C.E.M.TUITION

Student Name :______________________________

Review Topic : Probability

(HSC - PAPER 4)

Year 12 - 2 Unit

20. It has been estimated that the probability of a dog living beyond the age of 10 is 0·7 while that of a cat is 0·9. Alison owns two cats while Greg has one of each. Calculate the probability (using a tree diagram or otherwise) that:

(a) both of Alison's cats live past 10;

(b) one of Greg's animals lives beyond 10;

(c) at least one of Alison's animals lives beyond 10;

(d) all four animals survive past 10.

21. In Lower Warkworth the local doctor, based on years of data research, estimated that the probability of an adult catching influenza was 0·1 while the probability of a child catching the dreaded influenza was 0·3. The Blott family consists of Dad, Mum and two young Blotts. Calculate the probability that:

(a) both adults catch influenza;

(b) one child catches influenza;

(c) one adult and one child catches influenza;

(d) at least one family member catches influenza.

22. Two identical dice (faces numbered 1–6) are rolled. The two numbers on the uppermost faces are noted, and the larger number divided by the smaller with the remainder for each division written down. If there is no remainder a zero (0) is written down.

For example: $5 \div 2 = 2$ remainder $1 \Rightarrow$ write down 1
$5 \div 1 = 5$ remainder $0 \Rightarrow$ write down 0

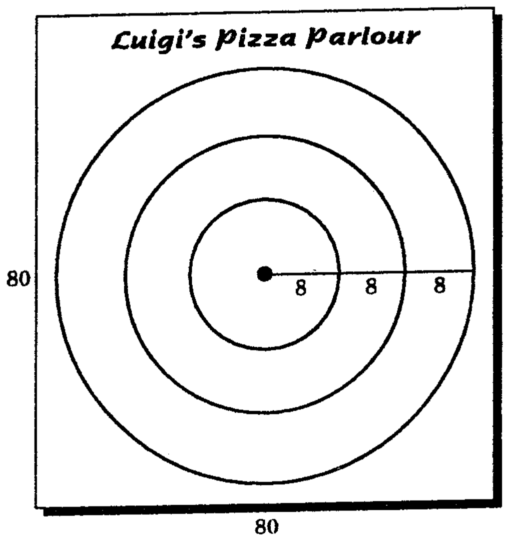
Find the probability that the remainder is:

(a) zero;
(b) three;
(c) not zero;
(d) at least one;
(e) even.

23. One hundred tickets are sold in a chook raffle. Gregory John purchases two tickets hoping to win both first and second prizes. Find the probability that GJ wins:

(a) first prize;
(b) both prizes;
(c) only second prize;
(d) a prize;
(e) no prize;
(f) at least one prize.

24. Luigi has a special incentive scheme at his pizza shop. On the side wall is a specially-designed target on an 80 × 80 cm board. The target consists of three concentric circles with radii 8 cm, 16 cm and 24 cm respectively. Each customer is given one dart to throw at the board. They win a free super supreme if they hit the inner circle, a medium pizza if they hit the middle area and a thickshake for the area in the outer circle. If they miss the circles they pay full price for their order. (Anyone who misses the board is allowed a rethrow.)



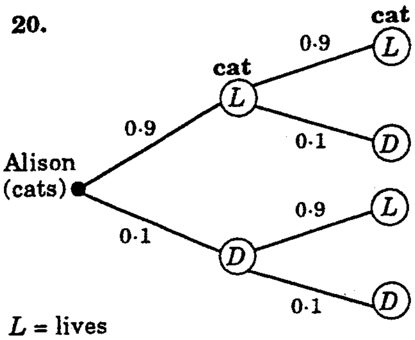
(a) Find the probability that:

- **(i)** Jill wins a super supreme;
- **(ii)** Peter wins a thickshake;
- **(iii)** Guisseppe wins a prize;
- **(iv)** Gwen pays full price.

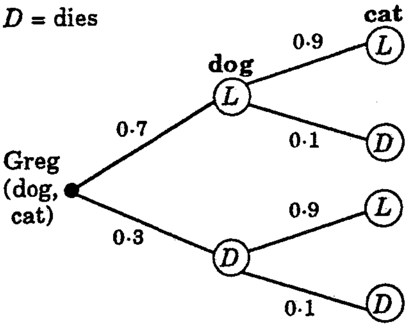
(b) If each customer is allowed 2 throws (and possibly can win 2 prizes) find the probability that:

- **(i)** Pietra wins 2 prizes;
- **(ii)** Janus wins 1 prize;
- **(iii)** Leigh fails to win at all;
- **(iv)** Scotty wins at least one prize.

20.



