C.E.M. TUITION

Student Name : ______________________________

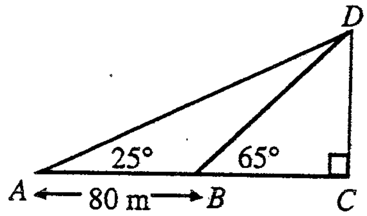
Review Topic : Trigonometric Ratios

(Paper 1)

Year 11 - 2 Unit

1.

Figure 5.30



(i) Show that $BD = \dfrac{80\sin 25°}{\sin 40°}$.

(ii) Show that $CD = \dfrac{80\sin 25° \sin 65°}{\sin 40°}$
and find the length of CD.

2. From the top of a cliff 40 metres high the angle of depression to a boat out at sea is $20°$. Find the distance from the top of the cliff to the boat.

3. A triangular airfield is bordered by three straight roads. The angles at the corners are $40°$, $65°$ and $75°$. The longest side of the airfield is 2.8 km. Find the shortest side and the area of the airfield correct to the nearest hectare.

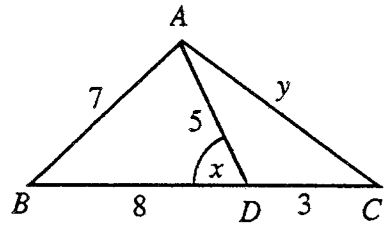
4. A boat leaves port and sails for 2.4 nautical miles on a bearing of 075°. It then changes direction and sails for 3.6 nautical miles on a bearing of 120°. Find the distance and bearing of the port from the boat.

5. **(i)** Find the exact value of

$\cos\frac{\pi}{6}\tan\frac{\pi}{3} - \sin^2\frac{\pi}{4}$

(ii) Find the exact values of x and y, where x is in radians.

Figure 5.31



6. (i) For θ acute, simplify
$\sqrt{1-\cos^2\left(\frac{\pi}{2}-\theta\right)}$

(ii) If $\cos x = -\frac{2}{3}$ and $\tan x > 0$ find the exact value of $\sin x$.

7. Solve the equations

(i) $\cos\left(x+\frac{\pi}{9}\right)=0 \quad , \quad 0\le x\le 2\pi$

(ii) $\sin x=-0.6 \quad , \quad 0°\le x\le 360°$

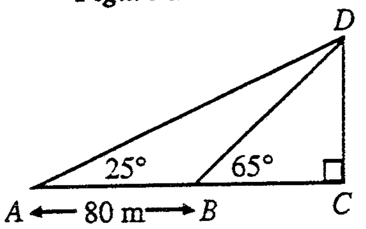
8. AB is a chord of length 16 cm in a circle with centre O and radius 10 cm.

(i) Find the size of $A\hat{O}B$ in radians.

(ii) Hence find the area of the minor segment cut off by AB.

1.

Figure 14.25



(i) In ΔABD,

$A\hat{D}B = 40°$ (Exterior $\angle$ is sum of interior opposite $\angle$s)

$\therefore \dfrac{BD}{\sin 25°} = \dfrac{80}{\sin 40°}$ (Sine rule)

$\therefore \quad BD = \dfrac{80\sin 25°}{\sin 40°}$

(ii) In ΔBCD,

$\dfrac{CD}{BD} = \sin 65°$

$CD = BD\sin 65°$

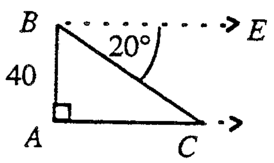
$= \dfrac{80\sin 25° \sin 65°}{\sin 40°}$

≈ 47.7

Hence $CD \approx 47.7$ m (to 3 sig. fig.).

2.

Figure 14.26



In ΔABC,

$B\hat{C}A = 20°$ (Alt. $\angle$s equal, $BE \parallel AC$)

$\therefore \dfrac{40}{BC} = \sin 20°$

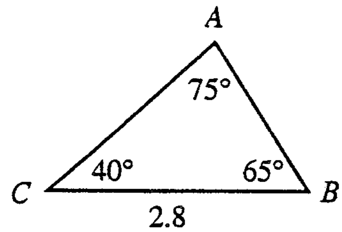
$\therefore BC = \dfrac{40}{\sin 20°}$

≈ 116.95

Hence the distance from the top of the cliff to the boat is 117 m (to 3 sig. fig.).

3. In ΔABC, the longest side is opposite the largest angle and the shortest side is opposite the smallest angle.

Figure 14.27



$\dfrac{AB}{\sin 40°} = \dfrac{2.8}{\sin 75°}$ (Sine rule)

$AB = \dfrac{2.8\sin 40°}{\sin 75°}$

≈ 1.8633

Hence the shortest side is 1.9 km (to 2 sig. fig.).

Area $\Delta ABC = \frac{1}{2} AB \,.\, BC \sin 65°$

$= \frac{1}{2} \times AB \times 2.8 \times \sin 65°$

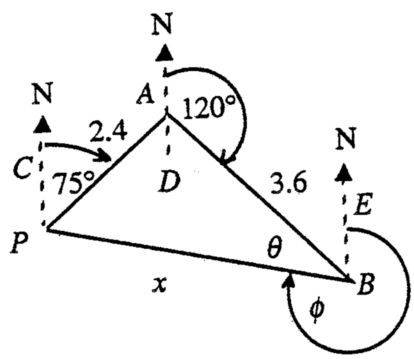
≈ 2.3642

$\therefore$ the area of the airfield is 236 ha (to nearest ha).

