# C.E.M.TUITION

Name : ______________________________

**Review Topic : Coordinate Geometry**

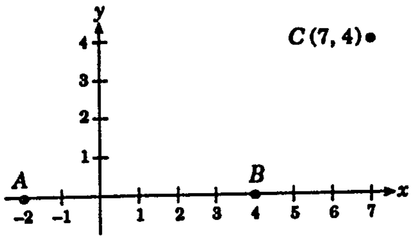
**(HSC - PAPER 1)**

Year 12 - Mathematics

**PHONE : 9666-3331** **FAX : 9316-4996**

**MOBILE: 0412 880 475**

**1.**




$A$, $B$, $C$ and $D$ have coordinates $(-2, 0)$, $(4, 0)$, $(7, 4)$ and $(1, 4)$ respectively.

(a) Draw this diagram and label the point $D$. Draw the lines $BC$ and $CD$.

(b) If $\angle BAD = \alpha°$, show that $\tan\alpha = \frac{4}{3}$.

(c) Prove that $\angle ABC = (180 - \alpha)°$. Why is $AD \parallel BC$?

(d) (i) Find the equation of the line $AC$ and the midpoint of the interval $BD$.

(ii) Show that $AC$ bisects $BD$. Why is this an expected result?

(e) Calculate the area of figure $ABCD$ and hence write down the area of $\Delta ACD$.

**2.** $A$, $B$ and $C$ are the points (2, 2), (4, 8) and (−2, 5) respectively.
Plot these points on your number plane.

**(a)** Find:
- **(i)** distance between $B$ and $C$;
- **(ii)** equation of the line containing $B$ and $C$;
- **(iii)** equation of the line perpendicular to $BC$ passing through $A$;
- **(iv)** the perpendicular distance from $A$ to $BC$;
- **(v)** the midpoint, $L$, of $AB$.

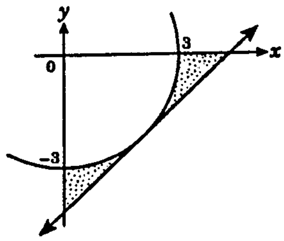
**(b)** Calculate the area of $\Delta ABC$.

**(c)** Derive the equation of the median, $CL$, of the $\Delta ABC$.

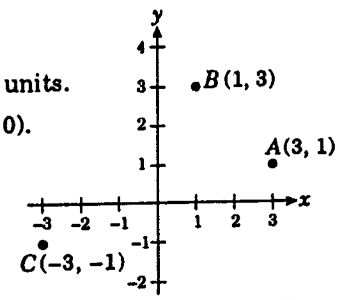
**3. (a)** Prove that the line $3x - 4y = 15$ is a tangent to the circle $x^2 + y^2 = 9$.

**(b)** Derive the equation of the line perpendicular to the tangent at the point of contact.

**(c)** Calculate the area in the fourth quadrant between the tangent and the circle (shaded on diagram).



**4.** $A$, $B$ and $C$ are the points (3, 1), (1, 3) and (−3, −1) respectively.




(a) Show that the length of $BC$ is $4\sqrt{2}$ units.

(b) Show that the midpoint of $AC$ is (0, 0).

(c) Derive the equation of the line $BC$.

(d) Show that the equation, $\ell$, of the line through $A$ parallel to $BC$ has equation $x - y - 2 = 0$.

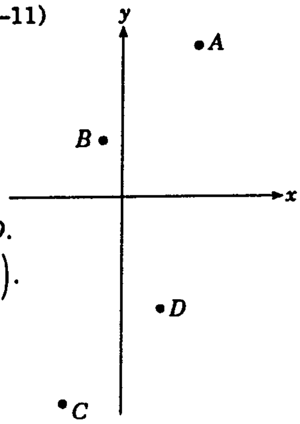
(e) (i) From the diagram plot the point $D$, the fourth vertex of parallelogram $ABCD$ (with diagonal $AC$). Write down the coordinates of $D$.

(ii) Show that $D$ lies on $\ell$.

(f) Find the area of parallelogram $ABCD$.

(g) Prove that the diagonals $AC$ and $BD$ bisect each other.

