C.E.M. TUITION

TRIAL HSC EXAMINATION 1995

MATHEMATICS

2/3 UNIT

COMMON PAPER

Total time allowed - THREE hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES :

- Attempt only 10 questions out of the 12 questions set.
- All questions are of equal value.
- All necessary working should be shown in every question.
- Full marks may not be awarded for badly arranged work.
- Standard integral are on the inside cover.
- Approved silent calculators may be used.
- You must hand in a blank page if a question is unanswered.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Question 1 **Marks**

(a) Simplify $\dfrac{2x-2y-xy+y^2}{x^3-y^3}$ **3**

(b) Solve for x : **6**

(i) $|x+7| = 3x-1$ (ii) $x > 2(x+1) - \frac{x-1}{3}$

(c) Express $\dfrac{6}{4-\sqrt{13}}$ in the form $a+b\sqrt{13}$ **3**

Question 2

(a) Differentiate with respect to x : **6**

[No simplification of the derivatives is needed]

(i) $3e^{6x}$ (ii) $\tan \frac{x}{5}$

(iii) $x^4(6x+5)^9$ (iv) $\dfrac{x^2+5}{4x-3}$

(b) Find the equation of the tangent to the curve **3**

$y = \dfrac{3}{\sqrt{x}}$ at the point where $x = 1$.

(c) Find the values of x for which the curve **3**

$y = x^3 - 3x^2 - 24x + 2$ is decreasing.

Question 3 **Marks**

(a) Solve the simultaneous equations : **2**

$$3x - 2y = 8$$
$$5x - 6y = 16$$

(b) (i) On the number plane, plot the point $A(-2,2), B(2,-2), C(3,3)$ and the origin. **3**

(ii) Show that ΔABC is isosceles. **3**

(iii) Find the equation of the line BC. **2**

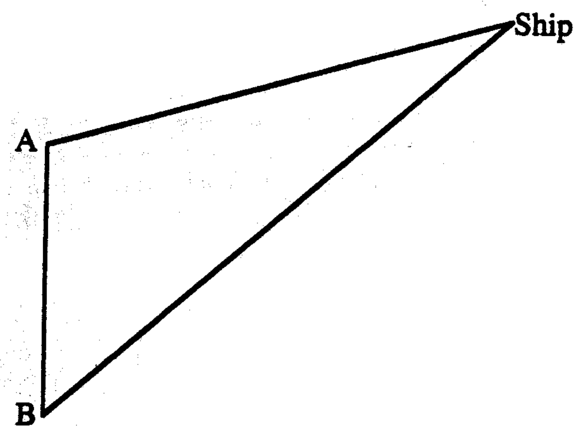
(iv) Find the perpendicular distance from point A to the line BC. **2**

Question 4

(a) Sketch the graph of $y = 3 - 2\sin \pi x$ for $0 \le x \le 2$. **5**

Show all relevant features.

(b) The bearing of a ship from a lighthouse, A is $075^0 T$ and its bearing from a second lighthouse, B 44km South of A is $040^0 T$.



(i) Copy the diagram and mark on it the information given. **1**

(ii) Find the distance of the ship from B. Give your answer to the nearest km. **3**

(iii) If the ship is sailing due south at a speed of 20 km/hr, how long will it take for it to be due east of B ? Give your answer to the nearest 10 minutes. **3**

Question 5 **Marks**

(a) If α and β are the roots of the equation $2x^2 - 3x - 4 = 0$, **4**

evaluate :

(i) $\alpha + \beta$ (ii) $\alpha\beta$ (iii) $\alpha^2 + \beta^2$

(b) If $2x^2 - 3x + 5 \equiv A(x+1)^2 + B(x+1) + C$, **3**

find A, B and C.

(c) In an arithmetic sequence, the fifth term is 26 and the ninth term is 2.

