

C.E.M.TUITION

Name : _____

Review Paper No. 2

Basic Arithmetic & Algebra

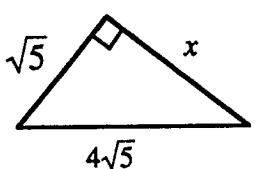
Year 11 - 2 Unit

1. Factorise

(i) $x^2 - y^2 + 2x + 2y$
(ii) $2x^3 + 54$

2. Simplify $\frac{x}{x-1} - \frac{2}{x+1} - \frac{2}{x^2-1}$

3.

Figure 1.2

Use Pythagoras' Theorem to find the value of x in simplest exact form.

4. If $S = \frac{a(r^n - 1)}{r - 1}$ find the value of S in simplest exact form when $a = 2$, $n = 4$, $r = \sqrt{3}$.

5. Solve simultaneously

$$x + 4y - 11 = 0$$

$$3x - 2y + 9 = 0$$

6. Solve

(i) $|2x - 1| = 4 - 3x$

(ii) $-2 < 3 - x \leq 4$

7. Solve $\frac{x}{x+2} = \frac{3}{x+4}$

8. In a rectangle of area 4 cm^2 the width is $x \text{ cm}$ and the length is 2 cm longer than the width.
- (i) Show that $x^2 + 2x - 4 = 0$.
 - (ii) Find the dimensions of the rectangle in simplest exact form.

$$\begin{aligned}1. \text{ (i)} \quad &x^2 - y^2 + 2x + 2y \\&= (x - y)(x + y) + 2(x + y) \\&= (x + y)(x - y + 2)\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad &2x^3 + 54 = 2(x^3 + 3^3) \\&= 2(x + 3)(x^2 - 3x + 9)\end{aligned}$$

$$\begin{aligned}2. \quad &\frac{x}{x-1} - \frac{2}{x+1} - \frac{2}{x^2-1} \\&= \frac{x(x+1) - 2(x-1) - 2}{(x-1)(x+1)} \\&= \frac{x^2 - x}{(x-1)(x+1)} \\&= \frac{x(x-1)}{(x-1)(x+1)} \\&= \frac{x}{(x+1)}\end{aligned}$$

$$\begin{aligned}3. \quad \text{Using Pythagoras' Theorem,} \\&x^2 + (\sqrt{5})^2 = (4\sqrt{5})^2 \\&x^2 + 5 = 16 \times 5 \\&x^2 = 75 \\&x = 5\sqrt{3}\end{aligned}$$

$$\begin{aligned}4. \quad &a = 2, \quad n = 4, \quad r = \sqrt{3} \\&S = \frac{a(r^n - 1)}{r - 1} \\&= \frac{2\left(\left(3^{\frac{1}{2}}\right)^4 - 1\right)}{\sqrt{3} - 1} \\&= \frac{2(9 - 1)}{\sqrt{3} - 1} \\&= \frac{16(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} \\&= \frac{16(\sqrt{3} + 1)}{3 - 1} \\&= 8(\sqrt{3} + 1)\end{aligned}$$

$$\begin{aligned}5. \quad &x + 4y - 11 = 0 \quad (1) \\&3x - 2y + 9 = 0 \quad (2)\end{aligned}$$

Take (1) + 2 × (2) to eliminate y:

$$\begin{array}{r}x + 4y - 11 = 0 \\ \oplus 6x - 4y + 18 = 0 \\ \hline 7x + 7 = 0 \\ x = -1\end{array}$$

Substitute for x in (1):

$$\begin{array}{l}4y - 12 = 0 \\ y = 3 \\ \therefore x = -1, \quad y = 3\end{array}$$

$$6. \text{ (i)} \quad |2x - 1| = 4 - 3x$$

Since LHS ≥ 0, RHS ≥ 0. Noting this restriction on x, we can square both sides.

$$\begin{array}{l}3x \leq 4 \text{ and } (2x - 1)^2 = (4 - 3x)^2 \\ (2x - 1)^2 - (4 - 3x)^2 = 0 \\ (3 - x)(5x - 5) = 0 \\ (3 - x)(x - 1) = 0 \\ \therefore x \leq \frac{4}{3} \text{ and } \{x = 3 \text{ or } x = 1\} \\ \therefore x = 1\end{array}$$

$$\text{(ii)} \quad -2 < 3 - x \leq 4$$

$$-5 < -x \leq 1$$

$$5 > x \geq -1$$

$$-1 \leq x < 5$$

$$\begin{aligned}7. \quad &\frac{x}{x+2} = \frac{3}{x+4} \\&x(x+4) = 3(x+2), \quad x \neq -2, \quad x \neq -4 \\&x^2 + 4x = 3x + 6 \\&x^2 + x - 6 = 0\end{aligned}$$

$$(x+3)(x-2) = 0$$

$$\therefore x = -3 \text{ or } x = 2$$

8. The sides are x and x + 2.

$$\text{(i)} \quad \text{Area is 4} \Rightarrow x(x+2) = 4$$

$$x^2 + 2x = 4$$

$$x^2 + 2x - 4 = 0$$

(ii) Using the quadratic formula :

$$\begin{aligned}x &= \frac{-b \pm \sqrt{\Delta}}{2a} \\a = 1 \\b = 2 \\c = -4\end{aligned}$$

$$\left. \begin{aligned}\Delta &= b^2 - 4ac \\&= 4 - 4(-4) \\&= 20\end{aligned} \right\} \Rightarrow$$

$$\sqrt{\Delta} = 2\sqrt{5}$$

$$x = \frac{-2 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2(-1 \pm \sqrt{5})}{2}$$

$$\therefore x = -1 + \sqrt{5} \text{ or } x = -1 - \sqrt{5}$$

But x cm is a length $\Rightarrow x \geq 0$

$$\therefore x = -1 + \sqrt{5}, \quad x + 2 = 1 + \sqrt{5}$$

Hence the dimensions are

$$(-1 + \sqrt{5}) \text{ cm by } (1 + \sqrt{5}) \text{ cm.}$$

1. Factorise

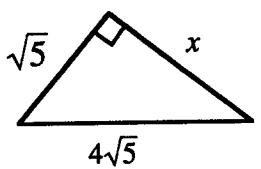
(i) $x^2 - y^2 + 2x + 2y$

(ii) $2x^3 + 54$

2. Simplify $\frac{x}{x-1} - \frac{2}{x+1} - \frac{2}{x^2-1}$

3.

