the geometric series $96+48+24+\dots$.
18. A courtyard is paved, and a pattern created, with bricks. The pattern consists of 8 rectangles and 7 rhombuses positioned as shown in the diagram.
    The first rectangle consists of 7 bricks and each subsequent rectangle uses twice as many as the previous one. The first rhombus uses 10 bricks and each subsequent rhombus uses twice as many as the previous one.
    (a) How many bricks are in the eighth rectangle?
    (b) How many bricks are used in all the rhombuses?
    (c) How many bricks are used altogether in the pattern?



Solutions to Exercises on Series and Applications (Chapter 7)

1. $u_3 - u_2 = u_2 - u_1$
$(9-x)-7 = 7-2x$
$2-x = 7-2x$
$2+x=7$
$x=5$

2. Arithmetic series: $a=3,\ d=6$
$u_n = a+(n-1)d$
$u_{13} = 3+(13-1)\cdot 6 = 75,$
$\therefore$ the thirteenth term is 75.

3. Arithmetic series: $a=57,\ d=-8$
$u_n = a+(n-1)d$
$u_{16} = 57+(16-1)\cdot -8 = -63,$
$\therefore$ the sixteenth term is −63.

4. $a=4,\ u_{19}=31$
$u_n = a+(n-1)d$
$u_{19} = 4+(19-1)d$
$31 = 4+18d$
$27 = 18d$
$d=1.5$
$u_{30} = 4+(30-1)\cdot 1.5 = 47.5,$
$\therefore$ the thirtieth term is $47\frac{1}{2}$.

5. $u_9=47,\ u_{17}=103$
$u_n = a+(n-1)d$
$u_9 = a+(9-1)d$
$47 = a+8d$ (i)
$u_{17} = a+(17-1)d$
$103 = a+16d$ (ii)
(ii)−(i) $56=8d$
$d=7$
Substitute in (i):
$47 = a+8\cdot 7$
$= a+56$
$= -9$
$\therefore$ the first term is −9 and the difference is 7.

6. Arithmetic series: $a=2,\ d=5$
$S_n = \frac{n}{2}[2a+(n-1)d]$
$S_{10} = \frac{10}{2}[2\cdot 2+(10-1)\cdot 5] = 245,$
$\therefore$ the sum of the first 10 terms is 245.

7. $d=-3,\ S_{12}=54$
$S_n = \frac{n}{2}[2a+(n-1)d]$
$S_{12} = \frac{12}{2}[2a+(12-1)\cdot -3]$
$54 = 6(2a-33)$
$9 = 2a-33$
$42 = 2a$
$a = 21,$
$\therefore$ the first term is 21.

8. $a=3,\ \ell=99,\ n=13$
$S_n = \frac{n}{2}[a+\ell]$
$S_{13} = \frac{13}{2}[3+99] = 663,$
$\therefore$ the sum of the first thirteen terms is 663.

9. Number of sequins = 16 + 18 + 20 + ...
Arithmetic series: $a=16,\ d=2,\ n=25.$
$S_n = \frac{n}{2}[2a+(n-1)d]$
$S_{25} = \frac{25}{2}[2\cdot 16+(25-1)\cdot 2] = 1000,$
$\therefore$ 1000 sequins will be used.

10. Arithmetic series: $a=270,\ d=35.$
$u_n = a+(n-1)d$
(a) $u_{18} = 270+(18-1)\cdot 35 = 865,$
$\therefore$ he will be paid \$865 in the last week.
(b) $S_n = \frac{n}{2}[2a+(n-1)d]$
$S_{18} = \frac{18}{2}[2\cdot 270+(18-1)\cdot 35]$
$= 10\,215,$
$\therefore$ Jerry will receive a total of \$10 215.

11. (a) 7 matches
(b) (i) 11 matches
(ii) 15 matches
(c) 7 + 11 + 15 + ...
Arithmetic series: $a=7,\ d=4$
$u_n = a+(n-1)d$
$u_{10} = 7+(10-1)\cdot 4 = 43$
43 matches are in the tenth row.
(d) $S_n = \frac{n}{2}[2a+(n-1)d]$
$S_{10} = \frac{10}{2}[2\cdot 7+(10-1)\cdot 4] = 250,$
250 matches will be used altogether.

12. $\frac{u_3}{u_2} = \frac{u_2}{u_1}$ | $\frac{x}{30} = \frac{30}{12}$ | $12x = 900$
$x = 75$

13. $a+b+5+\ldots$ is arithmetic
$b-a = 5-b$
$a = 2b-5$ (i)

$a+b-4+\ldots$ is geometric
$\frac{b}{a} = \frac{-4}{b}$
$b^2 = -4a$

$b^2 = -4(2b-5)$ [from (i)]

$b^2 + 8b - 20 = 0$

$(b+10)(b-2) = 0$

$b = -10$ or $b = 2$

Substitute in (i): $a = 2b - 5$

If $b = -10$, $a = 2 \cdot -10 - 5 = -25$

If $b = 2$, $a = 2 \cdot 2 - 5 = -1$,

$\therefore$ $a = -25$ and $b = -10$ or $a = -1$ and $b = 2$.

14. Geometric series: $a = 3$, $r = 4$.

$u_n = ar^{n-1}$

$u_{11} = 3 \cdot 4^{11-1} = 3\,145\,728$

15. $a = 4$, $u_8 = 8748$

$u_n = ar^{n-1}$

$u_8 = 4 \cdot r^{8-1}$

$8748 = 4r^7$

$2187 = r^7$

$r = 3$

$u_{12} = 4 \cdot 3^{12-1}$

$= 708\,588$,

$\therefore$ the twelfth term is 708 588.

16. $a = 7$, $r = -2$, $n = 20$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{20} = \frac{7((-2)^{20} - 1)}{-2 - 1} = -2\,446\,675$$

17. Geometric series: $a = 96$, $r = \frac{1}{2}$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_9 = \frac{96\left(1 - (\frac{1}{2})^9\right)}{1 - \frac{1}{2}} = 191.625$$

18. (a) The number of bricks in the rectangles form a geometric series: $a = 7$, $r = 2$.

$u_n = ar^{n-1}$

$u_8 = 7 \cdot 2^{8-1} = 896$,

$\therefore$ 896 bricks make up the 8th rectangle.

(b) The number of bricks in the rhombuses form a geometric series: $a = 10$, $r = 2$.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_7 = \frac{10(2^7 - 1)}{2 - 1} = 1270,$$

$\therefore$ 1270 bricks are used in the rhombuses.

(c) For the rectangles:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_8 = \frac{7(2^8 - 1)}{2 - 1} = 1785,$$

$\therefore$ total number of bricks $= 1270 + 1785 = 3055$,

$\therefore$ 3055 bricks are used in the pattern.