L = lives
D = dies



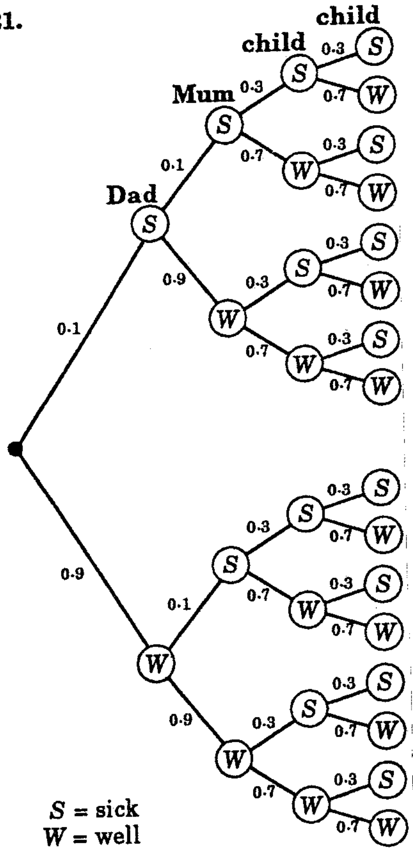
(a) P(both cats live) [Alison]
$= 0{\cdot}9 \times 0{\cdot}9 = 0{\cdot}81$

(b) P(one animal lives) [Greg]
$= P(LD) + P(DL)$
$= 0{\cdot}7 \times 0{\cdot}1 + 0{\cdot}3 \times 0{\cdot}9$
$= 0{\cdot}34$

(c) P(at least one of Alison's lives) $= 1 - P$(both die)
$= 1 - (0{\cdot}1 \times 0{\cdot}1) = 0{\cdot}99$

(d) Alison: P(both live) $= 0{\cdot}81$
Greg: P(both live)
$= 0{\cdot}7 \times 0{\cdot}9 = 0{\cdot}63$
P(all 4 live) $= 0{\cdot}81 \times 0{\cdot}63$
$= 0{\cdot}5103$ (Product Rule)

21.



> *Note* Entire tree is not needed for (a), (b) as Product Rule can be used—number of possible situations is small.

(a) P(both adults sick)
$= 0{\cdot}1 \times 0{\cdot}1 = 0{\cdot}01$

(b) P(one child sick)
$= P(SW) + P(WS)$
$= 0{\cdot}3 \times 0{\cdot}7 + 0{\cdot}7 \times 0{\cdot}3$
$= 0{\cdot}42$

(c) P(one adult, 1 child sick)
$= P(SWSW) + P(SWWS)$
$+ P(WSSW) + P(WSWS)$
$=$ ~~1×~~ $(0{\cdot}1 \times 0{\cdot}9 \times 0{\cdot}3 \times 0{\cdot}7) \times 4$
$=$ ~~0·0189~~ 0.0756

(d) P(at least one sick)
$= 1 - P$(all well)
$= 1 - 0{\cdot}9 \times 0{\cdot}9 \times 0{\cdot}7 \times 0{\cdot}7$
$= 0{\cdot}6031$

22.

+	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	1	0	1	0
3	0	1	0	1	2	0
4	0	0	1	0	1	2
5	0	1	2	1	0	1
6	0	0	0	2	1	0

(a) $P(0) = \frac{22}{36} = \frac{11}{18}$

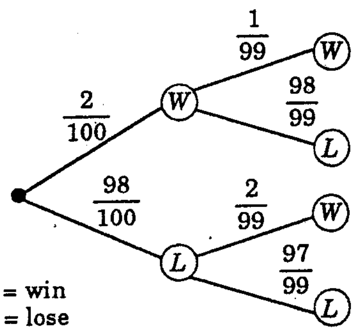
(b) $P(3) = 0$ (does not happen)

(c) $P(\neq 0) = 1 - P(0) = \frac{7}{18}$

(d) $P(\geq 1) = P(\neq 0) = \frac{7}{18}$

(e) $P(\text{even}) = P(2) = \frac{4}{36} = \frac{1}{9}$

23.



W = win
L = lose

(a) $P(\text{first prize}) = \frac{2}{100} = \frac{1}{50}$

(b) $P(WW)$
$= \frac{2}{100} \times \frac{1}{99}$
$= \frac{2}{9900} = \frac{1}{4950}$

(c) $P(LW) = \frac{98}{100} \times \frac{2}{99} = \frac{49}{2475}$
(only second implies lose, win)

(d) P(a prize)
$= P(WL) + P(LW) + P(WW)$
$= \frac{2}{100} \times \frac{98}{99} + \frac{98}{100} \times \frac{2}{99}$
$+ \frac{1}{4950}$ [from (b)]
$= \frac{197}{4950}$ [or $1 - P(LL)$]

(e) $P(LL) = \frac{98}{100} \times \frac{97}{99} = \frac{4753}{4950}$

(f) P(at least one prize)
$= 1 - P$(no prize)
$= 1 - \frac{4753}{4950} = \frac{197}{4950}$

24. We need to know the relative areas of each section.

A of inner circle
$= \pi r^2 = \pi 8^2 = 64\pi \text{ cm}^2$

A of middle section
$= \pi(16)^2 - 64\pi = 192\pi \text{ cm}^2$

A of outer ring
$= \pi(24)^2 - 192\pi = 384\pi \text{ cm}^2$

Also area of board
$= 80 \times 80 = 6400 \text{ cm}^2$

The probability of hitting any section will be the ratio of the area of each section relative to the area of the board.