(i) Find the values of the first term and the common difference. **3**

(ii) Find the sum of the first 30 terms. **2**

Question 6

(a) Find $\int xe^{4x^2}\, dx$ **2**

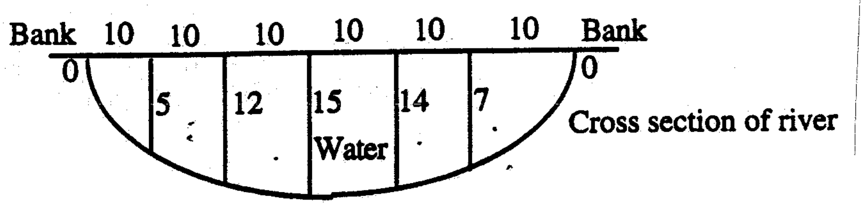
(b) Evaluate $\int_{\frac{\pi}{2}}^{\pi} \cos\left(\frac{x}{2}\right) dx$ **3**

(c) Find the value of k if $\int_1^k \frac{3}{x}\, dx = 3$ **3**

(d) Find $\frac{d}{dx}\left[\log_e\left(\frac{2+x}{x-3}\right)\right]$ and hence find $\int \frac{10\, dx}{(2+x)(x-3)}$ **4**

Question 7 **Marks**

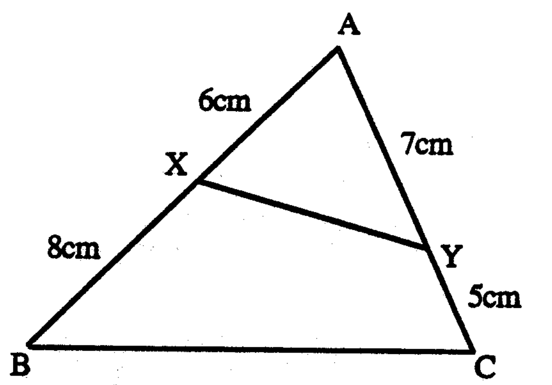
(a) A river 60 metres wide has its depth measured at 10 metres intervals across its width. The measurements from the bank to bank are given in the following diagram.



(i) Use Simpson's Rule to estimate the cross-sectional area of the river at this point . **4**

(ii) Hence find the volume of the water passing this point per second if water flows past at 5 m/s. Give answer to the nearest m^3. **1**

(b)



(i) Copy the diagram **1**

(ii) Prove that $\Delta AXY ||| \Delta ACB$. **2**

(iii) If $XY = 9$ cm, find the length of BC. **1**

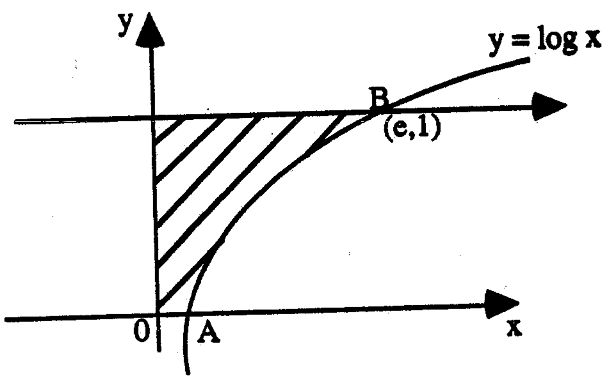
(c) A curve has a gradient function $\frac{dy}{dx} = 3x^2 + 6x$.

Given the curve passes through the point $(2, 5)$, find its equation. **3**

Question 8 **Marks**

(a) The shaded area in the diagram is rotated through 360^0 about the y–axis. **5**

Find the volume of the solid formed.



(b) On the birth of their first child, Jessica, Mr and Mrs Jones established a trust account for her. They deposited \$500 and this money was invested at 6% interest compounded annually.