**5.** Consider the points $A(4, 8)$, $B(-1, 3)$, $C(-3, -11)$ and $D(2, -6)$ on the number plane.




**(a)** Show that the gradient of $AD$ is 7.

**(b)** Show that the length of $AD$ is $10\sqrt{2}$.

**(c)** By investigating the gradient and length of $BC$ name the type of quadrilateral that best describes $ABCD$.

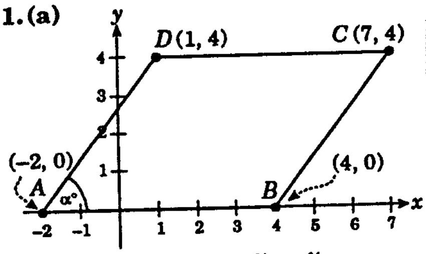
**(d)** Show that the midpoint of $AC$ is $\left(\frac{1}{2}, \frac{-3}{2}\right)$.

**(e)** Prove that the diagonals $AC$ and $BD$ bisect each other.

**(f)** Show that the equation of the line $AD$ is given by $7x - y - 20 = 0$.

**(g)** By first calculating the perpendicular distance of the point $B$ from the line $AD$, find the area of $ABCD$.

**1. (a)**




**(b)** Gradient $AD = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{4-0}{1-(-2)} = \dfrac{4}{3}.$

Now $m = \tan\alpha$, $\therefore \tan\alpha = \dfrac{4}{3}$.

**(c)** Gradient $BC = \dfrac{4-0}{7-4} = \dfrac{4}{3}$.

Then $\tan C\hat{B}X = \dfrac{4}{3}$, i.e. $C\hat{B}X = \alpha°$

Then $A\hat{B}C = (180-\alpha)°$
(angle sum of st. line)

*Also,* $AD \parallel BC$ as lines have the same gradient.

Alternatively, first state $AD \parallel BC$ because of gradients, and then follow with:

$\therefore A\hat{B}C = \alpha°$ (corr. ∠s, $AD \parallel BC$)

**(d) (i)** Equation of $AC$ is of form:

$$\boxed{\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}}$$

$\therefore \dfrac{y-4}{x-7} = \dfrac{0-4}{-2-7} = \dfrac{4}{9}$

$\therefore 9y - 36 = 4x - 28$

i.e. $4x - 9y + 8 = 0$.

Midpoint $BD$

$= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$= \left(\dfrac{4+1}{2}, \dfrac{0+4}{2}\right) = \left(\dfrac{5}{2}, 2\right).$

Midpoint $BD$ is $\left(\dfrac{5}{2}, 2\right)$.

**(ii)** $AC$ bisects $BD$ if $AC$ passes through midpoint of $BD$ *or* if midpoint of $BD$ lies on $AC$.

Subst. midpoint into $4x - 9y + 8 = 0$.

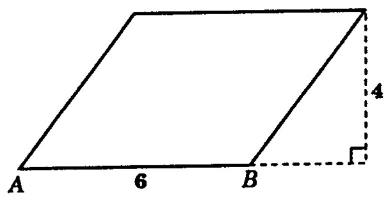
$\therefore$ LHS $= 4\left(\dfrac{5}{2}\right) - 9(2) + 8$

$= 10 - 18 + 8 = 0$.

Then midpoint satisfies eqn., i.e. $AC$ bisects $BD$.

This is expected, as $ABCD$ is a parallelogram (both pairs of opposite sides parallel, $AB \parallel DC$ is obvious); diagonals of a parallelogram bisect each other.

**(e)** We need length of $AB$, which is 6 units from diagram, and height which is 4 from diagram.




Area $= b \times h$

$= 6 \times 4 = 24$.

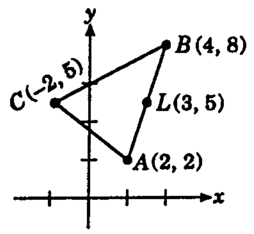
Area is 24 units$^2$.

Area $\Delta ACD = \dfrac{1}{2} \times 24$

$= 12$ units$^2$.

(Diagonals cut parallelogram into two equal triangles.)

**2.**




**(a) (i)** Distance $BC$

$= \sqrt{(4--2)^2 + (8-5)^2}$

$= \sqrt{45}$

$= 3\sqrt{5}$ units.

**(ii)** Equation of $BC$ is of form:

$\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}$

i.e. $\dfrac{y-5}{x+2} = \dfrac{8-5}{4+2}$

$= \dfrac{3}{6}$

$= \dfrac{1}{2}$, *

$\therefore 2(y-5) = 1(x+2)$ ☒

i.e. $2y - 10 = x + 2$

$\therefore x - 2y + 12 = 0$ is eqn.

**(iii)** $m_{BC} = \dfrac{1}{2}$ (from *),

$\therefore$ gradient perpendicular $= -2$.