Figure 1.2



Use Pythagoras' Theorem to find the value of x in simplest exact form.

4. If $S = \frac{a(r^n - 1)}{r - 1}$ find the value of S in simplest exact form when $a = 2$, $n = 4$, $r = \sqrt{3}$.

5. Solve simultaneously

$$x + 4y - 11 = 0$$

$$3x - 2y + 9 = 0$$

6. Solve

(i) $|2x - 1| = 4 - 3x$

(ii) $-2 < 3 - x \leq 4$

7. Solve $\frac{x}{x+2} = \frac{3}{x+4}$

8. In a rectangle of area 4 cm^2 the width is $x \text{ cm}$ and the length is 2 cm longer than the width.
- (i) Show that $x^2 + 2x - 4 = 0$.
 - (ii) Find the dimensions of the rectangle in simplest exact form.

1. (i) $x^2 - y^2 + 2x + 2y$
 $= (x - y)(x + y) + 2(x + y)$
 $= (x + y)(x - y + 2)$

(ii) $2x^3 + 54 = 2(x^3 + 3^3)$
 $= 2(x + 3)(x^2 - 3x + 9)$

2. $\frac{x}{x-1} - \frac{2}{x+1} - \frac{2}{x^2-1}$
 $= \frac{x(x+1) - 2(x-1) - 2}{(x-1)(x+1)}$
 $= \frac{x^2 - x}{(x-1)(x+1)}$
 $= \frac{x(x-1)}{(x-1)(x+1)}$
 $= \frac{x}{(x+1)}$

3. Using Pythagoras' Theorem,
 $x^2 + (\sqrt{5})^2 = (4\sqrt{5})^2$
 $x^2 + 5 = 16 \times 5$
 $x^2 = 75$
 $x = 5\sqrt{3}$

4. $a = 2, n = 4, r = \sqrt{3}$
 $S = \frac{a(r^n - 1)}{r - 1}$
 $= \frac{2((3^{\frac{1}{2}})^4 - 1)}{\sqrt{3} - 1}$
 $= \frac{2(9 - 1)}{\sqrt{3} - 1}$
 $= \frac{16(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$
 $= \frac{16(\sqrt{3} + 1)}{3 - 1}$
 $= 8(\sqrt{3} + 1)$

5. $x + 4y - 11 = 0 \quad (1)$
 $3x - 2y + 9 = 0 \quad (2)$

Take (1) + 2 × (2) to eliminate y:

$$\begin{array}{rcl} x + 4y - 11 & = & 0 \\ \oplus 6x - 4y + 18 & = & 0 \\ \hline 7x & + 7 & = 0 \\ x & = -1 & \end{array}$$

Substitute for x in (1):

$$\begin{array}{l} 4y - 12 = 0 \\ y = 3 \\ \therefore x = -1, y = 3 \end{array}$$

6. (i) $|2x - 1| = 4 - 3x$

Since LHS ≥ 0 , RHS ≥ 0 . Noting this restriction on x, we can square both sides.

$$\begin{array}{l} 3x \leq 4 \text{ and } (2x - 1)^2 = (4 - 3x)^2 \\ (2x - 1)^2 - (4 - 3x)^2 = 0 \\ (3 - x)(5x - 5) = 0 \end{array}$$

$$\begin{array}{l} (3 - x)(x - 1) = 0 \\ \therefore x \leq \frac{4}{3} \text{ and } \{x = 3 \text{ or } x = 1\} \\ \therefore x = 1 \end{array}$$

(ii) $-2 < 3 - x \leq 4$

$-5 < -x \leq 1$

$5 > x \geq -1$

$-1 \leq x < 5$

7. $\frac{x}{x+2} = \frac{3}{x+4}$
 $x(x+4) = 3(x+2), \quad x \neq -2, x \neq -4$
 $x^2 + 4x = 3x + 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$\therefore x = -3 \text{ or } x = 2$

8. The sides are x and $x + 2$.

(i) Area is 4 $\Rightarrow x(x+2) = 4$

$x^2 + 2x = 4$

$x^2 + 2x - 4 = 0$

(ii) Using the quadratic formula :

$$\left. \begin{array}{l} x = \frac{-b \pm \sqrt{\Delta}}{2a} \\ a = 1 \\ b = 2 \\ c = -4 \end{array} \right\} \quad \begin{array}{l} \Delta = b^2 - 4ac \\ = 4 - 4(-4) \\ = 20 \end{array}$$

$$\sqrt{\Delta} = 2\sqrt{5}$$

$$x = \frac{-2 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2(-1 \pm \sqrt{5})}{2}$$

$$\therefore x = -1 + \sqrt{5} \text{ or } x = -1 - \sqrt{5}$$

But x cm is a length $\Rightarrow x \geq 0$

$$\therefore x = -1 + \sqrt{5}, \quad x + 2 = 1 + \sqrt{5}$$

Hence the dimensions are

$$(-1 + \sqrt{5}) \text{ cm by } (1 + \sqrt{5}) \text{ cm.}$$