(Note: To avoid rounding errors, the value of AB should be stored in memory after calculation and recalled to carry out the calculation of the area of ΔABC.)

4.

Figure 14.28



$P\hat{A}D = 75°$ (Alt. $\angle$s equal, $CP \parallel AD$)

$D\hat{A}B = 60°$ (Straight $\angle$ at A is 180°)

$\therefore \; P\hat{A}B = P\hat{A}D + D\hat{A}B = 135°$

Then in ΔPAB, using the cosine rule,

$x^2 = (2.4)^2 + (3.6)^2 - 2 \times 2.4 \times 3.6\cos 135°$

≈ 30.939

$x \approx 5.562$

Hence the distance from the boat to port is 5.6 nm (to 2 sig. fig.).

The bearing of the boat from port is the reflex angle ϕ given by $\phi = 300° - \theta$, since $E\hat{B}A = 60° \begin{pmatrix}\text{Alt. and equal to} \\ D\hat{A}B,\ DA \parallel BE\end{pmatrix}$.

$$\frac{\sin\theta}{2.4} = \frac{\sin 135°}{x} \quad (\text{Sine rule } \Delta PAB)$$

$$\sin\theta = \frac{2.4\sin 135°}{x}$$

$$\approx 0.3051$$

$$\theta \approx 17.76°$$

$$300° - \theta \approx 282.24°$$

The bearing of the port from the boat is 282°.

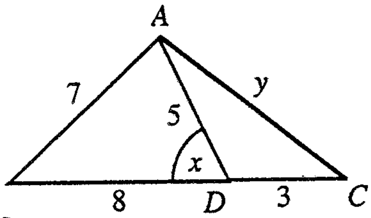
5. (i) $\cos\frac{\pi}{6}\tan\frac{\pi}{3} - \sin^2\frac{\pi}{4} = \frac{\sqrt{3}}{2}\times\sqrt{3} - \left(\frac{1}{\sqrt{2}}\right)^2$

$$= \frac{3}{2} - \frac{1}{2}$$

$$= 1$$

(ii)

Figure 14.29



In ΔABD,

$$\cos x = \frac{8^2 + 5^2 - 7^2}{2\times 8\times 5} \quad (\text{Cosine rule})$$

$$= \frac{1}{2}$$

$$x = \frac{\pi}{3}$$

$A\hat{D}C = \frac{2\pi}{3}$ (Straight $\angle BDC = \pi$)

In ΔACD, using the cosine rule,

$$y^2 = 5^2 + 3^2 - 2\times 5\times 3\cos\frac{2\pi}{3}$$

$$= 34 - 30\left(-\frac{1}{2}\right)$$

$$= 49$$

$$y = 7$$

6. (i) Using the identities $\cos^2\alpha + \sin^2\alpha = 1$ and $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$,

$$\sqrt{1 - \cos^2\left(\frac{\pi}{2} - \theta\right)} = \sqrt{\sin^2\left(\frac{\pi}{2} - \theta\right)}$$

$$= \sqrt{\cos^2\theta}$$

$$= |\cos\theta|$$

$$= \cos\theta$$

(ii) Since $\cos x < 0$ and $\tan x > 0$, x is a third quadrant angle and $\sin x < 0$.

$$\sin^2 x = 1 - \cos^2 x$$

$$= 1 - \left(-\frac{2}{3}\right)^2$$

$$= 1 - \frac{4}{9}$$

$$= \frac{5}{9}$$

$$\sin x = -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$$

7. (i) $\cos\left(x + \frac{\pi}{9}\right) = 0$, $0 \le x \le 2\pi$

$$x + \frac{\pi}{9} = \frac{\pi}{2},\ \frac{3\pi}{2}$$

$$x = \frac{\pi}{2} - \frac{\pi}{9},\ \frac{3\pi}{2} - \frac{\pi}{9}$$

$$x = \frac{7\pi}{18},\ \frac{25\pi}{18}$$

(ii) $\sin x = -0.6$, $0° \le x \le 360°$

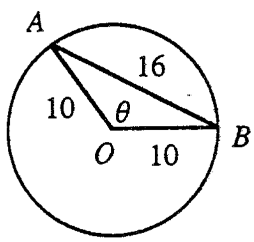
x lies in the third or fourth quadrants. The acute angle θ such that $\sin\theta = 0.6$ is $\theta \approx 36°52'$.

$$\therefore\ x \approx 180° + 36°52' = 216°52'$$

$$\text{or } x \approx 360° - 36°52' = 323°8'$$

8.

Figure 14.30



(i) In ΔAOB,

$$\cos\theta = \frac{10^2 + 10^2 - 16^2}{2\times 10\times 10} \quad (\text{Cosine rule})$$

$$= -\frac{56}{200}$$

$$\theta \approx 1.8546$$

$$\therefore A\hat{O}B \approx 1.9 \text{ radians}$$

(ii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$$\approx \frac{1}{2}\times 100\times 0.8946$$

$$= 44.73$$

Area of segment is $44.7\ \text{cm}^2$.