

C.E.M. TUITION

FINAL TRIAL HSC EXAMINATION 1998

MATHEMATICS

2/3 UNIT COMMON PAPER

Total time allowed - THREE hours

(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES :

- Attempt ALL questions.
- All questions are of equal value.
- All necessary working should be shown in every question.
- Full marks may not be awarded for badly arranged work.
- Standard integrals are on the inside cover.
- Approved silent calculators may be used.
- You must hand in a blank page if a question is unanswered.

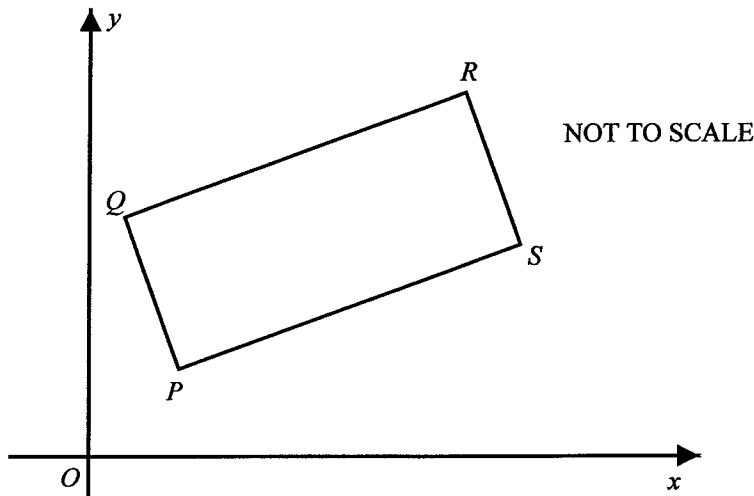
Question 1	Marks
(a) Find the value of $\pi \sqrt{\frac{l}{2g}}$ if $l = 3\sqrt{2}$, $g = 9.8$. Give your answer correct to two significant figures	2
(b) Find the exact value of $\cos 135^\circ$.	1
(c) Express as a single fraction : $x + 2 - \frac{x-1}{3}$	2
(d) Solve for r if $(r+5)^2 - (r-5)^2 = 12$	2
(e) Express $\frac{\sqrt{2}}{\sqrt{19-3\sqrt{2}}}$ in the form $a + \sqrt{b}$	3
(f) Find the x and y intercepts of the line $\frac{x}{3} - \frac{y}{6} = 1$	2

Continue next page

Question 2

Marks

(a)



In the rectangle $PQRS$, P and Q are the points $(4, 2)$ and $(2, 8)$ respectively. Given that the equation of PR is $y = x - 2$, show that

- (i) the equation of QR is $x - 3y + 22 = 0$. 3
- (ii) the coordinates of R is $(14, 12)$ 2
- (iii) the coordinates of S is $(16, 6)$ 2
- (iv) the area of the rectangle $PQRS$ is 80 units^2 3
- (b) If $P(x, y)$ is a point which moves in such a way that it is always a fixed distance PT away from the fixed point T , the midpoint of QS , write down the equation of the locus of P . 2

Continue next page

Question 3

Marks

(a) Differentiate the following functions :

6

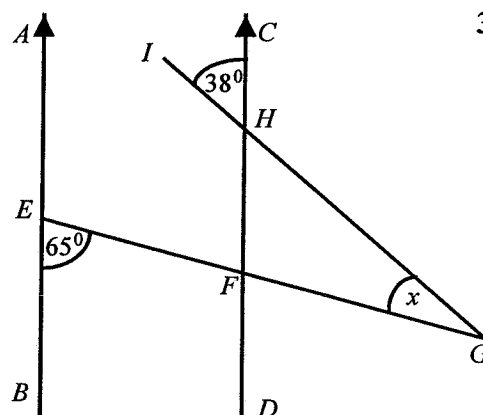
(i) $(2x^3 - 5)^7$

(ii) $\sin \sqrt{x}$

(iii) $e^{2x} + \ln 2x$

(b) Given that $AB \parallel CD$, and that

$\angle BEF = 65^\circ, \angle CHI = 38^\circ,$

find the value of x , giving reasons.

3

(c) Find :

3

(i) $\int (5x - 4)^{10} dx$

(ii) $\int_0^{\frac{\pi}{2}} (\cos x + x) dx$

Continue next page

Question 4

Marks

- (a) The table shows the values of a function $y = (\log_e x)^2$ for five values of x . Fill in the missing values of y . 3

x	1	1.25	1.5	1.75	2
y	0.0000	0.0498			0.4805

Use Simpson's rule with these five function values to estimate $\int_1^2 (\log_e x)^2 dx$.
Answer correct to three significant figures.

