C.E.M.TUITION

Student Name :________________________________

Review Topic : Trigonometric Ratios

(Preliminary - Paper 2)

Year 11 - 2 Unit

1. (a) Write an exact expression for:

(i) $\cos^2 60° - \sin^2 60°$ (ii) $\tan 30° \cos 30° \sin 30°$

(b) If θ lies between 0° and 360°, find all values of θ given that:

(i) $\cos\theta = \frac{\sqrt{3}}{2}$ (ii) $2\cos\theta = -\sqrt{3}$

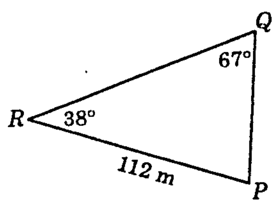
3. (a) Express $\sin 30° \cos 45° + \cos 30° \sin 45°$ as a fraction with rational denominator.

(b) Find all values of ϕ, $0° \leq \phi° \leq 360°$, where:

(i) $\tan \phi = -1$ (ii) $\sin \phi = -\cos \phi$

6. Given that $\sin\alpha = \frac{5}{13}$, and α is acute, find the value of $\cos\alpha$.

7. Use the Sine Rule to calculate the length of the fence PQ of this triangular paddock. Answer correct to the nearest metre.

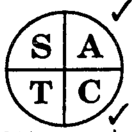


1. (a) (i) $\left(\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2$

$=\frac{1}{4}-\frac{3}{4}=-\frac{1}{2}$

(ii) $\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}\times\frac{1}{2}=\frac{1}{4}$

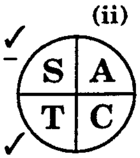
(b) (i) $\cos\theta=\frac{\sqrt{3}}{2}$



$\theta=30°$ and $(360-30)°$

$=30°$ and $330°$

(ii) $\cos\theta=-\frac{\sqrt{3}}{2}$



Acute angle is 30°
cos negative in 2nd, 3rd quadrants.

$\theta=(180-30)°,\ (180+30)°$

$=150°,\ 210°$

2. (a) $\dfrac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$ $\boxed{\tan 30°=\frac{1}{\sqrt{3}}}$

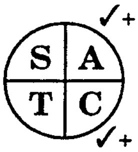
$=\dfrac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}=\dfrac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$=\frac{2}{\sqrt{3}}\div\frac{2}{3}=\frac{2}{\sqrt{3}}\times\frac{3}{2}$

$=\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$ (rationalise)

$=\frac{3\sqrt{3}}{3}=\sqrt{3}$

(b) (i) $\sec\theta=2$ (take reciprocal)
or $\cos\theta=\frac{1}{2}$



$\therefore\ \theta=60°$ or $(360-60)°$
$\theta=60°$ or $300°$

(ii) $\sin^2\theta=\frac{1}{2}$

$\therefore\ \sin\theta=\pm\frac{1}{\sqrt{2}}$

$\therefore\ \theta=45°,\ (180-45)°,\ (180+45)°,\ (360-45)°$

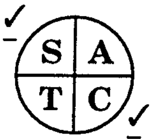
$\theta=45°,\ 135°,\ 225°$ or $315°$.

3. (a) $\frac{1}{2}\times\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}$

$=\frac{1+\sqrt{3}}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ (rationalise)

$=\frac{\sqrt{2}+\sqrt{6}}{4}$

(b) (i) $\tan\phi=-1$



Acute angle is 45°.
Tan ϕ is negative in 2nd and 4th quadrants.

$\therefore\ \phi=(180-45)°,\ (360-45)°$
$=135°$ or $315°$.

(ii) $\sin\phi=-\cos\phi$

÷ both sides by $\cos\phi$ → $\frac{\sin\phi}{\cos\phi}=-\frac{\cos\phi}{\cos\phi}$

$\therefore\ \tan\phi=-1$

from (i),
$\phi=135°$ or $315°$.

