C.E.M.TUITION

FINAL TRIAL HSC EXAMINATION 1996

MATHEMATICS

2/3 UNIT COMMON PAPER

Total time allowed - Three hours

(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES :

- Attempt ALL questions.
- All questions are of equal value.
- All necessary working should be shown in every question.
- Full marks may not be awarded for badly arranged work.
- Standard integrals are on the inside cover.
- Approved silent calculators may be used.
- You must hand in a blank page if a question is unanswered.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x,\ x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax},\ a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax,\ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax,\ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{(a^2 - x^2)}} \, dx = \sin^{-1} \frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{(x^2 - a^2)}} \, dx = \ln \left\{ x + \sqrt{(x^2 - a^2)} \right\},\ |x| > |a|$$

$$\int \frac{1}{\sqrt{(x^2 + a^2)}} \, dx = \ln \left\{ x + \sqrt{(x^2 + a^2)} \right\}$$

NOTE : $\ln x = \log_e x; \quad x > 0$

Question 1 : **Marks**

(a) Find the value of $3^{1.5} \times \sqrt{32+9.6}$ to 3 significant figures. **2**

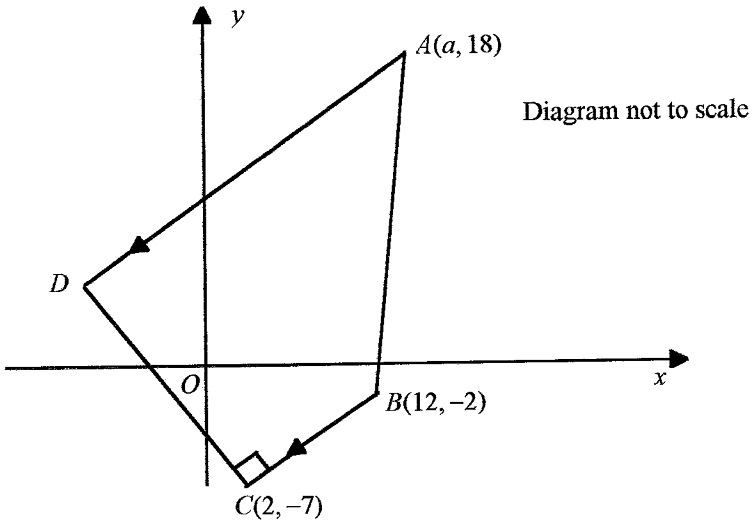
(b) A mansion at Point Piper was recently sold for \$11.5 million which represent 75% more than its original price in 1993. What was the original price in 1993 ? **2**

(c) Solve $|3x-1|=8$ **2**

(d) If $\frac{2x-y}{x+y} = 3$, find the value of $\frac{2x-y}{x-y}$ **3**

(e) Evaluate $(\cos 30^0 - \tan 45^0)(\sin 60^0 + \tan 45^0)$ without the use of your calculator. **3**

Question 2 : **Marks**



The diagram shows a trapezium in which $AD \| BC$ and $\angle ADC = \angle BCD = 90^0$.

The points A, B and C are $(a, 18)$, $(12, -2)$ and $(2, -7)$ respectively.

(a) Copy the diagram in your booklet — 1

Given that $AB = 2BC$,

(b) show that the value of a is 22 — 2

Find :

(c) the equation of AD — 2

(d) the equation of CD — 2

(e) the coordinates of D — 2

(f) the area of trapezium $ABCD$. — 3

Question 3 : **Marks**

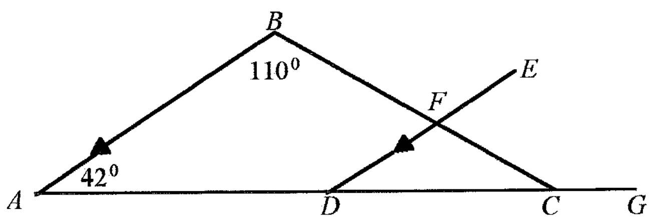
(a) Differentiate : **5**

(i) $\frac{3}{x^4}$

(ii) $x\sqrt{x+1}$

(iii) $\frac{e^{2x}}{1-x}$

(b)



In the diagram above, $AB \| DE$, $\angle BAC = 42^0$ and $\angle ABC = 110^0$. **4**

(i) Copy the diagram into your writing booklet.

(ii) Find $\angle BFD$ and $\angle FCG$ giving reasons.

(c) Andrew, Brian and Cathy play a game of cards. **3**
The probability that each of them will win a particular game are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ respectively. If they play three games in succession, use a tree diagram or otherwise, determine the probability that :

(i) Brian will win all three games.