(i) How much will this investment be worth after the interest is added, when Jessica turns 21 years old ? **1**

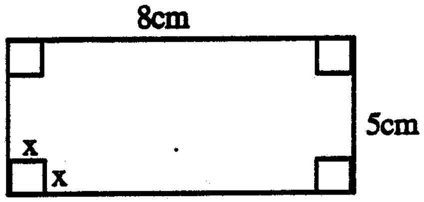
(ii) If the Joneses had decided to invest \$500, at the same compound interest rate, on each of Jessica's birthdays (starting at birth and continuing up to and including the 20th), what would be the value of the total investment on the day she turns 21 ? **4**

(c) State the domain and range of the function $y = \sqrt{2 - x}$. **2**

Question 9 **Marks**

(a) Find a number which, when added to each of 4, 10 and 18 gives three numbers in Geometric Progression. **3**

(b) A rectangle sheet of cardboard measures 8 cm by 5 cm. Small squares of side x cm are cut from each of the four corners of the sheet. The remaining sides are folded to form an open box.



(i) Show that the volume of the box is given $V = 4x^3 - 26x^2 + 40x$. **4**

(ii) Hence find the maximum volume of the box. **1**

(c) Solve for $0 \le x \le 2\pi$:

$$3\sec^2 x - 4 = 0$$ **4**

Question 10

(a) The velocity of a particle after t sec is $(64 - \frac{t^2}{3})$ m/s.

Find : (i) its initial velocity. **1**

(ii) the time at which the particle reverses its direction of motion. **1**

(iii) its distance from the starting point at that instant. **2**

(iv) the distance travelled in the first 10 seconds. **2**

(b) Find the possible values of k if $y = e^{-kx}$ satisfies the equation **4**

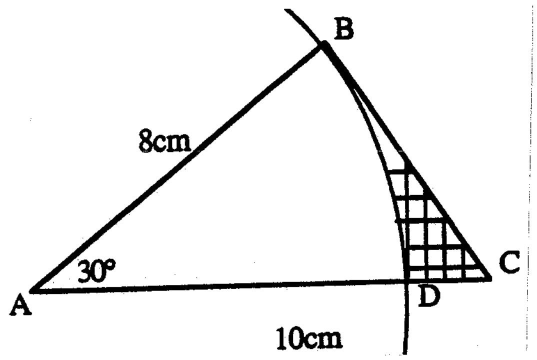
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 35y = 0.$$

Question 10 continues next page

Question 10 (continued) **Marks**

(c) ABC is a triangle with $\angle BAC = 30^0$, $AB = 8$ cm and $AC = 10$ cm. **2**

An arc is drawn through D, with centre A and radius 8 cm. D lies on AC.



Show that the shaded area BDC is given by $\left(20 - \frac{16\pi}{3} \right)$ cm^3.

Question 11

(a) Given that $2\log_n 2 + \log_n 25 = 2$, find the value of n. **2**

(b) (i) Sketch the graph of $y = e^x$. **1**

(ii) Find the equation of the tangent to $y = e^x$ at the point where the curve cuts the y–axis. **2**

(iii) Use the graph to show that the equation $e^x = x + 1$ has exactly one solution. **1**

(iv) Consider the equation $e^x = x + b$, where b is a real number. For what values of b does the equation have exactly two solutions ? **1**

(c) Three coins are identical in appearance. However two are "fair", and the third is "biased", so that the probability of tossing a head with it is $\frac{4}{7}$. With the aid of a tree diagram or otherwise, answer the following : **5**

