C.E.M. TUITION

Name :______________________________

Review Paper No. 2

Basic Arithmetic & Algebra

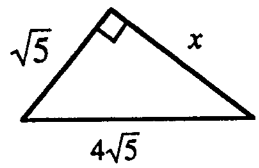
Year 11 - 2 Unit

1. Factorise
   (i) $x^2 - y^2 + 2x + 2y$
   (ii) $2x^3 + 54$

2. Simplify $\frac{x}{x-1} - \frac{2}{x+1} - \frac{2}{x^2-1}$

3.

Figure 1.2



Use Pythagoras' Theorem to find the value of x in simplest exact form.

4. If $S = \dfrac{a(r^n - 1)}{r - 1}$ find the value of S in simplest exact form when $a = 2$, $n = 4$, $r = \sqrt{3}$.

5. Solve simultaneously

$x+4y-11=0$

$3x-2y+9=0$

6. Solve

(i) $|2x-1|=4-3x$

(ii) $-2<3-x\leq 4$

7. Solve $\dfrac{x}{x+2}=\dfrac{3}{x+4}$

8. In a rectangle of area 4 cm^2 the width is x cm and the length is 2 cm longer than the width.

(i) Show that $x^2+2x-4=0$.

(ii) Find the dimensions of the rectangle in simplest exact form.

1. (i) $x^2 - y^2 + 2x + 2y$
$= (x-y)(x+y) + 2(x+y)$
$= (x+y)(x-y+2)$

(ii) $2x^3 + 54 = 2(x^3 + 3^3)$
$= 2(x+3)(x^2 - 3x + 9)$

2. $\dfrac{x}{x-1} - \dfrac{2}{x+1} - \dfrac{2}{x^2-1}$
$= \dfrac{x(x+1) - 2(x-1) - 2}{(x-1)(x+1)}$
$= \dfrac{x^2 - x}{(x-1)(x+1)}$
$= \dfrac{x(x-1)}{(x-1)(x+1)}$
$= \dfrac{x}{(x+1)}$

3. Using Pythagoras' Theorem,
$x^2 + (\sqrt{5})^2 = (4\sqrt{5})^2$
$x^2 + 5 = 16 \times 5$
$x^2 = 75$
$x = 5\sqrt{3}$

4. $a = 2, \quad n = 4, \quad r = \sqrt{3}$
$S = \dfrac{a(r^n - 1)}{r - 1}$
$= \dfrac{2\left((3^{\frac{1}{2}})^4 - 1\right)}{\sqrt{3} - 1}$
$= \dfrac{2(9-1)}{\sqrt{3}-1}$
$= \dfrac{16(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$
$= \dfrac{16(\sqrt{3}+1)}{3-1}$
$= 8(\sqrt{3}+1)$

5. $x + 4y - 11 = 0$ (1)
$3x - 2y + 9 = 0$ (2)
Take $(1) + 2 \times (2)$ to eliminate y:
$x + 4y - 11 = 0$
$\oplus\ 6x - 4y + 18 = 0$
$7x \quad + 7 = 0$
$x = -1$
Substitute for x in (1):
$4y - 12 = 0$
$y = 3$
$\therefore\ x = -1, \quad y = 3$

6. (i) $|2x - 1| = 4 - 3x$
Since LHS ≥ 0, RHS ≥ 0. Noting this restriction on x, we can square both sides.
$3x \leq 4$ and $(2x-1)^2 = (4-3x)^2$
$(2x-1)^2 - (4-3x)^2 = 0$
$(3-x)(5x-5) = 0$
$(3-x)(x-1) = 0$
$\therefore x \leq \frac{4}{3}$ and $\{x = 3 \text{ or } x = 1\}$
$\therefore x = 1$

(ii) $-2 < 3 - x \leq 4$
$-5 < -x \leq 1$
$5 > x \geq -1$
$-1 \leq x < 5$

7. $\dfrac{x}{x+2} = \dfrac{3}{x+4}$
$x(x+4) = 3(x+2), \quad x \neq -2,\ x \neq -4$
$x^2 + 4x = 3x + 6$
$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$
$\therefore x = -3$ or $x = 2$

8. The sides are x and $x+2$.
(i) Area is 4 $\Rightarrow$ $x(x+2) = 4$
$x^2 + 2x = 4$
$x^2 + 2x - 4 = 0$

(ii) Using the quadratic formula :

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\left.\begin{aligned} a &= 1 \\ b &= 2 \\ c &= -4 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} \Delta &= b^2 - 4ac \\ &= 4 - 4(-4) \\ &= 20 \end{aligned}$$

$$\sqrt{\Delta} = 2\sqrt{5}$$

$$x = \frac{-2 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2\left(-1 \pm \sqrt{5}\right)}{2}$$

$\therefore x = -1 + \sqrt{5}$ or $x = -1 - \sqrt{5}$

But x cm is a length $\Rightarrow \quad x \geq 0$

$\therefore x = -1 + \sqrt{5}, \quad x + 2 = 1 + \sqrt{5}$

Hence the dimensions are

$\left(-1 + \sqrt{5}\right)$ cm by $\left(1 + \sqrt{5}\right)$ cm.

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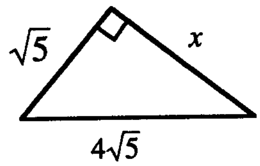

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