C.E.M. TUITION

Name : ______________________________

Review Topic : Kinematics

(HSC Course - Paper 2)

Year 12 - 2 Unit

6. A particle starts from the origin with a velocity of 4 ms^{-1}, and moves along the x axis so that its acceleration after time t is given by $a = 6t(t-2)$.

(a) Show that $v = 2t^3 - 6t^2 + 4$.

(b) Show that $x = \frac{t^4}{2} - 2t^3 + 4t$.

(c) Show that the particle is at the origin again after two seconds, and find its velocity at that instant.

7. A body, initially at rest at the origin 0, is given an acceleration a which at time t is given by $a = 6t - 12$.

(a) Show that the body stops after 4 seconds.

(b) Find the distance travelled in the first 4 seconds.

(c) Show that the body travels 64 metres in the first 6 seconds of motion.

8. A particle moves in a straight line such that its velocity after t seconds is given by $v = 3t^2 - 12$ metres per second.

(a) Find when the particle is at rest.

(b) Sketch the graph of the velocity v as a function of t.

(c) Does the particle ever slow down? Why?

(d) When does the particle return to the origin, 0, if the particle begins from the origin?

9. The acceleration of a moving body is given as $a = \sqrt{2t+1}$. If the body starts from rest from the origin, find:

(a) its velocity after 4 seconds;

(b) its distance from the starting point after 4 seconds.

10. A particle moves in a straight line such that its displacement $x(t)$ metres from a fixed point 0, at time t seconds is given by $x = 3 + \ln(t+1)$.

(a) Show that the particle starts from 3 metres to the right of 0.

(b) Determine whether the particle ever is at rest. Why?

(c) What happens to the acceleration as t approaches infinity?

6. (a) $a = 6t(t-2)$

$\therefore\ v = \int 6t(t-2)\,dt$

$= \int 6t^2 - 12t\,dt$

$= 2t^3 - 6t^2 + c$

Now, $t = 0$, $v = 4$

$\therefore\ 4 = 2(0)^3 - 6(0) + c$

$\therefore\ c = 4$

$\therefore\ v = 2t^3 - 6t^2 + 4$

(b) $x = \int v\,dt$

$= \int 2t^3 - 6t^2 + 4\,dt$

$= \frac{t^4}{2} - 2t^3 + 4t + k$

Now, $t = 0$, $x = 0$

$\therefore\ 0 = 0 + k$

$\therefore\ k = 0$

$\therefore\ x = \frac{t^4}{2} - 2t^3 + 4t.$

(c) Subs. $t = 2$ in

$x = \frac{t^4}{2} - 2t^3 + 4t$

$= \frac{(2)^4}{2} - 2(2)^3 + 4(2)$

$= 8 - 16 + 8$

$= 0$

$\therefore$ particle at origin after 2 seconds.

Subs. $t = 2$ in

$v = 2t^3 - 6t^2 + 4$

$= 2(2)^3 - 6(2)^2 + 4$

$= 16 - 24 + 4$

$= -4$

$\therefore$ velocity is 4 ms^{-1} towards the left after 2 seconds (negative direction).

7. (a) $a = 6t - 12$

$v = \int 6t - 12\,dt$

$= 3t^2 - 12t + c$

$t = 0,\ v = 0\ \therefore\ 0 = 0 + c$

$\therefore\ c = 0$

$\therefore\ v = 3t^2 - 12t$

Let $v = 0$,

$\therefore\ 3t^2 - 12t = 0$

$\therefore\ 3t(t-4) = 0$

$t = 0, 4.$

$\therefore$ stops again after 4 seconds.

(b) $x = \int v\,dt$

$\therefore\ x = \int_0^4 3t^2 - 12t\,dt$

$= \left[t^3 - 6t^2\right]_0^4$

$= \left[4^3 - 6(4)^2\right] - [0]$

$= 64 - 96$

$= -32$

$\therefore$ distance of 32 metres travelled in first 4 seconds.

(c) Subs. $t = 6$ in x

$x = t^3 - 6t^2$

$= 6^3 - 6(6)^2$

$= 0$

$\therefore$ back to origin 0 after 6 seconds

$\therefore$ travelled to -32 and then back to 0, i.e. travels 64 metres. (*Remember*, it stopped at $t = 4$.)

8. (a) $v = 3t^2 - 12$

$v = 0$,

$\therefore\ 3t^2 - 12 = 0$

$3t^2 = 12$

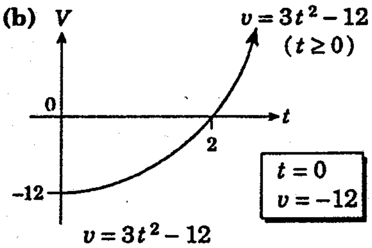
$t^2 = 4$

$\therefore\ t = \pm 2\ \ (t \neq -2)$

$\therefore\ t = 2$

$\therefore$ at rest after 2 seconds.