4. $AB^2=18{\cdot}5^2+23^2-2(18{\cdot}5)(23)\cos 146°$

$=1576{\cdot}761$ (Cosine Rule)

$\therefore AB=\sqrt{1576{\cdot}761}$

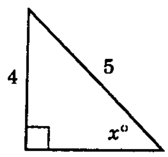
$=39{\cdot}70845$

$\approx 39{\cdot}7$ (1 dec. pl.)

$\therefore$ AB is $39{\cdot}7$ metres.

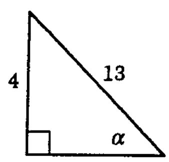
5. $\cos x°=\frac{3}{5}$

Using Pythagoras, other side is 4.



$\therefore\ \tan x°=\frac{4}{3}$

6. $\sin\alpha=\frac{5}{13}$



By Pythagoras,
$x^2=13^2-5^2$
$=144$
$x=12$ Other side is 12.

$\therefore\ \cos\alpha=\frac{12}{13}$ $\boxed{\frac{x}{13}}$

7. Let $PQ=x$ m, then using Sine Rule,

$\frac{x}{\sin 38°}=\frac{112}{\sin 67°}$

$\therefore\ x=\frac{112\sin 38°}{\sin 67°}$

$=74{\cdot}908986$

≈ 75 (nearest metre)

Length of fence PQ is 75 m.

8. Largest angle is opposite the longest side, that is, $\angle PRQ$.
Using the Cosine Rule,

$\cos P\hat{R}Q=\frac{9^2+11^2-17^2}{2\times 9\times 11}$

$=-\frac{87}{198}$

$=-0{\cdot}4393939$

$\therefore\ P\hat{R}Q=116°4'$

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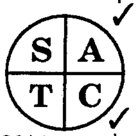

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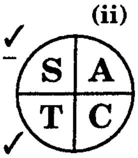

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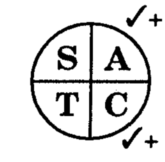

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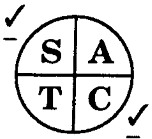

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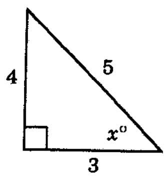

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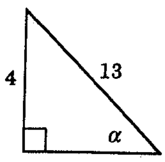

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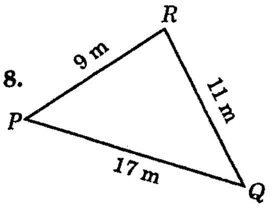
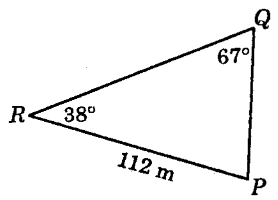
8. 

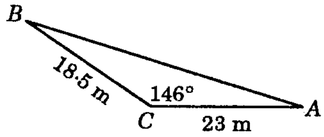
Use the Cosine Rule to calculate the largest angle in the triangle PQR with the sides as shown in the diagram. Answer correct to the nearest minute.

6. Given that $\sin\alpha = \frac{5}{13}$, and α is acute, find the value of $\cos\alpha$.

7. Use the Sine Rule to calculate the length of the fence *PQ* of this triangular paddock. Answer correct to the nearest metre.



4. Use the Cosine Rule to calculate the length of the side AB (correct to one decimal place).



5. If $\cos x° = \frac{3}{5}$, and $0° < x° < 90°$, calculate the value of $\tan x°$.

3. **(a)** Express $\sin 30° \cos 45° + \cos 30° \sin 45°$ as a fraction with rational denominator.

(b) Find all values of ϕ, $0° \le \phi° \le 360°$, where:

(i) $\tan \phi = -1$ (ii) $\sin \phi = -\cos \phi$

2. **(a)** Simplify to a surd with rational denominator $\frac{2\tan 30°}{1-\tan^2 30°}$.

(b) Find all values of θ, $0° \le \theta° \le 360°$ when:

(i) $\sec\theta = 2$ (ii) $\sin^2\theta = \frac{1}{2}$

1. (a) Write an exact expression for:
(i) $\cos^2 60° - \sin^2 60°$ (ii) $\tan 30° \cos 30° \sin 30°$

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