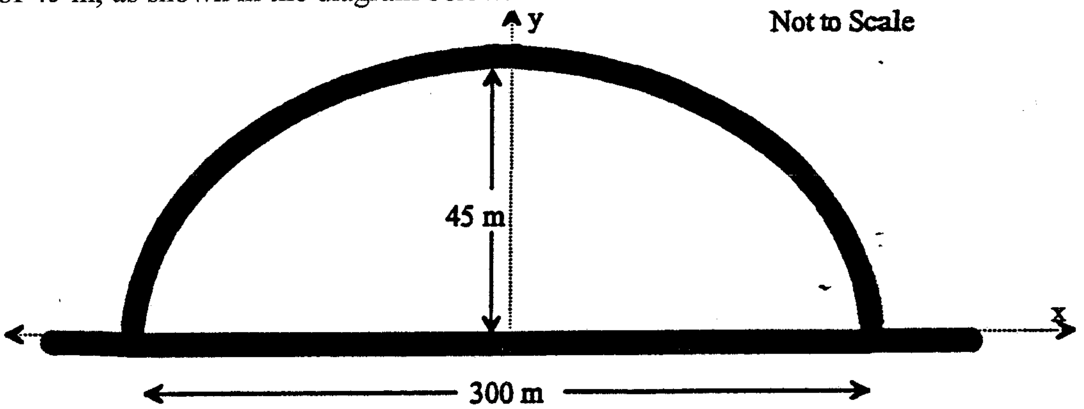
(i) If a coin is chosen at random, what is the probability that it is biased ?

(ii) If a coin is chosen at random and tossed once, what is the probability that a head is obtained ?

(iii) If a coin is chosen at random and tossed twice, what is the probability of obtaining at least one head ?

Question 12 **Marks**

(a) Some concrete bridges are supported by an overhead arch in the shape of a parabola. The arch of one such bridge has a span of 300 m and a clearance of 45 m, as shown in the diagram below. **5**



(i) By considering axes which have their origin at the centre of the bridge show that the arch can be represented by the equation

$$x^2 = 22\,500 - 500y$$

(ii) Give the coordinates of the ***vertex*** of this parabola.

(iii) Find the coordinates of the ***focus***, and the equation of the ***directrix***.

(b) A tank is to be emptied by means of a control valve. The valve operates so that V litres, the volume of fluid remaining in the tank, varies with time t, measured in minutes, according to the relation **4**

$$\frac{dV}{dt} = -kt, \text{ where } k \text{ is a constant.}$$

(i) Initially the tank contains 5000 litres of fluid. Show that after t minutes

$$V = 5000 - \frac{1}{2}kt^2$$

(ii) If $k = 1.44$, at what rate will the tank be emptying when $V = 2000$?

(iii) Find the time it takes to completely empty the tank ? (Give your answer to the nearest minute).

(c) Solve $2\sin^2 x - \sin x - 1 = 0$ for $0 \le x \le \pi$. **3**

[1][a] $\frac{2(x-y)-y(x-y)}{x^3-y^3}$ ....1 m

$= \frac{(2-y)(x-y)}{(x-y)(x^2+xy+y^2)}$ ....1 m

$= \frac{2-y}{x^2+xy+y^2}$ ....1 m

[b][i] Since $3x-1 \geq 0$

$x \geq \frac{1}{3}$ (Boundary Conditions)

Case 1 : $x+7=3x-1$
$2x=8$
$x=4$ ....1 m

Case 2 : $-(x+7)=3x-1$
$4x=-6$
$x=-\frac{3}{2}$ ....1 m

Check with boundary conditions
$x=4$ is the only solution.......1 m

[ii] $3x > 6(x+1)-(x-1)$ ....1 m
$3x > 6x+6-x+1$
$3x > 5x+7$ ....1 m
$-7 > 2x$
$x < -\frac{7}{2}$ ....1 m