- (b) Prove the identity $(\sin x + \cos x)(1 - \sin x \cos x) = \cos^3 x + \sin^3 x$ 3
- (c) The quadratic equation $x^2 + lx + m = 0$ has roots -2 and 6 . 4
Find :
- (i) the value of l and m .
- (ii) the range of values of n for which the equation $x^2 + lx + m = n$ has real roots.
- (d) After winning the game in Sale of the Century, Barry Jones has a chance to pick a prize from a board with 16 squares as shown below; 2

1	TRIP	3	4
5	6	7	8
9	10	BMW	12
13	14	15	16

To win the prize Barry must choose a matching pair and no two prizes are identical. He gets to choose again if there is no match. Assuming that Barry has already chosen boxes 2 and 11.

What is the probability that Barry :

- (i) wins the TRIP on his third pick ?
- (ii) wins the BMW on his fourth pick ?

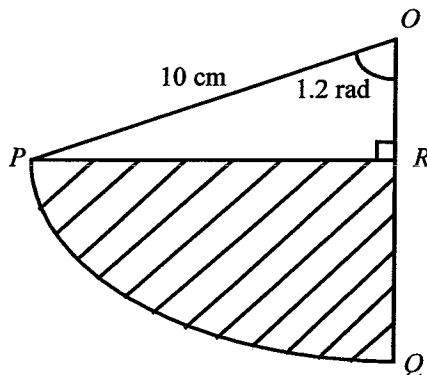
Continue next page

Question 5

Marks

(a)

8



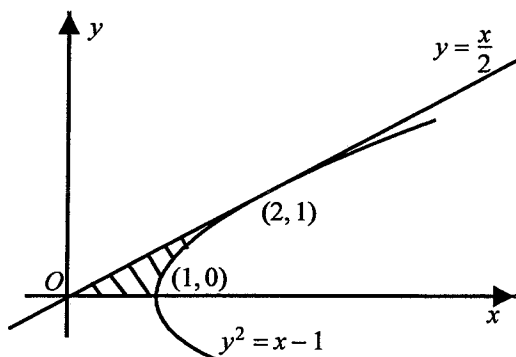
OPQ is a sector of the circle, centre O , radius 10 cm and $\angle POQ = 1.2$ radians.

Given that $PR \perp OQ$;

- (i) show that $QR = 6.38$ cm.
- (ii) calculate the perimeter of the shaded region.
- (iii) calculate the area of the shaded region.

(b)

4



The line $y = \frac{x}{2}$ is the tangent the curve $y^2 = x - 1$ at the point $(2, 1)$.

Calculate the volume formed by rotating the shaded area about the x -axis.

Continue next page

Question 6

Marks

- (a) In a certain arithmetic progression, the first term is a and the common difference is d . Given that the sum of the first four terms is equal to three times the fourth term,

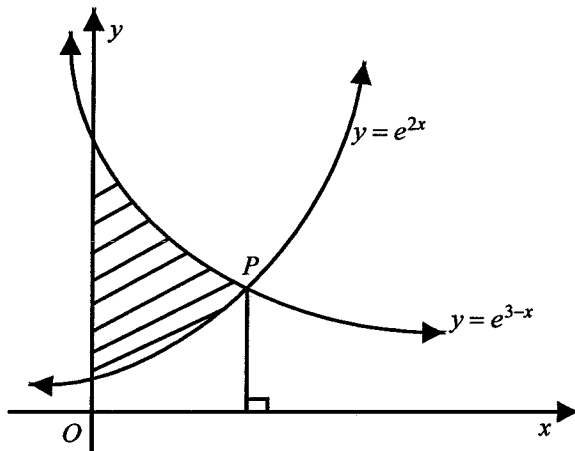
6

(i) show that $a = 3d$.

Given also that the eighteenth term is 60, calculate

- (ii) the value of a and d .
 (iii) the sum of the first eighteen terms.

(b)



4

In the diagram, P is the point of intersection of the curves $y = e^{2x}$ and $y = e^{3-x}$.

Find the x -coordinate of P and hence find, to two decimal places, the area of the region under the curve and the y -axis.

- (c) Sketch the parabola given by the equation $(x - 1)^2 = 12(y - 2)$

2

showing the coordinates of the vertex and focus.

