C.E.M.TUITION

Student Name :________________________________

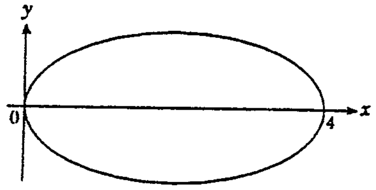
Review Topic : Trapezoidal and Simpson's Rule

(HSC Course - Paper 2)

Year 12 - 2 Unit

1995

7.



The loop of the curve $y^2 = x(4-x)^2$ has an area of A units^2.
(*Note* symmetry about x axis.)

(a) Complete the table of values for $y = (4-x)\sqrt{x}, \quad 0 \le x \le 4.$

x	0	1	2	3	4
y					

(b) Use the table of values and Simpson's Rule with 5 function values to evaluate $\int_0^5 (4-x)\sqrt{x}\,dx$.

(c) Calculate A correct to 2 decimal places.

8. A bike rider racing around an oval track has her speed, v, in km/h recorded at 3 minute intervals as recorded in the following table:

time (t) mins	0	3	6	9	12
speed (v) km/h	0	14	18	16	15

(a) Explain why the distance x, in km travelled by the rider over the 12 minutes, is given by $x = \int_0^{\frac{1}{5}} v\, dt$.

(b) Calculate x using the Trapezoidal Rule with four subintervals.

9. Complete this table of values for $y = \dfrac{e^x + e^{-x}}{e^x}$.

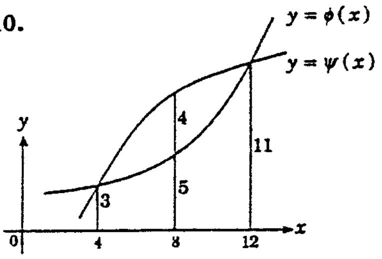
x	0	0·5	1
y			

(a) Using Simpson's Rule with three function values, estimate the value of $\displaystyle\int_0^1 \frac{e^x + e^{-x}}{e^x}\,dx$ correct to 2 decimal places.

(b) Show that $\dfrac{e^x + e^{-x}}{e^x} = 1 + e^{-2x}$.

(c) Calculate the area under the curve $y = \dfrac{e^x + e^{-x}}{e^x}$ between $x = 0$ and $x = 1$ by integration. Find the difference between the actual area and the approximate area correct to two decimal places.

10.



This diagram shows two unknown curves, $y = \phi(x)$ and $y = \psi(x)$, drawn for $4 \le x \le 12$. The ordinates are marked on the diagram.

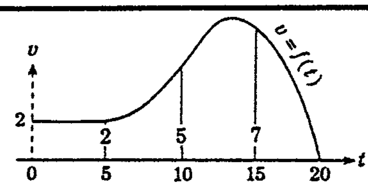
(a) Complete these tables of value for $y = \phi(x)$ and $y = \psi(x)$.

x	4	8	12
$\phi(x)$			

x	4	8	12
$\psi(x)$			

(b) Using the Trapezoidal Rule and 3 function values, calculate the area contained between the curves $y = \phi(x)$ and $y = \psi(x)$.

11.



(a) Use Simpson's Rule with five function values to calculate the area under the curve $v = f(t)$ between $t = 0$ and $t = 20$.

(b) If the graph represents the velocity of a particle moving in a straight line, such that at time t the velocity is v ms^{-1}, calculate the distance travelled in the first 20 seconds.

12. Use the Trapezoidal Rule with three function values to evaluate

$$\int_0^1 \sin \pi x \, dx.$$

Compare this answer with the actual value of the integral.

7. (a)

x	0	1	2	3	4
y	0	3	2·828	1·732	0

$h = 1 - 0 = 1$ (TODD Simpson)

(b) $\int_0^5 (4-x)\sqrt{x}\,dx = \frac{h}{3}\left[\textit{first} + \textit{last} + 2\ \textit{odd} + 4\ \textit{even}\right]$ (*FLOE*)

$= \frac{1}{3}\left[f(0)+f(4)+2f(2)+4\left[f(1)+f(3)\right]\right]$

$= \frac{1}{3}\left[0+0+2(2\cdot828)+4(3+1\cdot732)\right]$

$= 8\cdot772\,016\,9.$

(c) Total $A = 2 \times \int_0^4 (4-x)\sqrt{x}\,dx$

$= 2 \times 8\cdot772\,016\,9$

$= 17\cdot544\,034.$

∴ Area is 17·54 units^2.

> $\int_0^4 (4-x)\sqrt{x}\,dx$ represents the area above the x axis and below the curve.

8.

t	0	3	6	9	12
v	0	14	18	16	15

$v = f(t)$

(a) $x = \int_0^{\frac{1}{5}} v\,dt$ 12 mins = $\frac{1}{5}$ hour

Velocity is $\frac{dx}{dt}$ (rate of change of displacement),

hence distance will be $\int v\ dt$.

This integral is measuring the distance covered over $\frac{1}{5}$ of an hour as velocity has been measured in km/hour.

(b) $x = \frac{1}{2}(3)\left[f(0)+2f(3)+2f(6)+2f(9)+f(12)\right]$

$= 1\cdot5\left[0+2(14)+2(18)+2(16)+15\right]$

$= 166\cdot5.$

Rider has ridden 166·5 km.