[c] $\frac{6}{4-\sqrt{13}} \times \frac{4+\sqrt{13}}{4+\sqrt{13}}$ ....1 m

$= \frac{6(4+\sqrt{13})}{16-13}$ ....1 m

$= 8+2\sqrt{13} \equiv a+b\sqrt{13}$ .....1 m

Therefore $a=8, b=2$

[2][a][i] $18e^{6x}$........................ 1 m

[ii] $\frac{1}{5}\sec^2\frac{x}{5}$ .... 1 m

[iii] Use the product rule

$= x^4.9(6x+5)^8.6+(6x+5)^9.4x^3$

$= 54x^4(6x+5)^8+4x^3(6x+5)^9$

$= 2x^3(39x+10)(6x+5)^8$ ..... 2 m

[iv] Use the quotient rule

$= \frac{(4x-3)2x-(x^2+5).4}{(4x-3)^2}$

$= \frac{4x^2-6x-20}{(4x-3)^2}$ .... 2 m

[b] $\frac{dy}{dx} = -3.\frac{1}{2}x^{-\frac{3}{2}}$ .... 1 m

$= \frac{-3}{2x\sqrt{x}}$

When $x=1, y=3$ and $\frac{dy}{dx}=-\frac{3}{2}$
(1m)

Equation of the tangent is

$y-3=-\frac{3}{2}(x-1)$
$2y-6=-3x+3$
$3x+2y-9=0$.................... 1 m

[c] $\frac{dy}{dx} = 3x^2-6x-24$ .... 1 m

Curve is decreasing when $y' < 0$

i.e. $3x^2-6x-24<0$

$x^2-2x-8<0$

$(x-4)(x+2)<0$ .... 1 m

Test : $x=0$ i.e $-8<0$ is true

Therefore $-2<x<4$ .... 1 m

[3][a] $9x-6y=24$ .... (i)
$5x-6y=16$ .... (ii)

(i) - (ii) $4x=8$
$x=2$ .... 1 m

Sub. in (ii) , $y=-1$ .... 1 m

Coordinates are $(2,-1)$

[b] [i] 1 m for labelling axes
0.5 m for each correct point

[ii] $AC=\sqrt{(-2-3)^2+(2-3)^2}$
$=\sqrt{25+1}=\sqrt{26}$ .... 1 m

$BC=\sqrt{(2-3)^2+(-2-3)^2}$
$=\sqrt{26}$ .... 1 m

Therefore, ΔABC is isosceles.

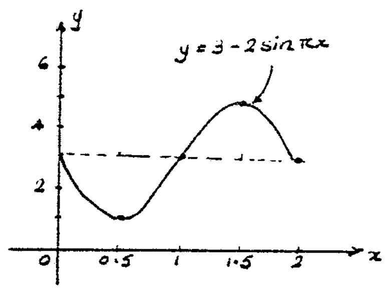
[iii] Grad. of $BC=\frac{3+2}{3-2}=5$ .... 1 m
Equation of BC is
$y+2=5(x-2)$
$5x-y-12=0$ .... 1 m

[iv] $d=\frac{|5(-2)-1.2-12|}{\sqrt{5^2+1^2}}$ .... 1 m

$= \frac{|-24|}{26}$

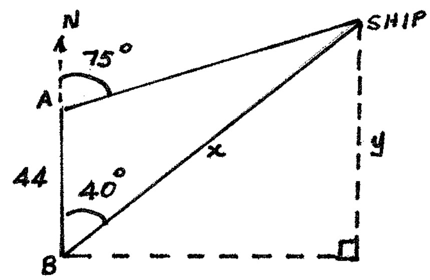
$= \frac{12\sqrt{26}}{13}$ units .... 1 m

[4] [a] Period $=\frac{2\pi}{\pi}=2$ rad.
Amplitude = 2



(5 m)

[b] [i]



1 m

[ii] Let x be the distance from the ship to B. Using Sine Rule :

$\frac{x}{\sin 105^0} = \frac{44}{\sin 35^0}$................. 1 m

$x=\frac{44\sin 105^0}{\sin 35^0}$ .... 1 m

≈ 74.09777261................. 1 m
= 74 km (to the nearest km)

[iii] Let the distance travelled south by y.