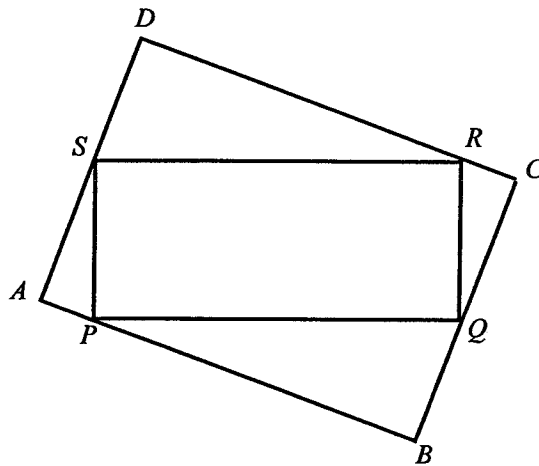
Continue next page

Question 7

Marks

(a)

7

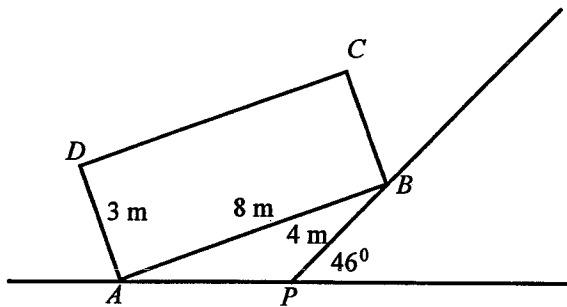


$PQRS$ is an inscribed rectangle inside a second rectangle $ABCD$ with sides AB, BC, CD, DA passing through vertices P, Q, R, S respectively.

- (i) Copy this diagram into your exam booklet.
- (ii) Prove that $\triangle PBQ$ is similar to $\triangle QCR$
- (iii) Hence or otherwise, show that $PQ \cdot CR = BQ \cdot QR$
- (iv) If $PQRS$ is a square, prove that $ABCD$ is also a square.

(ii)

5



A rectangular crate measuring 3 m by 8 m rests on a ramp inclined at 46° to the horizontal. $PB = 4$ m.

- (a) Using Sine rule, find $\angle BAP$ to the nearest degree.
- (b) Hence find the distance D is above the horizontal.

Continue next page

Question 8

Marks

- (a) Consider the curve given by $y = 6x^2 - 9x - x^3 - 1$. 8
- (i) Find the coordinates of the two stationary points.
 - (ii) Find the coordinates of any point(s) of inflexion.
 - (iii) Determine the nature of the stationary points.
 - (iv) Sketch the curve for the domain $0 \leq x \leq 4$.
- (b) The number of cryptosporidium, $N(t)$, in a water supply at a time t seconds 4
is given by the equation :

$$N(t) = 10^6 e^{0.02t}$$

- (i) Show that $\frac{dN(t)}{dt} = 0.02N(t)$.
- (ii) Determine how long it will take for the number to **double**.
- (iii) At what rate is the number of cryptosporidium increasing when $t = 30$ sec.

Continue next page

Question 9

Marks

- (a) A particle starts from rest at O and moves in a straight line so that, t seconds later, its velocity v m/s is given by $v = 2t - t^2$. 8

Given that the particle comes instantaneously to rest at P , find

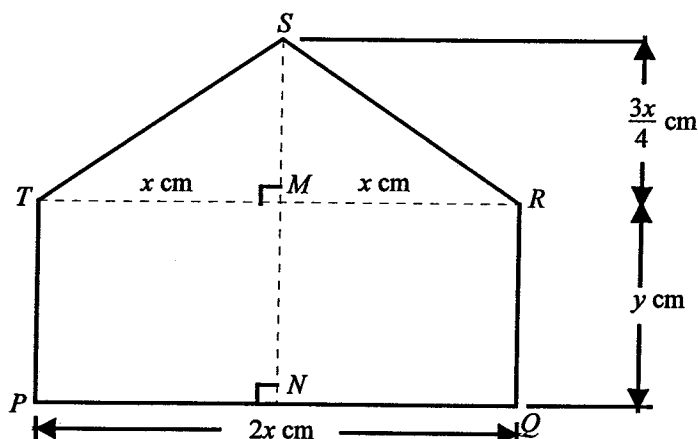
- (i) the time taken to reach P
 - (ii) the acceleration of P
 - (iii) the distance OP
 - (iv) the total distance travelled by the particle during the first four seconds of the motion.
- (b) When Shannon entered University, her parents borrowed \$40 000 to pay for her education. They plan to repay the loan by making 60 equal monthly repayments. Interest is charged at the rate of 7.5% per annum on the balance owing. 4
- (i) Show that immediately after making two monthly repayments of $\$M$, the balance is given by:
$$$(40\,501.56 - 2.00625M)$$
 - (ii) Calculate the value of each monthly repayment.