$\boxed{m_1 \times m_2 = -1}$

Eqn. is of form:

$y - y_1 = m(x - x_1)$

$\boxed{(2, 2)}$

$\therefore y - 2 = -2(x-2)$

$= -2x + 4$,

$\therefore\ 2x+y-6=0.$

(iv) $p=\left|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

$A=1 \quad x_1=2$
$B=-2 \quad y_1=2$
$C=12$

$=\left|\dfrac{1(2)+(-2)(2)+12}{\sqrt{1^2+(-2)^2}}\right|$

$=\left|\dfrac{2-4+12}{\sqrt5}\right|$

$=\dfrac{10}{\sqrt5}=\dfrac{10\sqrt5}{5}=2\sqrt5.$

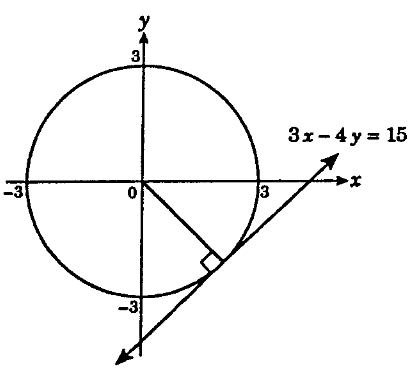
Perp. distance is $2\sqrt5$ units.

(v) $L=\left(\dfrac{2+4}{2},\ \dfrac{2+8}{2}\right)$
$=(3,\ 5).$

(b) Area
$=\dfrac{1}{2}\times BC\times$ perp. distance
$=\dfrac{1}{2}\times 3\sqrt5\times 2\sqrt5$
$=15$ units$^2$.

(c) By observation, equation of $CL$ is $y=5$ (check diagram).

3. (a) Circle has centre (0, 0), radius = 3 units. Radius drawn to point of contact of tangent is right angle.




If perpendicular distance from (0, 0) to $3x-4y=15$ is 3 units, then $3x-4y=15$ is a tangent.

$p=\left|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

$A=3 \quad x_1=0$
$B=-4 \quad y_1=0$
$C=-15$

$=\left|\dfrac{0+0-15}{\sqrt{9+16}}\right|$

$=\left|\dfrac{-15}{\sqrt{25}}\right|$

$=\left|\dfrac{-15}{5}\right|=3$ (= radius).

$\therefore\ 3x-4y=15$ is tangent to circle.

(b) $3x-4y=15$
$\therefore\quad 4y=3x-15$
$y=\dfrac{3}{4}x-\dfrac{15}{4}$
$\therefore\ m=\dfrac{3}{4}.$

Perpendicular gradient
$=\dfrac{-4}{3}$    $\boxed{m_1m_2=-1}$

Eqn. of perpendicular is of form:
$y-0=\dfrac{-4}{3}(x-0)$
$\therefore\quad 3y=-4x,$
i.e. $4x+3y=0$ is required equation.

(c) Shaded $A$
= area of $\Delta$ − area of $\dfrac{1}{4}$ circle
$=\left(\dfrac{1}{2}\times5\times\dfrac{15}{4}\right)-\dfrac{1}{4}\pi(3)^2$
$=\dfrac{3}{8}[25-6\pi]$ units$^2$.

*Note* Line $3x-4y=15$ cuts $x$ axis at $5 (y=0)$ and $y$ axis at $\dfrac{-15}{4} (x=0)$.

4. $A(3,1),\ B(1,3),\ C(-3,-1)$

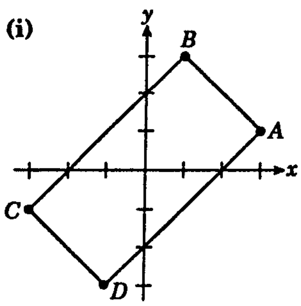
(a) $BC=\sqrt{(1--3)^2+(3--1)^2}$
$=\sqrt{32}$
$=4\sqrt2$ units.

(b) Midpoint of $AC$
$=\left(\dfrac{3+-3}{2},\ \dfrac{1+-1}{2}\right)$
$=(0,\ 0).$

(c) Equation of $BC$ has form:
$\dfrac{y-3}{x-1}=\dfrac{-1-3}{-3-1}=\dfrac{-4}{-4}=1$ *
$\therefore\ y-3=x-1$
i.e. $x-y+2=0.$

(d) Line parallel to $BC$ has same gradient, i.e. $m=1$ *.
Eqn. is of form:
$y-1=1(x-3)\ \ [A(3,1)]$
$\therefore\ y-1=x-3,$
i.e. $x-y-2=0$ is equation of $\ell$.

