C.E.M.TUITION

Name :________________________________

Review Topic : Displacement, velocity & acceleration

Time Graphs

(HSC - PAPER 1)

Year 12 - Mathematics

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For corrections refer to pages:

2.7 VELOCITY–TIME GRAPHS

Velocity–time graphs are very useful for helping with calculations of both acceleration and displacement. Figure 2.14 shows the velocity–time graph that represents the motion of the object used for Figure 2.11.

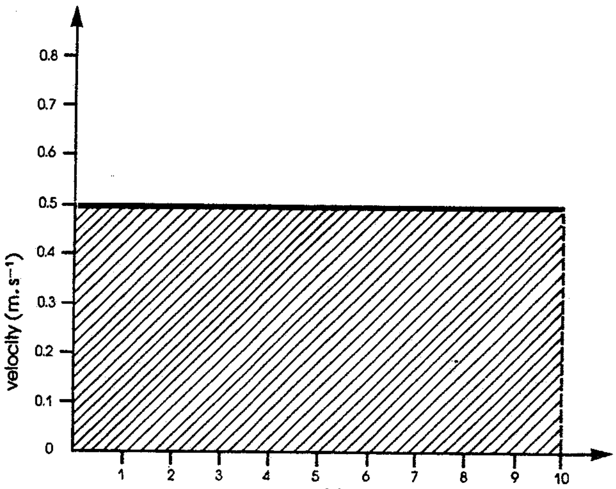

Figure 2.14 Velocity-time graph for constant velocity

The object has a constant velocity of $0.5\,m.s^{-1}$.

> **A constant velocity is shown by a horizontal line on a velocity–time graph.**

Note: Make sure that you take the following steps when you draw your graphs.

1. **Number axes in even increments.**
2. **Label axes with quantities and units.**
3. **Use a ruler and graph paper to plot points and draw lines accurately.**

The shaded area between the line and the time axis in Figure 2.14 gives the value of the displacement. In this case

$$S = \text{area} = 0.5 \times 10 = 5\,m$$

As seen in Figure 2.11 this is the displacement from the origin after 10 seconds.

> **The area under the curve in a velocity–time graph is equal to the displacement.**

In Figure 2.12 we saw an object which carried out several segments of uniform motion. Figure 2.15 shows the velocity–time graph for the same object. Read the description of that object's motion to verify that the velocity–time graph fits the motion.