Continue next page

Question 10

Marks

(a)



The diagram shows a piece of cardboard consisting of a rectangle $PQRT$ and an isosceles triangle RST , where $PQ = 2x$ cm, $QR = y$ cm and $SM = \frac{3x}{4}$ cm. Given that the perimeter of $PQRST$ is 30 cm,

(i) show that the area, A cm², of $PQRST$ is given by 3

$$A = 30x - \frac{15x^2}{4}.$$

(ii) the maximum of A . 2

(iii) the length of the perpendicular SN from S to PQ when A is a maximum 2

(b) Calculate the value of the ratio $\frac{36 - 18 + 9 - \dots}{36 + 12 + 4 + \dots}$ as a simple fraction 2

if both the numerator and denominator are infinite series.

(c) Find the equation of the tangent to the curve $y = \log_e \left(\frac{x}{x-1} \right)$ at $x = 2$. 3

$$(1)(a) \pi \sqrt{\frac{3\sqrt{2}}{9.8}} = 1.5 \text{ (to 2 s.f.)} \quad \checkmark \checkmark$$

$$(b) \cos 135^\circ = -\frac{1}{\sqrt{2}} \quad \checkmark \checkmark$$

$$(c) x+2 - \frac{x-1}{3} = \frac{3(x+2) - (x-1)}{3} \quad \checkmark$$

$$= \frac{3x+6-x+1}{3} = \frac{2x+7}{3} \quad \checkmark$$

$$(d) (r+5)^2 - (r-5)^2 = 12$$

$$r^2 + 10r + 25 - r^2 + 10r - 25 \quad \checkmark$$

$$20r = 12$$

$$r = \frac{3}{5} \quad \checkmark$$

$$(e) \frac{\sqrt{2}}{\sqrt{19} - 3\sqrt{2}} \times \frac{\sqrt{19} + 3\sqrt{2}}{\sqrt{19} + 3\sqrt{2}} \quad \checkmark$$

$$\frac{\sqrt{38} + 6}{19 - 18} = 6 + \sqrt{38} \quad \checkmark \checkmark$$

$$(f) \text{ Let } x=0, y=-6 \quad \checkmark$$

$$\text{Let } y=0, x=3 \quad \checkmark$$

$$(2) (a) (i) m_{OP} \times m_{QR} = -1$$

$$m_{OP} = \frac{8-2}{2-4} = -3$$

$$m_{QR} = \frac{1}{3} \quad \checkmark$$

$$y - y_1 = m(x - x_1) \quad \checkmark$$

$$y - 8 = \frac{1}{3}(x - 2)$$

$$3y - 24 = x - 2 \quad \checkmark$$

$$x - 3y + 22 = 0$$

(ii) Solve simultaneously:

$$x - 3y + 22 = 0$$

$$x - y - 2 = 0$$

$$-2y + 24 = 0$$

$$y = 12 \quad \checkmark$$

Substituting $y = 12$

$$12 = x - 2$$

$$x = 14 \quad \checkmark$$

(iii) Find the midpoint of PR i.e.

$$\left(\frac{4+14}{2}, \frac{2+12}{2}\right) = (9, 7) \quad \checkmark$$

Let the coordinates of S be (a, b) ,

using midpoint formula of QS again

$$\left(\frac{a+2}{2}, \frac{b+8}{2}\right) = (9, 7)$$

$$a+2 = 18 \Rightarrow a = 16$$

$$b+8 = 14 \Rightarrow b = 6 \quad \checkmark$$

Coordinates of S is $(16, 6)$

$$(iv) PQ = \sqrt{(4-2)^2 + (2-8)^2} = \sqrt{40} \quad \checkmark$$

$$PS = \sqrt{(4-16)^2 + (2-6)^2} = \sqrt{160} \quad \checkmark$$

$$\text{Area} = PQ \times PS$$

$$= \sqrt{40} \times \sqrt{160} = 80 \text{ sq. units} \quad \checkmark$$

(b) Midpoint of QS is $(9, 7)$

$$PT = \sqrt{(9-4)^2 + (7-2)^2} = \sqrt{50} \quad \checkmark$$

Therefore the equation of the locus of P is

$$(x-9)^2 + (y-7)^2 = 50 \quad \checkmark$$

$$(3)(a)(i) \frac{dy}{dx} = 7(2x^3 - 5)^6 \times 6x^2 \checkmark$$

$$= 42x^2(2x^3 - 5)^6 \checkmark$$

$$(ii) \frac{dy}{dx} = (\cos \sqrt{x}) \left(\frac{1}{2} x^{-\frac{1}{2}} \right) \checkmark$$