(b)



$v = 3t^2 - 12$

$= 3(t^2 - 4)$

$\therefore\ v = 3(t-2)(t+2).$

(c) Check $a = \frac{dv}{dt} = 6t > 0,$ as $t \geq 0$

$\therefore$ acceleration is always positive,

$\therefore$ particle never slows down.

(d) $v = 3t^2 - 12$

$x = \int v\,dt$

$= \int 3t^2 - 12\,dt$

$= t^3 - 12t + c.$

Now, $t = 0, x = 0$

$\therefore\ 0 = 0 + c$

$\therefore\ c = 0$

$\therefore\ x = t^3 - 12t$

Now, $x = t(t^2 - 12)$

$\therefore$ subs. $x = 0$

$\therefore\ t(t^2 - 12) = 0$

$t = 0,\quad t^2 - 12 = 0$

$\therefore\ t = 0,\quad t^2 = 12$

i.e. $t = 0$,

$t = \pm\sqrt{12}$ or $\pm 2\sqrt{3}$

$\therefore\ t = 0, 2\sqrt{3}\ \ (t \neq -2\sqrt{3})$

$\therefore$ particle starts at origin, and passes again after $2\sqrt{3}$ seconds.

9. (a) $v = \int \sqrt{2t+1}\,dt$

$\therefore\ v = \int (2t+1)^{\frac{1}{2}}\,dt$

$= \frac{(2t+1)^{\frac{3}{2}}}{\frac{3}{2} \cdot 2} + c$

$\therefore\ v = \frac{\sqrt{(2t+1)^3}}{3} + c$

Now, $t = 0$, $v = 0$

$\therefore\ 0 = \frac{\sqrt{1^3}}{3} + c$

$\therefore\ c = -\frac{1}{3}$

$\therefore\ v = \frac{\sqrt{(2t+1)^3}}{3} - \frac{1}{3}.$

Subs. $t = 4$ in

$v = \frac{\sqrt{(2t+1)^3}}{3} - \frac{1}{3}$

$= \frac{\sqrt{9^3}}{3} - \frac{1}{3}$

$= \frac{27}{3} - \frac{1}{3}$

$= 9 - \frac{1}{3}$

$= 8\frac{2}{3}$

$\therefore$ velocity is $8\frac{2}{3}$ ms^{-1}.

(b) $x=\int\left[\frac{(2t+1)^{\frac{3}{2}}}{3}-\frac{1}{3}\right]dt$

$=\frac{1}{3}\int\left[(2t+1)^{\frac{3}{2}}-1\right]dt$

$=\frac{1}{3}\left[\frac{(2t+1)^{\frac{5}{2}}}{\frac{5}{2}.2}-t\right]+c$

$=\frac{1}{3}\left[\frac{\sqrt{(2t+1)^5}}{5}-t\right]+c.$

Now $t=0, x=0$

$\therefore\ 0=\frac{1}{3}\left[\frac{1}{5}\right]+c$

$\therefore\ c=-\frac{1}{15}$

$\therefore\ x=\frac{1}{3}\left[\frac{\sqrt{(2t+1)^5}}{5}-t\right]-\frac{1}{15}.$

Subs. in $t=4$

$\therefore\ x=\frac{1}{3}\left[\frac{243}{5}-4\right]-\frac{1}{15}$

$=14\cdot8$ (1 decimal place)

$\therefore$ distance is 14·8 metres.

10.(a) $x=3+\ln(t+1)$

Subs. $t=0$ $\therefore\ x=3+\ln 1$

$=3$ $\boxed{\ln 1=0}$

$\therefore$ particle is 3 metres to the right of 0.

(b) $v=\frac{dx}{dt}=\frac{1}{t+1}$

Let $v=0$ $\therefore\ \frac{1}{t+1}=0$

there is no solution,

$\therefore\ v\neq 0$

$\therefore$ particle never rests, as velocity can never be zero.

(c) Now $v=(t+1)^{-1}$

$\therefore\ a=\frac{dv}{dt}=-(t+1)^{-2}.1$

$=\frac{-1}{(t+1)^2}$

Now, if $t\to\infty$

$\therefore\ (t+1)^2\to\infty$

$\therefore\ \frac{1}{(t+1)^2}\to 0,\ a\to 0$

$\therefore$ acceleration gets smaller and smaller (and therefore the velocity approaches a constant).