(ii) Brian wins more games than Andrew or Cathy.

Marks

Question 4 :

(a) A function is defined by :

$$f(x)\begin{cases} = 1 - x & \text{for } -2 \le x < 0 \\ = x + 1 & \text{for } 0 \le x \le 2 \end{cases}$$

(i) Write down the values of $f\left(\frac{1}{2}\right)$ and $f\left(-\frac{1}{2}\right)$ **1**

(ii) Show that $f(-x) = f(x)$ and state whether this function is odd, even or neither. **2**

(iii) By first graphing the above function, write down a single expression for $f(x)$. **2**

(b) If $\frac{dy}{dx} = e^{2x-1}$, find the equation of the curve that passes through $\left(\frac{1}{2}, 1\right)$. **2**

(c) Find the range values of x for which **2**

$$x^2 > \frac{9x+5}{2}$$

(d) Given that $3x^2 - 11x + 3 \equiv A(x-2)(x-1) + B(x-1) + C$ **3**

for all values of x, find the values of A, B and C.

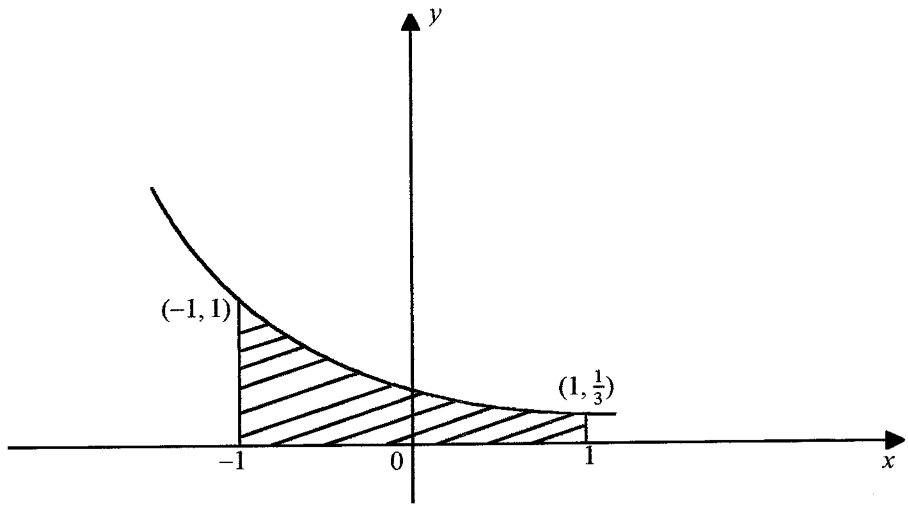
Question 5 : **Marks**

(a) Evaluate :

(i) $\int_1^5 \frac{1}{\sqrt{3x+1}}\,dx$ **3**

(ii) $\int_0^{\frac{\pi}{3}} (\sin x - \cos 3x)\,dx$ **3**

(b)



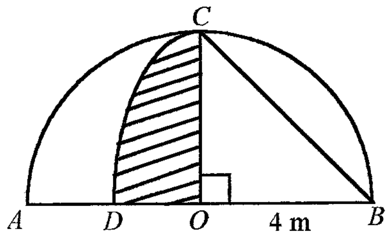
The diagram shows part of the curve $y = \frac{1}{x+2}$. Calculate :

(a) the area of the shaded region, **3**

(b) the volume obtained when this region is rotated through 360^0 about the x-axis. **3**

Question 6 : **Marks**

(a)



The diagram shows a semi-circle ABC, with centre O and radius 4 m, such that $\angle BOC = 90^0$. Given that CD is an arc of circle, centre B, calculate :

(i) the length of the arc CD. **3**

(ii) the area of the shaded region. **2**

(b) Sketch the graph of a function $y = f(x)$, which satisfy the following conditions : **4**

$f(-5) = 2$ $f(-2) = -4$ $f(0) = 0$ $f(1) = 2$ $f(2) = 0$	$f'(x) < 0$ for $x < -5$ $f'(x) = 0$ for $x = -5$ $f'(x) < 0$ for $-5 < x < -2$ $f'(x) = 0$ for $x = -2$ $f'(x) > 0$ for $-2 < x < 1$ $f'(x) = 0$ for $x = 1$ $f'(x) < 0$ for $x > 1$

(c) Find the equation of the tangent and normal to the curve **3**

$y = \dfrac{1}{1-x}$ at the point $(0, 1)$.