$$= \frac{1}{2\sqrt{x}} \cos \sqrt{x} \checkmark$$

$$(iii) \frac{dy}{dx} = 2e^{2x} + \frac{2}{2x} \checkmark$$

$$= 2e^{2x} + \frac{1}{x} \checkmark$$

$$(b) \angle FHG = 38^\circ \text{ (vert. opp. angles)} \checkmark$$

$$\angle DEF = 180^\circ - 65^\circ = 115^\circ$$

(Cointerior angles are supplementary) \checkmark

$$\angle GFH = 115^\circ \text{ (Vert. opp. angles)}$$

$$x = 27^\circ \text{ (Angle sum of triangle)} \checkmark$$

$$(c) (i) \frac{(5x-4)^{11}}{11 \times 5} + c$$

$$= \frac{(5x-4)^{11}}{55} + c \checkmark$$

$$(ii) \left[\sin x + \frac{x^2}{2} \right]_0^{\frac{\pi}{2}} \checkmark$$

$$= \left[1 + \frac{\left(\frac{\pi}{2} \right)^2}{2} \right] - 0$$

$$= 1 + \frac{\pi^2}{8} \checkmark$$

(4) (a)

1.5	1.75
0.1644	0.3132

Using Simpson's rule:

$$\text{Area} \approx \frac{h}{3} [y_1 + y_5 + 4(y_2 + y_4) + 2y_3] \checkmark$$

$$= \frac{0.25}{3} [0 + 0.4805 + 4(0.0498 + 0.1644) + 2(0.3132)]$$

$$\text{Area} \approx 0.188 \checkmark$$

(b)

$$\text{LHS} = \sin x - \sin^2 x \cos x + \cos x - \sin x \cos^2 x$$

$$= \sin x - \sin x \cos^2 x + \cos x - \sin^2 x \cos x \checkmark$$

$$= \sin x(1 - \cos^2 x) + \cos x(1 - \sin^2 x) \checkmark$$

$$= \sin x(\sin^2 x) + \cos x(\cos^2 x) \checkmark$$

$$= \sin^3 x + \cos^3 x = \text{RHS.}$$

$$(c)(i) P(-2) = 0$$

$$4 - 2l + m = 0$$

$$2l - m = 4 \dots\dots (i)$$

$$P(6) = 0$$

$$36 + 6l + m = 0$$

$$6l + m = -36 \dots\dots (ii) \checkmark$$

Solving simultaneously (i) and (ii)

$$l = -4 \text{ and } m = -12 \checkmark$$

(ii) For real roots of the equation:

$$x^2 - 4x - 12 - n = 0$$

$$\Delta \geq 0$$

$$\Delta = b^2 - 4ac = (-4)^2 - 4(1)(-12 - n) \checkmark$$

$$16 + 48 + 4n \geq 0$$

$$n \geq -16 \checkmark$$

$$(d) (i) \frac{1}{14} \checkmark \quad (ii) \frac{12}{14} \times \frac{1}{13} = \frac{6}{91} \checkmark$$

$$(5) (a) (i) \cos 1.2^\circ = \frac{OR}{10}$$

$$OR = 10 \times 0.3623577 = 3.623577 \checkmark$$

$$QR = 10 - OR = 6.376423 \approx 6.38 \text{ cm } \checkmark$$

$$(ii) \text{ Perimeter} = PR + QR + \text{Arc } QP$$

$$\text{Arc } QP = 10 \times 1.2 = 12 \text{ cm } \checkmark$$

$$PR = 10 \sin 1.2^\circ = 9.3203909 \text{ cm } \checkmark$$

$$\text{Perimeter} = 9.320.. + 6.376.. + 12$$

$$P = 27.7 \text{ cm (to 1 d.p.) } \checkmark$$

(iii)

$$\text{Area of sector} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 100 \times 1.2 \checkmark$$