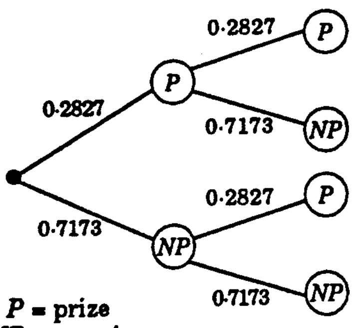
(a) (i) P(Super Supreme)
$= P$(hitting centre)
$= \dfrac{64\pi}{6400} = 0{\cdot}0314$ (4 dp)

(ii) P(thickshake)
$= P$(outer ring)
$= \dfrac{384\pi}{6400} = 0{\cdot}1885$ (4 dp)

(iii) P(prize) $= P$(hitting anywhere inside large circle)
$= \dfrac{\pi(24)^2}{6400} = 0{\cdot}2827$ (4 dp)

(iv) P(no prize)
$= 1 - P(\text{prize}) = 0{\cdot}7173$ (4dp)

(b) $P(\text{prize}) = 0{\cdot}2827$
$P(\text{no prize}) = 0{\cdot}7173$



P = prize
NP = no prize

(i) $P(P, P)$
$= (0{\cdot}2827)^2$
$= 0{\cdot}0799$ (4dp)

(ii) P(one prize)
$= P(P, NP) + P(NP, P)$
$= 0{\cdot}2827 \times 0{\cdot}7173 + 0{\cdot}7173 \times 0{\cdot}2827$
$= 0{\cdot}4056$ (4 dp)

(iii) P(no prizes)
$= P(NP, NP)$
$= (0{\cdot}7173)^2$
$= 0{\cdot}5145$ (4dp)

(iv) P(at least one prize)
$= 1 - P$(no prizes)
$= 1 - 0{\cdot}5145$
$= 0{\cdot}4855$ (4dp)

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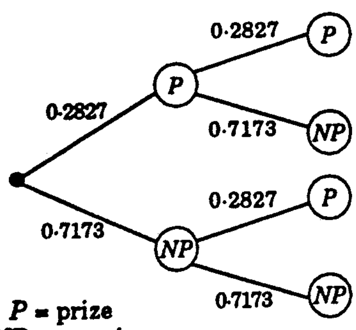

P = prize
NP = no prize

(i) $P(P, P)$
$= (0 \cdot 2827)^2$
$= 0 \cdot 0799$ $(4dp)$

(ii) $P(\text{one prize})$
$= P(P, NP) + P(NP, P)$
$= 0 \cdot 2827 \times 0 \cdot 7173 + 0 \cdot 7173 \times 0 \cdot 2827$
$= 0 \cdot 4056$ $(4dp)$

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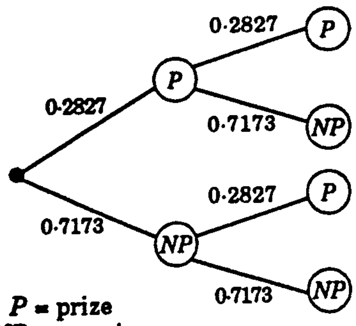

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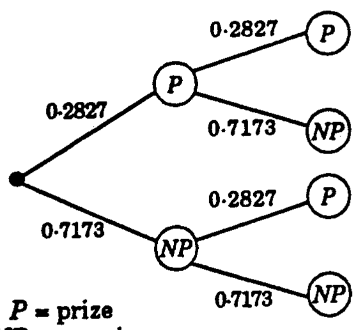

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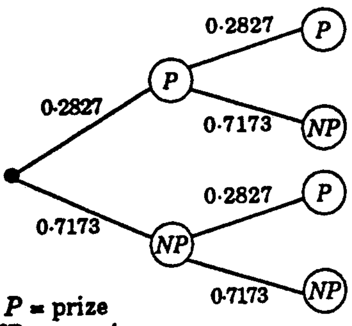

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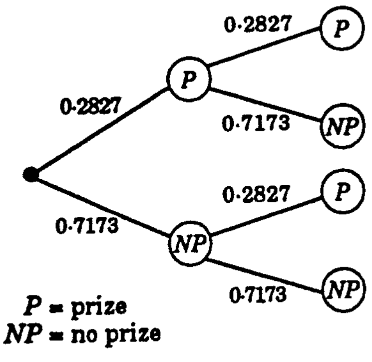

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