Therefore, $\sin 50^0=\frac{y}{x}$ .... 1 m

$y = x\sin 50^0 \approx 56.76218696$

Now, Speed $= \frac{\text{Distance}}{\text{Time}}$

Therefore, Time $= \frac{\text{Distance}}{\text{Speed}} = \frac{y}{20}$

$= 2.838109348$ ..... 1 m

$= 2$ h 50 min (to the nearest min) (1 m)

[5][a] [i] $\alpha + \beta = -\frac{b}{a} = \frac{3}{2}$ ..... 1 m

[ii] $\alpha\beta = \frac{c}{a} = \frac{-4}{2} = -2$ .... 1 m

[iii] $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (1.5)^2 - 2(-2)$

$= 6.25$ .... 2 m

[b] Expand R.H.S.

$= A(x^2 + 2x + 1) + Bx + B + C$

$= Ax^2 + (2A + B)x + (A + B + C)$

$\equiv 2x^2 - 3x + 5$ .... 1 m

Therefore, $A = 2$

$2A + B = -3 \Rightarrow B = -7$........... 1 m

$A + B + C = 5 \Rightarrow C = 10$ .... 1 m

[c][i] A.P. $T_n = a + (n-1)d$

$T_5 = a + 4d = 26$ ....(i)

$T_9 = a + 8d = 2$ ....(ii) 1 m

(ii) - (i) $4d = -24$

$d = -6$ .... 1 m

Sub. into (i) $a = 50$ .... 1 m

[ii] $S_n = \frac{n}{2}[2a + (n-1)d]$ ...... 1 m

$S_{30} = 15[100 + 29(-6)]$

$= -1110$ .... 1 m

[6][a] $\frac{1}{8}\int xe^{4x^2}\,dx$

$= \frac{1}{8}e^{4x^2} + c$ .... 2 m

[b] $2\left[\sin\frac{x}{2}\right]_{\frac{\pi}{2}}^{\pi}$ .... 1 m

$= 2[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}]$ .... 1 m

$= 2[1 - \frac{1}{\sqrt{2}}]$ .... 1 m

[c] $3\int_1^k \frac{1}{x}\,dx = 3$

$\left[\ln x\right]_1^k = 1$ .... 1 m

$[\ln k - \ln 1] = 1$

$\ln k - 0 = 1$ .... 1 m

$k = e^1$ .... 1 m

[d] $[\ln(2+x) - \ln(x-3)]$ .... 0.5 m

using $\ln(\frac{a}{b}) = \ln a = \ln b$

$\frac{dy}{dx} = \frac{1}{2+x} - \frac{1}{x-3}$ .... 1 m

$= \frac{x-3-2-x}{(2+x)(x-3)}$ .... 0.5 m

$= \frac{-5}{(2+x)(x-3)}$

Therefore,

$I = \int \frac{10dx}{(2+x)(x-3)}$

$= -2\int \frac{-5\,dx}{(2+x)(x-3)}$ .... 1 m

$= 2\ln\left(\frac{2+x}{x-3}\right) + c$ .... 1 m

[7][a] $h = 10, y_F = 0, y_L = 0$

$A \approx \frac{h}{3}[(y_F + y_L) + 4(\text{Sum of Odds} + 2(\text{Sum of Evens})]$...... 1 m

$\approx \frac{10}{3}[0 + 0 + 4(5 + 15 + 7) + 2(12 + 14)]$ .... 1 m

$= 533\frac{1}{3}$ m^2

[ii] V = Cross-sectional area x depth

$V = 533\frac{1}{3} \times 5$

$= 2666\frac{2}{3}$ m^3

[b][i] Clear and tidy diagram(1 m)

[ii] Proof : $\angle A$ is common.