Shaded area

$$= \text{Area of sector} - \text{Area of } \triangle OPR$$

$$= 60 - \frac{1}{2} \times 10 \times 3.6235775 \times \sin 1.2 \checkmark$$

$$A = 43.12 \text{ cm}^2 \text{ (to 2 d.p.) } \checkmark$$

(b) Volume about the x - axis is

$$= \pi \left[\int_0^2 \left(\frac{x}{2}\right)^2 dx - \int_1^2 x - 1 dx \right] \checkmark$$

$$= \pi \left[\int_0^2 \frac{x^2}{4} dx - \int_1^2 x - 1 dx \right]$$

$$= \pi \left\{ \left[\frac{x^3}{12} \right]_0^2 - \left[\frac{x^2}{2} - x \right]_1^2 \right\} \checkmark$$

$$= \pi \left\{ \left[\frac{8}{12} - 0 \right] - \left[\left(\frac{4}{2} - 2 \right) - \left(\frac{1}{2} - 1 \right) \right] \right\} \checkmark$$

$$= \pi \left\{ \frac{1}{6} \right\}$$

$$\frac{\pi}{6} \text{ units}^3 \checkmark$$

$$(6)(a)(i) S_4 = \frac{4}{2}[2a + (4-1)d]$$

$$= 4a + 6d \checkmark$$

$$T_4 = a + 3d$$

$$3 \times T_4 = 3a + 9d$$

$$4a + 6d = 3a + 9d \checkmark$$

Therefore, $a = 3d$ as required.

$$(ii) T_{18} = 60; a = 3d$$

$$a + 17d = 60$$

$$3d + 17d = 60$$

$$20d = 60 \checkmark$$

$$d = 3$$

$$60 = a + 17 \times 3$$

$$60 = a + 51 \checkmark$$

$$a = 9$$

$$(iii) S_{18} = \frac{18}{2}(a + l) \checkmark$$

$$= 9(9 + 60) = 621 \checkmark$$

(b) To find the point of intersection solve the equations:

$$y = e^{2x} \text{ and } y = e^{3-x}$$

$$e^{2x} = e^{3-x}$$

$$2x = 3 - x$$

$$3x = 3$$

$$x = 1 \checkmark$$

$$\text{Area} = \int_0^1 e^{3-x} - e^{2x} dx$$

$$= \left[\frac{e^{3-x}}{-1} - \frac{e^{2x}}{2} \right]_0^1 \checkmark$$

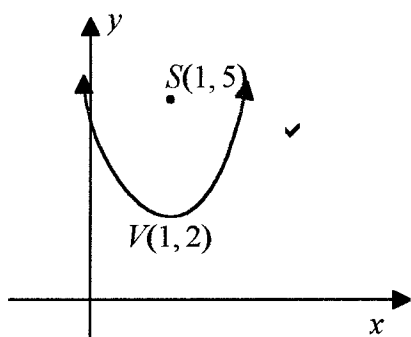
$$= \left[\left(\frac{e^2}{-1} - \frac{e^2}{2} \right) - \left(\frac{e^3}{-1} - \frac{e^0}{2} \right) \right] \checkmark$$

$$= \left[-e^2 - \frac{e^2}{2} - \left(-e^3 - \frac{1}{2} \right) \right]$$

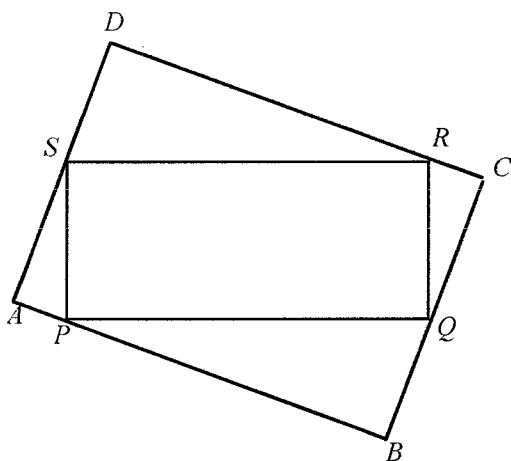
$$= 9.50 \text{ sq. units (to 2 d.p.)} \checkmark$$

(c) $(x-h)^2 = 4a(y-k)$

$$V(1,2); a=3 \checkmark$$



(7) (a) (i)