Question 7 : **Marks**

(a) If α and 2 are the roots of the quadratic equation **3**

$$2x^2 - kx + 2 = 0$$

Find the value of k and the other root α.

(b) The population of a town over t years is given by the formula **2**

$P = P_0 e^{0.0124t}$ where P_0 is its initial population

How long will it take to ***triple*** its initial population (to the nearest year).

(c) Consider the curve given by $y = 3x^4 - 4x^3 - 12x^2$ **7**

(i) Find the coordinates of the three stationary points.

(ii) Determine the nature of the stationary points.

(iii) Find all values of x for which $\dfrac{d^2y}{dx^2} = 0$

(iv) Sketch the curve for the domain $-2 \le x \le 3$.

Question 8 : **Marks**

(a) A 62 m length of rope is cut into pieces whose lengths are in arithmetic progression with a common difference of d metres. Given that the lengths of the shortest and longest pieces are 0.5 m and 3.5 m respectively, find the number of pieces and the value of d. **3**

(b) A sum of $\$P$ is borrowed at $r\%$ per month compound interest. If the money is paid back with monthly instalments of $\$M$ over n years,

(i) show that after the ***first*** month, the amount owing is **2**

$(PR - M)$ dollars where $R = 1 + \frac{r}{100}$

(ii) show that after the ***second*** month, the amount owing is **2**

$PR^2 - M(R+1)$

(iii) deduce the formula for the amount owing after the ***first year*** **1**

(iv) deduce the formula for the amount owing after n years. **1**

(v) find the monthly instalments on a sum of \$15 000 borrowed over 5 years at 12% p.a. compound interest. **3**

Question 9 : **Marks**

A particle P moves along a horizontal straight line so that its displacement, x m from a fixed point O, t seconds after motion has begun, is given by

$$x = 28 + 4t - 5t^2 - t^3.$$

(a) Obtain expressions, in terms of t, for the velocity and acceleration of P, and **2**

(b) state the initial velocity and the initial acceleration of P. **2**

A second particle Q moves along the same horizontal straight line as P and starts from O at the same instant that P begins to move. The initial velocity of P is 2 ms^{-1} and its acceleration, a ms^{-2}, t seconds after motion has begun, is given by

$$a = 2 - 6t.$$

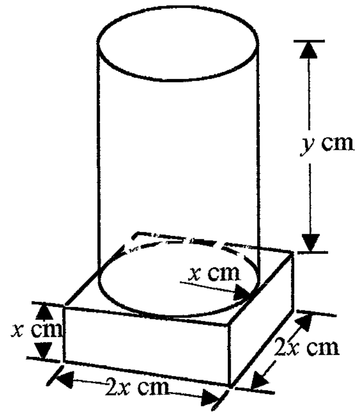
Find :

(c) the value of t at the instant when P and Q collide and **8**

(d) determine whether or not P and Q are travelling in the same direction.

Question 10 : **Marks**

The diagram shows a solid body which consists of a right circular cylinder fixed, with no overlap, to a rectangular block. The block has a square base of side $2x$ cm and a height of x cm. The cylinder has a radius of x cm and a height of y cm.



(a) Given that the total volume of the solid is 27 cm^3, **3**

express y in terms of x.

(b) Hence show that the total surface area, A cm^2, of the solid is given by **3**

$$A = \frac{54}{x} + 8x^2$$

Find : (i) the value of x for which A has a stationary value, **3**

(ii) the value of A and of y corresponding to this value of x. **2**

(c) Determine whether the stationary value of A is a maximum or minimum. **1**

End of Paper

(1)(a) 33.5 (to 3 s.f.)

(b) \$6.6 million (1 d.p.)

(c) $x = 3$ or $-\frac{7}{3}$

(d) $\frac{9}{5}$

(e) $-\frac{1}{4}$

(2) (a) Diagram (b) Proof

(c) $x - 2y + 14 = 0$

(d) $2x + y + 3 = 0$

(e) $D(-4, 5)$

(f) 270 units^2

(3)(a)(i) $-\frac{12}{x^5}$

(ii) $\sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$

(iii) $\frac{e^{2x}(3-2x)}{(1-x)^2}$

(b) (i) Diagram

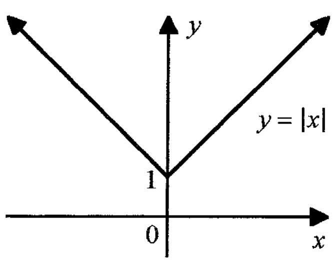
(ii) $\angle BFD = 70^0$(Cointerior angles $AB || DE$)

$\angle FCG = 152^0$(Ext.angle of ΔABC)

(c) (i) $\frac{1}{27}$ (ii) $\frac{7}{27}$

(4) (a) (i) $1\frac{1}{2}, 1\frac{1}{2}$ (ii) Proof; Even fn.