$\frac{AX}{AC} = \frac{6}{12} = \frac{1}{2}$

$\frac{AY}{AB} = \frac{7}{14} = \frac{AX}{AC}$

Therefore, $\Delta AXY \,|||\, \Delta ABC$

(Two sides in the same ratio and the included angle equal)... 2 m

[iii] $\frac{BC}{XY} = \frac{2}{1}$

$BC = 18$ .... 1 m

[c] $\frac{dy}{dx} = 3x^2 + 6x$

$y = x^3 + 3x^2 + c$ .... 1 m

$5 = 8 + 12 + c$

$c = -15$ .... 1 m

$y = x^3 + 3x^2 - 15$........... 1 m

[8][a] $V = \int \pi x^2 dy$

If $y = \ln x$, then $x = e^y$ 1m

$V = \int_0^1 (e^y)^2\,dy$................. 1 m

$= \pi\int_0^1 (e^{2y})\,dy$

$= \frac{\pi}{2}[e^{2y}]_0^1$ .... 1 m

$= \frac{\pi}{2}[e^2 - 1]$ ....1 m

$= 10.04$ (to 2 d.p.) .... 1 m

[b] [i] $A = PR^n$

$= 500(1.06)^{21}$

$= \$1699.78$ ....1 m

[ii] $A_{21} = 500(1.06)^{21}$

$A_{20} = 500(1.06)^{20}$

......

$A_1 = 500(1.06)^1$ .... 1 m

Total amount at age 21 is

$A_1 + A_2 + ... + A_{20} + A_{21}$

$= 500[1.06 + (1.06)^2 + ... + (1.06)^{21}$

$= 500 \times \frac{1.06(1.06^{21}-1)}{1.06-1}$

$= \$21\,196.15$ .... 3 m

[c] $D: \{x \le 2\}$ .... 1 m

$R: \{y \ge 0\}$ .... 1 m

[9][a] Let x be the number

$4 + x, 10 + x, 18 + x$ is in G.P.

$\frac{10+x}{4+x} = \frac{18+x}{10+x}$ .... 1 m

$(18 + x)(4 + x) = (10 + x)^2$

$72 + 22x + x^2 = 100 + 20x + x^2$ .1 m

$2x = 28$

$x = 14$.................................. 1 m

[b][i] $V = L \times B \times H$
$= (8-2x)(5-2x).x$
$= (40-26x+4x^2).x$
$= 4x^3 - 26x^2 + 40x$...... 1m

[ii] $V' = 12x^2 - 52x + 40$
$V'' = 24x - 52$.................. 1m
Max. occurs when $V' = 0$
$12x^2 - 52x + 40 = 0$
$4(3x-10)(x-1) = 0$
$x = 1$ or $\frac{10}{3}$............................ 1m
Since $x \neq \frac{10}{3}$ (unrealistic value),
then $x = 1$, when$x = 1$, $V'' < 0$(1m)
therefore maximum.
$V = 4 - 26 + 40 = 18$ cm^3........ 1m

[c] $3\sec^2 x - 4 = 0$
$\sec^2 x = \frac{4}{3}$
$\sec x = \pm\sqrt{\frac{4}{3}}$ 1m
$\cos x = \pm\frac{\sqrt{3}}{2}$................ 1m
$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ 2m

[10][i] When $t = 0, v = 64$ m/s(1m)

[ii] Particle comes to rest when
$v = 0$, i.e. $t = 8\sqrt{3}$ 1m

[iii] $x = \int_0^{8\sqrt{3}} 64 - \frac{t^2}{3}\, dt$

$= \left[64t - \frac{t^3}{9}\right]_0^{8\sqrt{3}}$ 1m

$= \frac{1024\sqrt{3}}{9} \approx 591.21$

$x = \int_0^{10} 64 - \frac{t^2}{3}\, dt$

$= \frac{4760}{9}$ 1m

Total distance travelled is
$1024\sqrt{3} + \frac{1024\sqrt{3}}{9} - \frac{4760}{9}$
$= 653.52$ (to 2 d.p.) 1m