(ii) Let $\angle BQP = x$

$$\angle BPQ = 90 - x$$

$$\angle CQR = 180 - 90 - x = 90 - x \text{ (straight line)}$$

$$\angle QRC = 180 - 90 - (90 - x) = x$$

(Angle sum of $\triangle RCQ$) \checkmark

In $\triangle PBQ, \triangle QCR$

$$\angle B = \angle C = 90 \text{ (Property of a rectangle)} \checkmark$$

$$\angle BPQ = \angle RQC = x \text{ (proven above)}$$

Remaining angles are equal (Angle sum of triangle)

Therefore, $\triangle BPQ \parallel \triangle RQC$ (Equiangular) \checkmark

(iii) Now taking ratios from similar triangles

$$\frac{PQ}{QR} = \frac{BQ}{CR} \checkmark$$

Therefore, $PQ \times CR = BQ \times QR$ as required.

(iv) If $PQRS$ is a square, i.e. $PQ = QR$

thus $CR = BQ$ (from part (iii)); similarly \checkmark

$BP = PA$, thus P, Q, R and S are midpoints

and $ABCD$ is also a square (All sides equal and all angles are equal). \checkmark

(b)(i) Since $\angle BPA = 180^\circ - 46^\circ = 134^\circ$
(Straight line)

Using Sine rule,

$$\frac{4}{\sin \angle BAP} = \frac{8}{\sin 134^\circ} \checkmark$$

$$\angle BAP = 21^\circ \text{ (to the nearest deg.)} \checkmark$$

(ii) Let the perpendicular from D to AP at Q be h .

$$\angle DAQ = 180^\circ - 90^\circ - 21^\circ \text{ (Straight line)}$$

$$= 69^\circ \checkmark$$

$$\sin 69^\circ = \frac{h}{3} \checkmark$$

$$h = 2.8 \text{ m (to 1 d.p.)}$$

$$(8)(a) (i) \frac{dy}{dx} = 12x - 9 - 3x^2$$

Stationary points occur when $y' = 0$

$$3x^2 - 12x + 9 = 0 \checkmark$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$\therefore x = 3 \text{ or } 1$$

$$y = -1 \text{ or } -5 \checkmark$$

$$\frac{d^2y}{dx^2} = 12 - 6x$$

When $x = 3, f'(3) < 0,$

\therefore Max. t.p. at $(3, -1) \checkmark$

When $x = 1, f'(1) > 0,$

\therefore Min t.p. at $(1, -5) \checkmark$

(ii) Points of inflexion when $\frac{d^2y}{dx^2} = 0$

$$\therefore 12 - 6x = 0$$

$$\therefore x = 2, y = -3 \checkmark$$

Testing for concavity

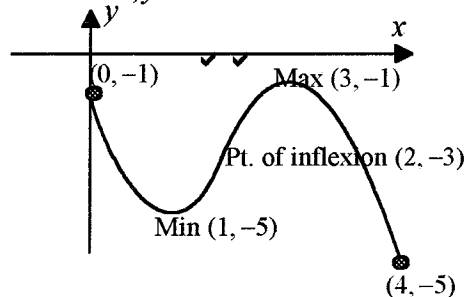
$$f''(2^-) > 0, f''(2^+) < 0$$

\therefore change in concavity and $(2, -3)$ is a point of inflexion. \checkmark

(iv) For the domain $0 \leq x \leq 4,$

$$x = 0, y = -1$$

$$x = 4, y = -5$$



$$(b)(i) N(t) = 10^6 e^{0.02t}$$

$$\frac{dN(t)}{dt} = 0.02(10^6 e^{0.02t})$$

$$= 0.02N(t) \text{ as } N(t) = 10^6 e^{0.02t} \checkmark$$

$$(ii) 2 \times 10^6 = 10^6 e^{0.02t}$$

$$2 = e^{0.02t}$$

$$\ln 2 = \ln e^{0.02t} \checkmark$$

$$\ln 2 = 0.02t$$

$$t \approx 35 \text{ sec (nearest sec)} \checkmark$$

$$(iii) \frac{dN}{dt} = 0.02N$$

$$= 0.02 \times 10^6 \times e^{0.02 \times 30}$$

$$= 36442 \text{ cryptos/sec} \checkmark$$

(9) (a) (i) When $v = 0$

$$t(t-2) = 0 \quad \checkmark$$

$$\therefore t = 0 \text{ or } t = 2$$

$$\therefore t = 2 \text{ sec} \quad \checkmark$$

(ii) $a = 2 - 2t \quad \checkmark$

When $t = 2$, $a = -2 \text{ m/s}^2 \quad \checkmark$

(iii) $x = t^2 - \frac{t^3}{3} + c$

When $t = 0$, $x = 0$, $c = 0 \quad \checkmark$

At P , $x = 4 - \frac{8}{3} = 1\frac{1}{3} \text{ m} \quad \checkmark$

(iv) $x = \left| 16 - \frac{64}{3} \right| = 5\frac{1}{3} \text{ m} \quad \checkmark$

$$\therefore \text{distance travelled is } 2 \times 1\frac{1}{3} + 5\frac{1}{3} = 8 \text{ m} \quad \checkmark$$