(iii)



(b) $y = \frac{e^{2x-1}+1}{2}$

(c) $x < -\frac{1}{2}$ or $x > 5$

(d) $A = 3, B = -2, C = -5$

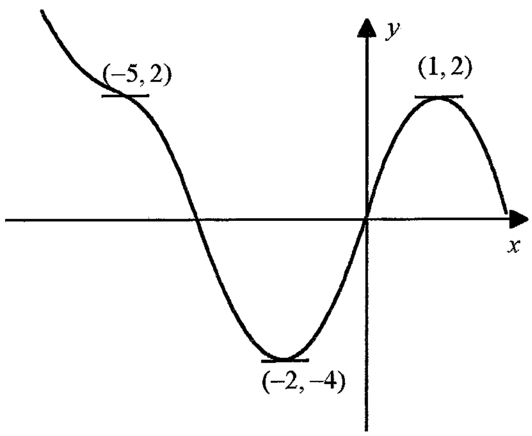
(5) (a) (i) $\frac{4}{3}$ (ii) $\frac{1}{2}$

(b) (i) $\ln 3$ units^2 (ii) $\frac{2\pi}{3}$ units^3

(6)(a)(i) $\sqrt{2}\,\pi \approx 4.44$ cm (2 d.p.)

(ii) $4(\pi - 2)$ cm^2

(b)



(c) $m_T = 1$, tangent: $x - y + 1 = 0$

normal : $x + y - 1 = 0$

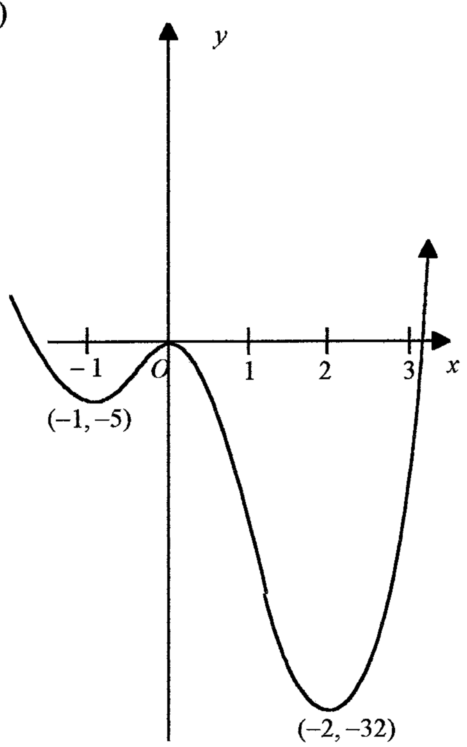
(7) (a) $k = 5, \alpha = \frac{1}{2}$

(b) 89 years

(c) (i), (ii) $(-1, -5)$ min. t.p.
$(0, 0)$ max. t.p. $(2, -32)$ min. t.p.

(iii) Pt. of inflexion at $x = \frac{1 \pm \sqrt{7}}{3}$

(iv)



(8)(a)(i) $n = 31, d = \frac{1}{10}$

(b)(i) At the end of first month

Int.= $\frac{r}{100} \times P$

$A_1 = P + \frac{\Pr}{100} - M = P\left(1 + \frac{r}{100}\right) - M$

$= PR - M$ as required.

(ii) At the end of second month
Interest = $\frac{r}{100} \times A_1$

$A_2 = A_1 + \frac{A_1 r}{100} - M = A_1\left(1 + \frac{r}{100}\right) - M$

$= (PR - M)R - M = PR^2 - MR - M$

$= PR^2 - M(R + 1)$ as required.

(iii) $A_{12} = PR^{12} - M(1 + R + .. + R^{11})$

(iv) $A_{12n} = PR^{12n} - M(1 + R + ... + R^{12n-1})$

(v) $M = \$333.67$

(9)(a) $v = 4 - 10t - 3t^2$; $a = -10 - 6t$

(b) $v = 4$ m/s; $a = -10$ m/s^2

(c) Hint: Equate $x_P = x_Q$

At $t = \frac{3}{2}$ s, $v_P < 0$ and $v_Q < 0$,

particle is travelling in the same direction.

(10) (a) $y = \dfrac{27 - 4x^3}{\pi x^2}$

(b)(i) $x = \frac{3}{2}$

(ii) $A = 54$ cm^2, $y = 1.91$ cm

(c) minimum value.