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CENTRE OF EXCELLENCE

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MATHS TUITION

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YEAR 11 – MATHEMATICS

LESSON & TUTORIAL - 2
TOPIC : INEQUALITIES & ABOSLUTE VALUES

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2. INEQUALITIES AND ABSOLUTE VALUES:

2.1 Inequalities:

Explain in words the four inequality symbols and other associated symbols below:

$<$ means

$\leq$ means

$>$ means

$\geq$ means

$\cap$ means

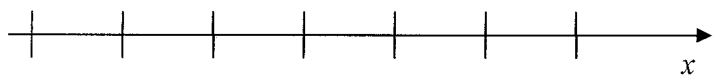
$\cup$ means

$\{x : x > 0\}$ means the set of all x such that x is greater than 0.

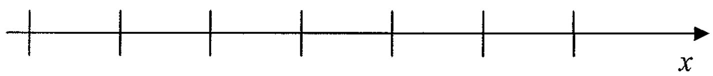
Examples:

Represent each of the following sets on the number lines provided:

1. $\{x : x \geq 1\}$



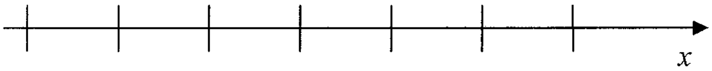
2. $\{x : x < -1\}$



3. $\{x : 1 \leq x \leq 4\}$

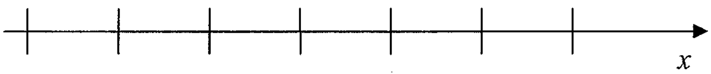
4. $\{x : -1 < x < 2\}$

5. $\{x : -2 < x \leq 2\}$

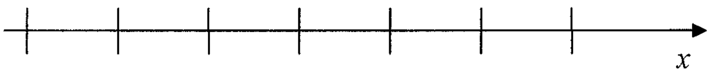


6. $\{x : x \leq 0\} \cup \{x : x \geq 2\}$

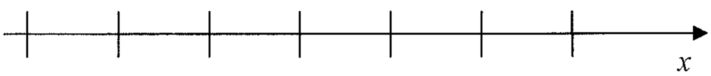
7. $\{x : x < -1\} \cup \{x : x > 3\}$



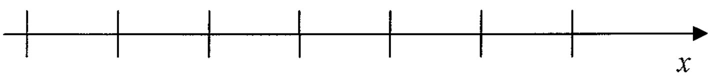
8. $\{x : x < 1\} \cap \{x : x \leq -1\}$



9. $\{x : x > 0\} \cap \{x : -1 \leq x \leq 2\}$



10. $\{x : -3 < x < 3\} \cap \{x : -2 \leq x \leq 2\}$



2.2 Absolute values:

The absolute value symbol is denoted by $|\ \ |$.

Therefore, $|2|$ or $|-2|$ means that the distance from O is exactly 2 units regardless of its position. $|x|$ may also be defined by $\sqrt{x^2}$.

Examples:

Simplify the following:

(1) $|3|+|5|-|-4|$

$\boxed{4}$

(2) $2|-3|\times|-5|$

$\boxed{30}$

(3) $\dfrac{|12|\times|-30|}{-4|15|}$

$\boxed{-6}$

(4) $\dfrac{|-8|+3|-2|}{-2|3-10|}$

$\boxed{-1}$

(5) $\dfrac{4|x|+2|-y|}{|x-y|}$ if $x=5$ and $y=-3$

$\boxed{3\frac{1}{4}}$

2.3 Equations involving absolute values:

If $|2| = |-2|$, then $|x| = 2 \Rightarrow x = 2$ or -2.

Hence, in solving equations or inequations with absolute values, replace the | | symbol with $\pm$ signs.

Examples:

Solve the following for x:

(1) $|x| = 5$

$\boxed{\pm 5}$

(2) $|x-1| = 3$

$\boxed{4 \text{ or } -2}$

(3) $|2-x| = 5$

$\boxed{7 \text{ or } -3}$

(4) $|2x-1| = x+3$ (Hint: check your answer by substitution).

$\boxed{4 \text{ or } -\frac{2}{3}}$

(5) $|x-1| = 2x+3$ (Hint: check your answer by substitution).

$\boxed{-\frac{2}{3} \text{ only}}$

Note: When solving equations with absolute values on both sides, consider:

(i) both positive results only and (ii) a positive result against a negative result only.