(b) (i) Let amount owing after n periods be A_n .

$$\therefore A_1 = 40000(1.00625) - M \quad \checkmark$$

$$A_2 = A_1(1.00625) - M$$

$$= (40000(1.00625) - M)(1.00625) - M$$

$$= 40000(1.00625)^2 - M(1 + 1.00625) \quad \checkmark$$

$$= \$ (40501.56 - 2.00625M) \text{ as required.}$$

(ii) $\therefore A_{60} = 40000(1.00625)^{60} -$

$$M(1 + 1.00625 + 1.00625^2 + \dots + 1.00625^{59})$$

At the end of the 60 months, $A_n = 0$

$$M = \frac{40000(1.00625)^{60}}{(1 + 1.00625 + 1.00625^2 + \dots + 1.00625^{59})} \quad \checkmark$$

$$= \$801.52 \quad \checkmark$$

(10) (a) (i) Area of $TRPQ = 2xy$

Area of $STR = \frac{1}{2} \times \frac{3x}{4} \times 2x = \frac{6x^2}{8}$

$$SR^2 = \left(\frac{3x}{4}\right)^2 + x^2$$

$$= \frac{9x^2}{16} + x^2$$

$$= \frac{9x^2 + 16x^2}{16}$$

$$SR = \frac{5x}{4} \quad \checkmark$$

Perimeter of $PQRST$ is 30 cm

$$\therefore QR = y = \frac{30 - \left(2x + \frac{10x}{4}\right)}{2}$$

$$= \frac{30 - \left(\frac{8x + 10x}{4}\right)}{2}$$

$$= \frac{30 - \frac{18x}{4}}{2}$$

$$= \frac{120 - 18x}{8}$$

$$= \frac{60 - 9x}{4} \quad \checkmark$$

Area of $TRPQ$ is $2x \left(\frac{60 - 9x}{4}\right)$

Area of $PQRST$ is

$$A = \frac{60x - 9x^2}{2} + \frac{6x^2}{8}$$

$$= \frac{120x - 18x^2 + 3x^2}{4}$$

$$= \frac{120x - 15x^2}{4} \quad \checkmark$$

$$\frac{dA}{dx} = \frac{1}{4}(120 - 30x) = 0$$

$$\therefore 30x = 120$$

$$\therefore x = 4 \quad \checkmark$$

$f'(4) < 0$, \therefore maximum when $x = 4$.

$$\therefore A = 120 - 60 = 60 \text{ cm}^2 \quad \checkmark$$

$$(iii) SM = \frac{3x}{4} = \frac{3 \times 4}{4} = 3 \quad \checkmark$$

$$QR = \frac{120 - 18x}{8} = \frac{120 - 18 \times 4}{8} \\ = 6$$

$$\therefore SN = SM + QR = 9 \text{ cm} \quad \checkmark$$

$$(b) \text{ Limiting sum} = \frac{a}{1-r}$$

$$\text{Numerator} = \frac{36}{1 - \left(-\frac{1}{2}\right)} = 24$$

$$\text{Denominator} = \frac{36}{1 - \frac{1}{3}} = 54 \quad \checkmark$$

$$\therefore \text{Ratio} = \frac{24}{54} = \frac{4}{9} \quad \checkmark$$

$$(c) y = \ln\left(\frac{x}{x-1}\right) = \ln x - \ln(x-1)$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x-1} \\ = \frac{(x-1) - x}{x(x-1)} = \frac{-1}{x(x-1)} \quad \checkmark$$

$$\text{At } x = 2, \frac{dy}{dx} = \frac{-1}{2(2-1)} = -\frac{1}{2}$$

\therefore the equation of the tangent to the curve at $x = 2$ is

$$y - \ln 2 = -\frac{1}{2}(x - 2) \quad \checkmark$$

$$2y - 2 \ln 2 = -x + 2$$

$$x + 2y - 2(\ln 2 + 1) = 0 \quad \checkmark$$