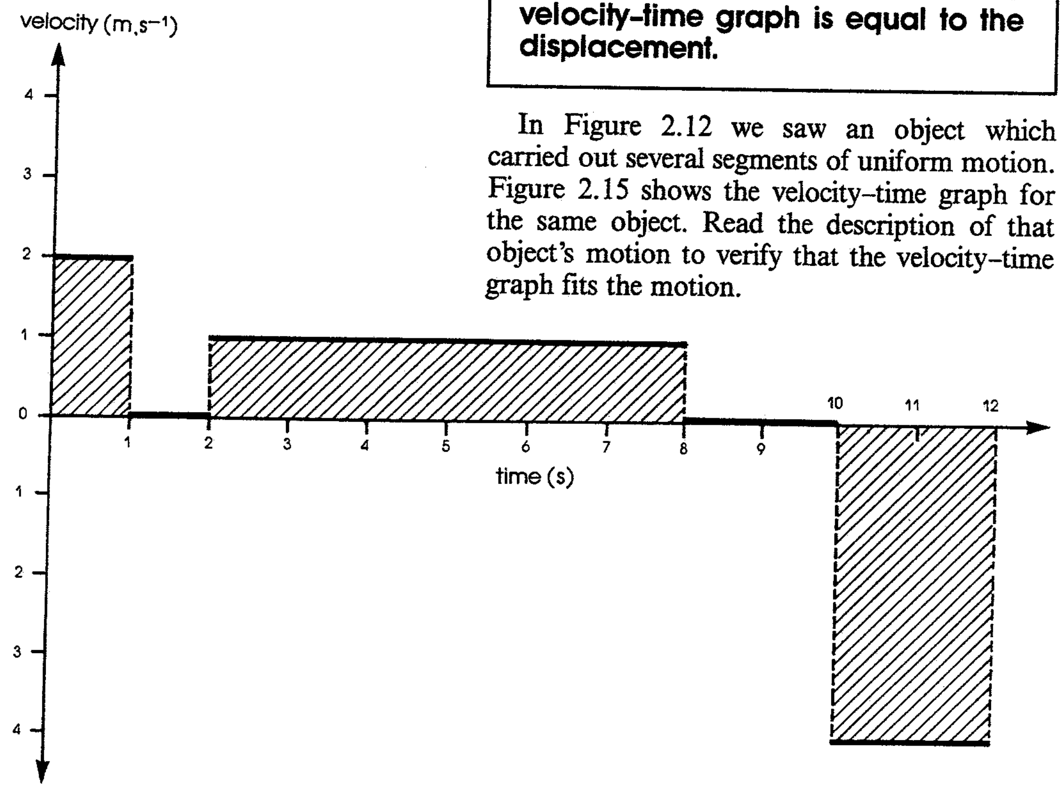

Figure 2.15 Velocity-time graph

Again, we can calculate the object's displacement by looking at the area under the curve.

From 0 to 1 second the area is $2 \times 1 = 2$. So the displacement is 2 m. This corresponds to point A in Figure 2.12.

From 1 to 2 seconds the object doesn't move. It has zero velocity and has zero change in displacement.

From 2 to 8 seconds, the area is $1 \times 6 = 6$. So the displacement in the segment is 6 m. Its total displacement so far is $2 + 6 = 8$ m. This corresponds to point C in Figure 2.12.

From 8 to 10 seconds there is a segment of zero velocity and zero change in displacement.

From 10 to 12 seconds the displacement is $-4 \times 2 = -8$ m. This area is below the time axis and gives a negative displacement.

For the entire motion, the total **displacement** is $2 + 6 - 8 = 0$ m.

But the total **distance** travelled is $2 + 6 + 8 = 16$ m.

Accelerated motion is shown in Figure 2.16.

This graph describes the flight of a rocket. At time $t = 0$ the rocket is stationary ($v = 0$). When it is lit, the rocket accelerates for 6 seconds until it reaches a velocity of 20 m.s^{-1} upward. At that time, its fuel runs out (point A) and it begins to slow down (due to gravity). It takes another 2 seconds (until $t = 8$) for it to stop ($v = 0$) at the top of its flight (point B). After that time it begins to fall back to earth with an increasing velocity in a negative direction. At point C it reaches ground where it stops (point D) 12 seconds after take-off.

Displacement can be found by calculating the area under the curve and the accelerations can be found from the slopes of the lines.

The displacement for the first 6 seconds is found using the formula for the area of a triangle.

$$
\begin{aligned}
A &= \tfrac{1}{2} \times b \times h \\
&= \tfrac{1}{2} \times 6 \times 20 \\
\underset{\sim}{S} &= 60\text{ m upwards}
\end{aligned}
$$

The displacement from A to B can be found similarly.

$$
\begin{aligned}
A &= \tfrac{1}{2} \times b \times h \\
&= \tfrac{1}{2} \times 2 \times 20 \\
\underset{\sim}{S} &= 20\text{ m upwards}
\end{aligned}
$$