> Four strips, 0 – 3, 3 – 6, 6 – 9, and 9 – 12. Strip width = 3

9. $y = \frac{e^x + e^{-x}}{e^x}$

x	0	0·5	1
y	2	1·367 9	1·135 3

$a = 0,\ b = 1$

$\frac{a+b}{2} = 0\cdot5$

(a) $\int_0^1 \frac{e^x+e^{-x}}{e^x}\,dx = \frac{1}{6}(1-0)\left[f(0)+f(1)+4f(0\cdot5)\right]$

$= \frac{1}{6}\left[2+1\cdot135\,3+4(1\cdot367\,9)\right]$

$= 1\cdot434\,483\,3 \quad \doteq 1\cdot43$ (2 decimal places).

(b) $\frac{e^x+e^{-x}}{e^x} = \frac{e^x}{e^x}+\frac{e^{-x}}{e^x} = 1+e^{-2x}$

(c) Approximate area under the curve, using Simpson's Rule, is 1·43 units² [from (a)].

$$\int_0^1 \frac{e^x+e^{-x}}{e^x}\,dx=\int_0^1 1+e^{-2x}\,dx$$
$$=\left[x-\frac{1}{2}e^{-2x}\right]_0^1 =\left[1-\frac{1}{2}e^{-2}\right]-\left[0-\frac{1}{2}e^0\right]$$
$$=\left[1-0{\cdot}067\,667\,6\right]+\frac{1}{2}$$
$$=1{\cdot}432\,332\,4 \quad \approx 1{\cdot}43 \text{ (2 dec. places)}$$

Difference is zero when considering areas correct to 2 decimal places. However, from (a), approximate area is 1·4344833, while by integration the area is 1·4323324. Difference is 0·00215094, which is negligible.

10. (a)

x	4	8	12
$\phi(x)$	3	5	11

x	4	8	12
$\psi(x)$	3	9	11

(b) Area contained between the curves is the area under higher curve, $y=\psi(x)$, less the area under the lower curve, $y=\phi(x)$.

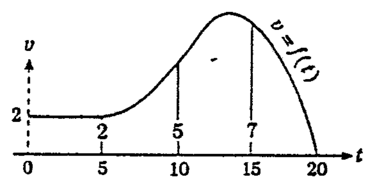
$$A=\int_4^{12}\psi(x)\,dx-\int_4^{12}\phi(x)\,dx,$$
using $A=\frac{h}{2}[\text{first}+\text{last}+2(\text{remaining})]$

$$\int_4^{12}\psi(x)\,dx=\frac{4}{2}[\psi(4)+\psi(12)+2\psi(8)]=2[3+11+2(9)]=64.$$
$$\int_4^{12}\phi(x)\,dx=\frac{4}{2}[\phi(4)+\phi(12)+2\phi(8)]=2[3+11+2(5)]=48.$$

Area $=64-48=16$. Area between curves is 16 units².

11. (a)

t	0	5	10	15	20
$f(t)$	2	2	5	7	0



(TODD Simpson)

$$A=\frac{h}{3}[\text{first}+\text{last}+2\text{ odd}+4\text{ even}] \qquad (FLOE)$$
$$=\frac{h}{3}\left[f(0)+f(20)+2f(10)+4\left(f(5)+f(15)\right)\right]$$

$h=5-0$ $=5$

$$=\frac{5}{3}[2+0+2(5)+4(2+7)]$$
$$=\frac{5}{3}[48]$$
$$=80.$$

Area under the curve is 80 units².

(b) This area represents the distance travelled by the particle as $x=\int_0^{20} v\,dt$. Distance travelled in the first 20 seconds is

$$\int_0^{20} v\,dt=\int_0^{20} f(t)\,dt=80.$$

Distance travelled in the first 20 seconds is 80 metres.

12. Put $I = \int_0^1 \sin \pi x \, dx$, where $f(x) = \sin \pi x$.
Two subintervals are 0 – 0·5, 0·5 – 1.

Now $I = \frac{1}{2}(b-a)\left[f(a)+f(b)\right]$

x	0	0·5	1
$f(x)$	0	1	0

$I_1 = \frac{1}{2}(0\cdot5-0)\left[f(0)+f(0\cdot5)\right]$

$= 0\cdot25[0+1]$

$= 0\cdot25.$

$I_2 = \frac{1}{2}(1-0\cdot5)\left[f(0\cdot5)+f(1)\right]$

$= 0\cdot25[1+0]$

$= 0\cdot25.$

$\therefore \quad I = I_1 + I_2 = 0\cdot5 \quad \int_0^1 \sin \pi x \, dx = 0\cdot5.$

For actual value, $\int_0^1 \sin \pi x \, dx = \left[-\frac{1}{\pi}\cos \pi x\right]_0^1$

$= -\left[\frac{1}{\pi}\cos \pi - \frac{1}{\pi}\cos 0\right]$

$= -\left[\frac{1}{\pi}(-1) - \frac{1}{\pi}(1)\right]$

$= -\left[\frac{-2}{\pi}\right]$

$= \frac{2}{\pi}$

$= 0\cdot636\,619\,7$

$\approx 0\cdot637$ (3 dec. places).

Actual value is 0·637, which is 0·137 greater than the value found using the Trapezoidal Rule.