(e) (i)




From diagram, coords. of $D$ are (−1, −3).

| |
|---|
| →2<br>↓2<br>B to A |

(ii) Substitute (−1, −3) into $x-y-2=0$.
LHS $=-1+3-2$
$=0=$ RHS.
$D(-1,-3)$ lies on $\ell$.

(f) Gradient $AB=\dfrac{3-1}{1-3}$
$=\dfrac{2}{-2}=-1.$

Hence $AB\perp BC$ $(1\times-1=-1)$,
$\therefore$ figure $ABCD$ is a rectangle.
Area $=L\times W$
$=BC\times AB$
$=4\sqrt2\times2\sqrt2$
$=16$ units$^2$.

$AB=\sqrt{(3-1)^2+(1-3)^2}$
$=\sqrt{4+4}=\sqrt8$
$=2\sqrt2.$

**(g)** Midpoint $AC = (0, 0)$ from (b).

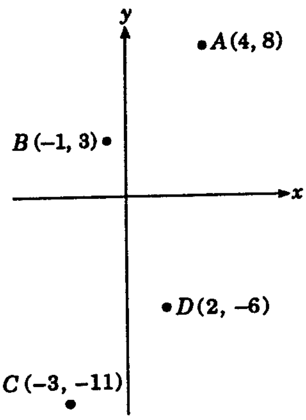
Midpoint $BD$

$$= \left(\frac{1+-1}{2}, \frac{3+-3}{2}\right)$$
$$= (0, 0).$$

Diagonals $AC$ and $BD$ both have midpoints at $(0, 0)$,

$\therefore$ diagonals bisect each other.

**5.**




**(a)** $m_{AD} = \dfrac{8-(-6)}{4-2} = \dfrac{14}{2} = 7.$

**(b)** $AD = \sqrt{(4-2)^2+(8+6)^2}$
$= \sqrt{4+196} = \sqrt{200}$
$= 10\sqrt{2}.$

**(c)** $m_{BC} = \dfrac{3-(-11)}{-1-(-3)} = \dfrac{14}{2}$
$= 7 = m_{AD}.$

$BC = \sqrt{(-1+3)^2+(3+11)^2}$
$= \sqrt{200} = 10\sqrt{2} = AD.$

$ABCD$ is a parallelogram as one pair of opposite sides is both parallel and equal.

**(d)** $MP_{AC} = \left(\dfrac{4-3}{2}, \dfrac{8-11}{2}\right)$
$= \left(\dfrac{1}{2}, \dfrac{-3}{2}\right).$

**(e)** $MP_{BD} = \left(\dfrac{-1+2}{2}, \dfrac{3-6}{2}\right)$
$= \left(\dfrac{1}{2}, \dfrac{-3}{2}\right).$

As midpoints of both diagonals are the same, the diagonals bisect each other.

**(f)** Eqn. of $AD$ is of form:
$y-8 = 7(x-4)$
$= 7x-28,$
$\therefore\ 7x-y-20=0$ is eqn. of $AD$.

**(g)** $p = \left|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

$A = 7$ $\quad x_1 = -1$
$B = -1$ $\quad y_1 = 3$
$C = -20$

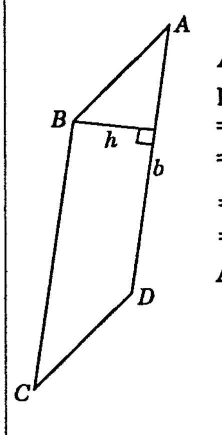
$= \left|\dfrac{7(-1)+(-1)(3)-20}{\sqrt{7^2+(-1)^2}}\right|$

$= \left|\dfrac{-7-3-20}{\sqrt{50}}\right|$

$= \dfrac{30}{\sqrt{50}} \quad = \dfrac{3\sqrt{50}}{5}$

$= \dfrac{3.\cancel{5}\sqrt{2}}{\cancel{5}} = 3\sqrt{2}.$

$$\boxed{\frac{30}{\sqrt{50}} \times \frac{\sqrt{50}}{\sqrt{50}} = \frac{3\sqrt{50}}{5} = \frac{3}{5}\sqrt{50}}$$




Area of parallelogram
= base × height
= $AD$ × perp. ht.
$= 10\sqrt{2} \times 3\sqrt{2}$
$= 60.$

Area is 60 units$^2$