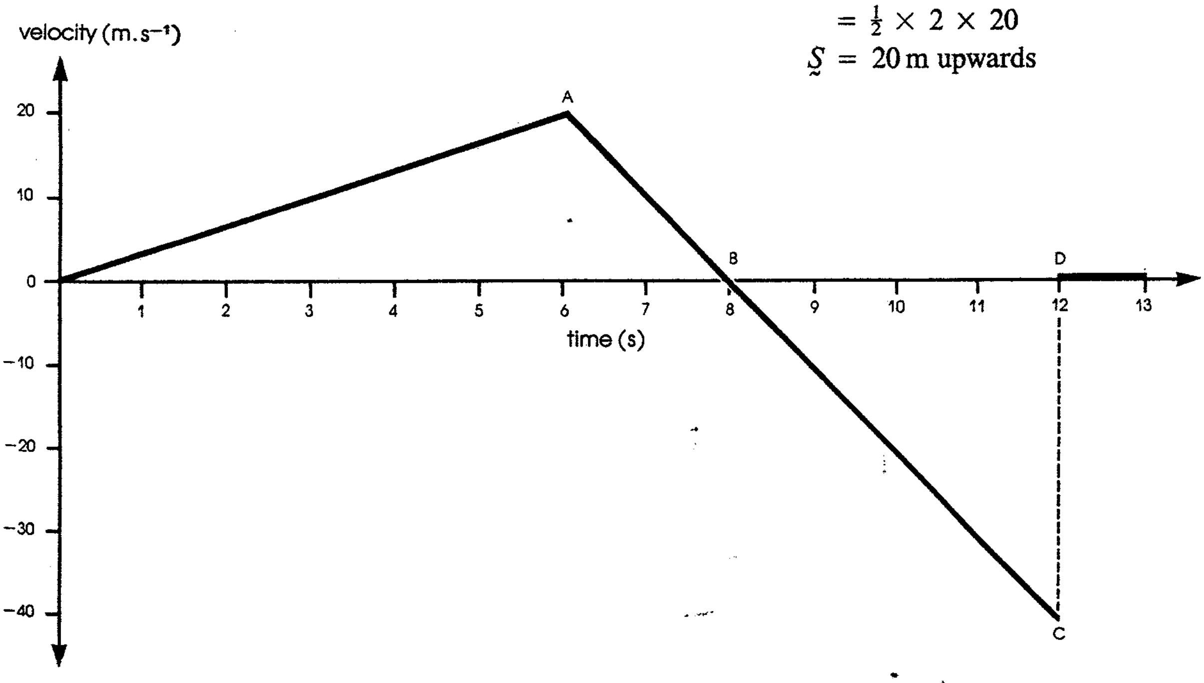

Figure 2.16 The flight of a rocket

The total displacement after 8 seconds is $60 + 20 = 80\,\text{m}$ upward. This is the maximum height.

During the 4 second time interval 8 seconds to 12 seconds, the rocket falls through a distance given by

$$A = \tfrac{1}{2} \times b \times h$$
$$= \tfrac{1}{2} \times 4 \times -40$$
$$= -80$$
$$\underset{\sim}{S} = 80\,\text{m downwards}$$

which is below the curve. (Note that it is not mathematically correct to assign a negative value to an area. The negative sign just represents the direction of the displacement.)

The total displacement for the entire journey is $80 - 80 = 0$. The rocket has returned to its starting point.

We can now look at the slopes of the lines in Figure 2.16 to calculate the acceleration during each interval:

From 0 to A, slope $= \text{rise/run}$
$= 20/6$
$= 3\tfrac{1}{3}$
$\underset{\sim}{a} \doteqdot 3.3\,\text{m.s}^{-2}$ upward

From A to B, slope $= \text{rise/run}$
$= -20/2$
$= -10$
$\underset{\sim}{a} = 10\,\text{m.s}^{-2}$ downward

From B to C, slope $= \text{rise/run}$
$= -40/4$
$= -10$
$\underset{\sim}{a} = 10\,\text{m.s}^{-2}$ downward.