(6) $|3x-1| = |x+2|$

$\boxed{\frac{3}{2} \text{ or } -\frac{1}{4}}$

(7) $|x-4| = |2x+1|$

$\boxed{1 \text{ or } -5}$

(8) $|3x-2| = |x+5|$

$\boxed{\frac{7}{2} \text{ or } -\frac{3}{4}}$

2.4 Inequations with and without absolute values:

2.4.1 Note the following results:

(i) $6 > 2$ but $-6 < -2$.

So, remember to switch the inequality sign **when multiplying or dividing by a negative number.**

(ii) $6 > 2$ but $\frac{1}{6} < \frac{1}{2}$.

So, remember to switch the inequality sign **when taking reciprocals on both sides of the inequation.**

Examples:

Solve the following inequations and graph your solution set on a number line:

(1) $x + 3 < 6$

$\boxed{x < 3}$

(2) $2x - 3 \geq 10$

$\boxed{x \geq 6\frac{1}{2}}$

(3) $|x + 3| \leq 6$

$\boxed{-9 \leq x \leq 3}$

2.4.2 To solve inequations involving absolute values with x on both sides:

Take note that prior knowledge about sketching of graphs with absolute values is useful.

Examples:

(1) (a) Sketch the graphs of $y = x + 2$ and $y = |2x+1|$ on the same set of axes.

(b) Find the points of intersection of the two graphs by algebraic methods.

$\boxed{(1,3) \text{ or } (-1,1)}$

(c) Hence solve $|2x+1| \leq x+2$

$\boxed{-1 \leq x \leq 1}$

(2) (a) Sketch the graphs of $y = x + 1$ and $y = |2x - 3|$.

(b) From the graphs or otherwise, find the points of intersection.

$(4,5)$ or $\left(\frac{2}{3}, 1\frac{2}{3}\right)$

(c) Hence solve $|2x - 3| > x + 1$

$x < \frac{2}{3}$ or $x > 4$

PAST EXAMINATION QUESTIONS:

HSC 05

(1)

(e) Find the values of x for which $|x-3| \leq 1$. **2**

$\boxed{2 \leq x \leq 4}$

HSC 04

(1)

(f) **Find the values of x for which $|x+1| \leq 5$.** **2**

$\boxed{-6 \leq x \leq 4}$

HSC 03

(4) (a) Solve $|x-3| = 7$.

$\boxed{x = -4 \text{ or } 10}$

HSC 2002

(4)

(a) Solve $|x-1| \geq 3$ and graph your solution on the number line. **2**

$\boxed{x \geq 4 \text{ or } x \leq -2;\ \text{Number line solution}}$

<u>HSC 2001</u> **<u>Marks</u>**

(1) (b) Solve $|x+3|<2$. **2**

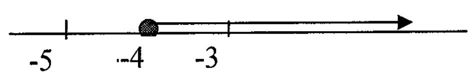
Graph your solution on a number line.

$\boxed{-5<x<-1}$

<u>HSC 2000</u>

(1) (b) Solve $x+7\geq 3$ and graph the solution on the number line. **2**

$\boxed{x\geq -4}$;



(f) Solve $|x-5|=3$. **2**

$\boxed{x = 2 \text{ or } 8}$

HSC '99

(1) (c) Solve $3-2x \geq 7$.

$\boxed{x \leq -2}$

HSC '98

(1) (b) Simplify $|-5| - |8|$

$\boxed{-3}$

HSC '97

(4)

(b) (i) Sketch the graph of $y = x^2 - 6$, and label all intercepts with the axes. **6**

(ii) On the same set of axes, carefully sketch the graph of $y = |x|$.

(iii) Find the x coordinates of the two points where the graphs intersect.

$\boxed{x = -3 \text{ or } 3}$

(iv) Hence solve the inequality $x^2 - 6 \le |x|$.

$\boxed{-3 \le x \le 3}$

HSC '96

(1) (f) Sketch the solution of $|x+2| \le 3$ on a number line.

$\boxed{-5 \le x \le 1;\ \text{Number line}}$

HSC '95

(1) (f) Solve $5-3x<7$.

$\boxed{x>-\frac{2}{3}}$

HSC '94

(1) (d) Solve $|x-1|=4$.

$\boxed{x=-3 \text{ or } 5}$

HSC '93

(1) (f) Find the values of x which satisfy the inequality $5-3x<17$.

$\boxed{x>-4}$

HSC '91

(1) (f) Mark on the number line the values of x for which $|x+1|\le 3$.

$\boxed{-4\le x\le 2;\ \text{Number line}}$