[b] $y = e^{-kx}$

$\frac{dy}{dx} = -ke^{-kx}$

$y'' = k^2 e^{-kx}$ 1m

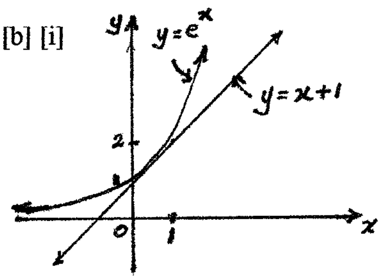
Therefore,
$k^2 e^{-kx} + 2(-ke^{-kt}) - 35e^{-kx} = 0$
$e^{-kx}(k^2 - 2k - 35) = 0$...... .1m
$(k-7)(k+5) = 0$ 1m
$k = 7$ or -5 .... 1m

[c] Area of BDC = Area ABC - Area ABD
$A = \frac{1}{2} \times 8 \times 10 \times \sin 30^0 -$
$\frac{1}{2} \times 8^2 \times \frac{\pi}{6}$................. 2m

$= 20 - \frac{16\pi}{3}$

[11][a] $\log_n 2^2 + \log_n 25 = 2$
$\log_n(4 \times 25) = 2$......... 1m
Therefore, $100 = n^2$
$n = 10$ 1m

[b] [i]



[ii] $m = y' = e^x$
At $x = 0$, $m = e^0 = 1, y = 1$...... 1m
Equation of the tangent is
$y - 1 = 1(x - 0)$
$x - y + 1 = 0$ 1m

[iii] Refer to the above graph

[iv] If the y–intercept is greater than zero than the line will cut the curve at two points, $b > 0$ 1m

[c][i] Let P(B) = Probability of choosing a biased coin = $\frac{1}{3}$ 1m
P(B_H) = Probability of tossing head on a biased coin
$= \frac{4}{7}$
P(F) = Probability of choosing a fair coin = $\frac{2}{3}$

[ii] P(FH) + P(BH) 1m
$= \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{4}{7}$
$= \frac{11}{21}$ 1m

[iii]
3×P(FHT) + 2×P(BHT) + P(BHH)
$= 3 \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} + 2 \times \frac{1}{3} \times \frac{4}{7} \times \frac{3}{7}$
$+ \frac{1}{3} \times \frac{4}{7} \times \frac{4}{7}$ 1m

$= \frac{227}{294}$.............................. 1m

[12][a][i] Using the equation of the parabola
$(x-h)^2 = -4a(y-k)$
where $(h,k) = (0,45)$
$(x-0)^2 = -4a(y-45)$ but the parabola passes through $(150,0)$
Therefore,
$(150-0)^2 = -4a(0-45)$
$22500 = 180a$
$a = 125$
The equation is
$x^2 = -500(y-45)$
$x^2 = -500y + 22500$ 2m

[ii] Vertex is (0,45) 1m

[iii] Focus is (0,-80)................ 1m
Directrix is $y = 170$ 1m

[b][i] $V = 5000 - \frac{1}{2}kt^2$
$\frac{dV}{dt} = -\frac{1}{2} \times 2kt$
$= -kt$ 1m

[ii] $2000 = 5000 - 0.5 \times 1.44 \times t^2$
$0.72t^2 = 5000 - 2000$
$t^2 = \frac{3000}{0.72}$
$t = 4166\frac{2}{3}$ mins 1m
$\frac{dV}{dt} = -1.44 \times 4166\frac{2}{3}$
$= -6000$ litres/min 1m

[iii] When $V = 0$, $t^2 = \frac{5000 \times 2}{1.44}$
$t = 6944.\dot{4}$ mins 1m

[d] $(2\sin x + 1)(\sin x - 1) = 0$ .. 1m
$2\sin x = -1$ or $\sin x = 1$
$\sin x = -0.5$ or $x = \frac{\pi}{2}$ 1m
(Note $0 \le x \le \pi$)
Therefore, the only solution is
$x = \frac{\pi}{2}$ 1m