QUESTIONS

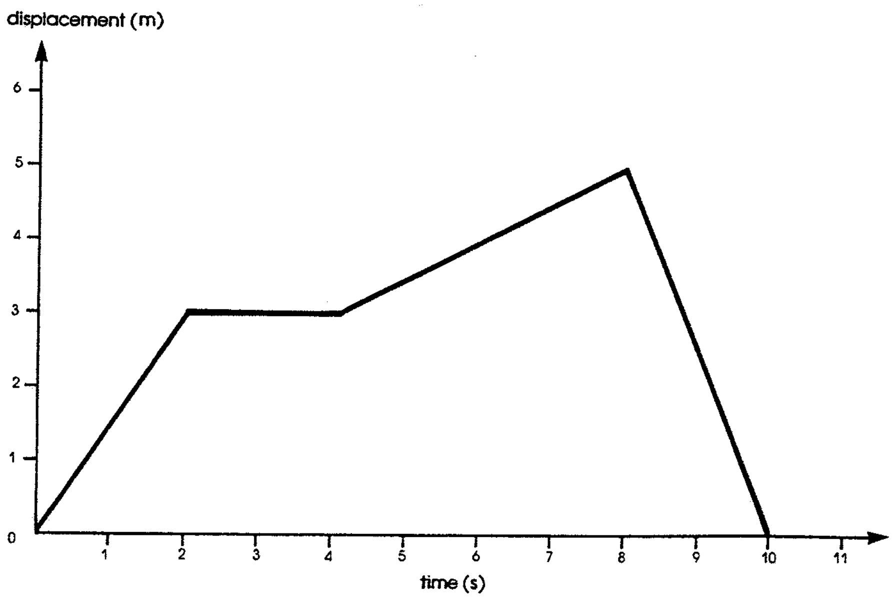

Figure 2.17

1. (a) In Figure 2.17 find:
   (i) The displacement after 3 seconds.
   (ii) The time taken to be 4 m from the origin.
   (b) What is the average velocity in each of these time intervals:
   (i) 0 s–2 s
   (ii) 2 s–4 s
   (iii) 4 s–8 s
   (iv) 8 s–10 s
   (c) Draw a velocity–time graph to represent the motion of the object shown in Figure 2.17.
   (d) Describe, in words, the motion of this object.

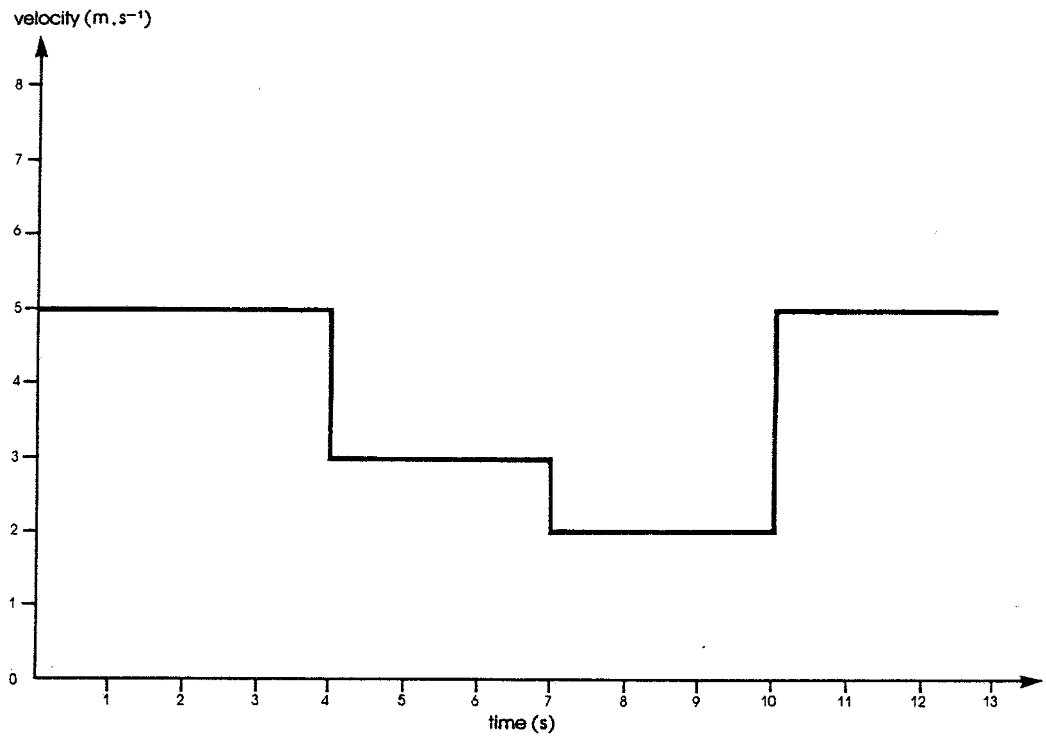

Figure 2.18

2. (a) In Figure 2.18 find:
   - (i) The velocity during the first 3 seconds.
   - (ii) The time(s) when the velocity is $3\,m.s^{-1}$.
   - (iii) The displacement during the first 3 seconds.
   - (iv) The displacement after 13 seconds.

(b) Draw a displacement–time graph to describe the motion of the object shown in Figure 2.18. **(Hint:** find the displacement after 4 seconds, 7 seconds, 10 seconds, and 13 seconds, making sure that you add the area of the previous sections; ie the displacement after 7 seconds is the total area under the graph between $t = 0$ and $t = 7$ seconds.)

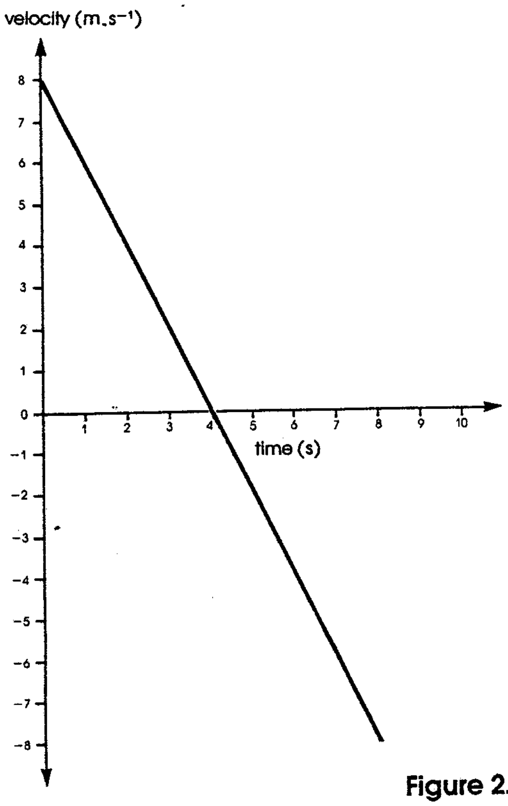

Figure 2.19

3. (a) From Figure 2.19 find:
   (i) initial velocity
   (ii) velocity after 4 seconds
   (iii) velocity after 7 seconds.
   (b) Find the displacement in the time intervals:
   (i) 0 to 4 seconds
   (ii) 4 to 8 seconds
   (iii) 0 to 8 seconds.
   (c) What is the acceleration of this object?
   (d) Sketch the displacement–time graph for this object.

Note that when the word 'sketch' is used, your diagram does not have to be as accurate as when the word 'graph' is used. (Figure 2.13 shows some sketches.)

4. A girl roller skates 20 m in 10 seconds. She then falls over and takes 3 seconds to get up. Her brother then pushes her and she takes 2 seconds to go 3 m backwards where she falls. After 2 seconds she gets up and skates forward, chasing her brother. She only takes 10 seconds to cover the 30 m before she catches him.
   (a) Draw a displacement–time graph to represent the motion of the girl.
   (b) Find her velocity during the first 10 seconds.
   (c) What is her displacement from the origin after catching her brother?
   (d) What is the total distance that she travelled?
   (e) Draw a velocity–time graph for the girl.

5. A car takes 20 seconds to accelerate from rest to a speed of $72\,km.h^{-1}$. It travels at that speed for 2 minutes and then takes a further 10 seconds to slow down and stop on a hill. After remaining stationary for 30 seconds, it rolls backwards, reaching a speed of $3.6\,km.h^{-1}$ after 5 seconds.
   (a) Draw a velocity–time graph to represent the car's motion. (**Hint**: convert **all** units to metres and seconds before you start.)
   (b) Calculate the area between the graph and the X-axis for each straight line segment.
   (c) Draw a displacement–time graph for this motion.

2.7 Velocity–Time Graphs

1. (a) (i) 3 m (ii) 6 s
   (b) (i) 1.5 $m.s^{-1}$, (ii) 0, (iii) 0.5 $m.s^{-1}$, (iv) $-2.5\ m.s^{-1}$
2. (a) (i) 5 $m.s^{-1}$, (ii) 4–7 s, (iii) 15 m, (iv) 50 m
3. (a) (i) 8 $m.s^{-1}$, (ii) 0, (iii) $-6\ m.s^{-1}$
   (b) (i) 16 m, (ii) −14 m, (iii) ~~0~~ 2
   (c) $-2\ m.s^{-2}$
4. (b) 2 $m.s^{-1}$ forward
   (c) 47 m
   (d) 53 m