C.E.M. TUITION

Name : ______________________________

Review Topic : Kinematics

(HSC Course - Paper 1)

Year 12 - 2 Unit

1. A point moving in a straight line is distant x metres from the origin 0 at time t seconds, where $x = t^3 - 3t^2 + 3t + 1$.
   (a) Find the velocity and acceleration at any time t.
   (b) Find the initial velocity and acceleration.
   (c) At what time is the velocity zero?
   (d) At what time is the acceleration zero? Find its velocity and position then.
   (e) Between what times will the velocity be positive?

2. The velocity $v(t)$ ms^{-1} of a particle moving in a straight line at any time t seconds, $t \geq 0$, is $v(t) = 2t - 4$.

(a) When is the particle at rest?

(b) If it is known that the particle passes through the origin after 5 seconds, find an expression for $x(t)$ and hence its initial position.

3. A particle moves such that its velocity after t seconds is given by $v = 3t^2 - 6t$ metres per second. Find:

(a) the distance travelled in the third second;

(b) its acceleration when $t = 2$;

(c) its velocity when the acceleration ceases.

4. The acceleration a metres per second per second of a moving object is given at time t seconds ($t \geq 0$) by $a = 6t - 18$. If the particle starts from the origin, with a velocity of 24 metres per second, find:

(a) when and where the particle comes to rest;

(b) its position and velocity when the acceleration is zero.

5. A particle moves in a straight line, and, at any time t seconds, its displacement from a fixed origin on the line is x metres, where $x = 2 + 2\cos 2t, \quad 0 \le t \le 2\pi$.

(a) Draw the graph of x as a function of t.

(b) Express the velocity v, in terms of t.

(c) For what values of t is the particle stationary?

(d) Show that the acceleration of the particle at the origin is 8 ms^{-2}.

1. (a) $x = t^3 - 3t^2 + 3t + 1$

$v = \frac{dx}{dt} = 3t^2 - 6t + 3$

$\therefore$ velocity $v = 3t^2 - 6t + 3$

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 6t - 6$

$\therefore$ acceleration $a = 6t - 6$.

(b) Initial $\therefore t = 0$

$\therefore$ subs. $t = 0$ in

$v = 3t^2 - 6t + 3$
$= 3$

$\therefore$ initially, velocity is 3 ms^{-1}.

Also, subs $t = 0$ in $a = 6t - 6$

$\therefore a = -6$

$\therefore$ initially, acceleration is -6 ms^{-2}.

(c) $v = 0$ $\therefore 3t^2 - 6t + 3 = 0$

$\therefore t^2 - 2t + 1 = 0$

$(t-1)^2 = 0$

$t = 1$

$\therefore$ velocity zero after 1 second.

(d) $a = 0$ $\therefore 6t - 6 = 0$

$6t = 6$

$t = 1$

Now, from (c),

when $t = 1$ $\therefore v = 0$

Also, subs. $t = 1$ in

$x = t^3 - 3t^2 + 3t + 1$
$= (1)^3 - 3(1)^2 + 3(1) + 1$
$= 1 - 3 + 3 + 1$
$= 2$

$\therefore$ acceleration zero after one second, when velocity is 0 ms^{-1} and its position is 2 m to the right of origin.

(e) $v > 0$ $\therefore 3t^2 - 6t + 3 > 0$

$\therefore t^2 - 2t + 1 > 0$

$(t-1)^2 > 0$

Now, $(t-1)^2 > 0$ for all t except $t = 1$ [which gives $(1-1)^2 = 0$],

$\therefore$ velocity positive

$0 \le t < 1$ and $t > 1$.

2. (a) $v(t) = 2t - 4$

at rest $\therefore v(t) = 0$

$\therefore 2t - 4 = 0$

$2t = 4$

$t = 2$

$\therefore$ particle at rest after 2 seconds.

(b) $x(t) = \int v\, dt$

$\therefore x(t) = \int 2t - 4\, dt$

$\therefore x(t) = t^2 - 4t + c$

Now, $x = 0$ when $t = 5$

$\therefore 0 = 5^2 - 4(5) + c$

$\therefore 0 = 25 - 20 + c$

$0 = 5 + c$

$c = -5$

$\therefore x = t^2 - 4t - 5$

$\therefore x(t) = t^2 - 4t - 5$

Now, initial position

$\therefore t = 0$,

$\therefore$ subs. $t = 0$ in x

i.e. $x = 0^2 - 4(0) - 5$
$= -5$

$\therefore$ initially, particle is 5 metres to the left of the origin 0.

3. (a) $v = 3t^2 - 6t$

$\therefore x = \int_2^3 3t^2 - 6t\, dt$

third second is from $t = 2$ to $t = 3$

$= \left[t^3 - 3t^2\right]_2^3$
$= (27 - 27) - (8 - 12)$
$= 0 - (-4)$
$= 0 + 4$
$= 4$

$\therefore$ distance travelled is 4 m.

(b) $v = 3t^2 - 6t$

$\therefore a = 6t - 6$

Subs. $t = 2$ in $a = 6t - 6$
$= 6(2) - 6$
$= 6$

$\therefore$ acceleration is 6 ms^{-2}.

(c) $a = 6t - 6$

acceleration ceases, $\therefore a = 0$

$\therefore 6t - 6 = 0$

$6t = 6$

$\therefore t = 1$

Now, subs. $t = 1$ in

$v = 3t^2 - 6t$
$= 3(1)^2 - 6(1)$
$= 3 - 6$
$= -3$

$\therefore$ velocity is -3 ms^{-1} when acceleration ceases (at 1 second).

4. (a) $a = 6t - 18$

$v = \int 6t - 18\, dt$

$\therefore v = 3t^2 - 18t + c$

Now, $t = 0$, $v = 24$

$\therefore 24 = 3(0)^2 - 18(0) + c$

$\therefore c = 24$

$\therefore v = 3t^2 - 18t + 24$

Comes to rest $\therefore v = 0$

$\therefore 3t^2 - 18t + 24 = 0$

$\therefore t^2 - 6t + 8 = 0$

$(t-4)(t-2) = 0$

$t = 4, 2.$

Particle comes to rest after 2 seconds and again after 4 seconds.

Now, $x = \int 3t^2 - 18t + 24\, dt$
$= t^3 - 9t^2 + 24t + k$

But, 'starts from origin',

$\therefore x = 0,\ t = 0$ in

$x = t^3 - 9t^2 + 24t + k$

$\therefore 0 = k$

$\therefore k = 0$

$\therefore x = t^3 - 9t^2 + 24t$

Now, subs. $t = 4$ in

$x = t^3 - 9t^2 + 24t$
$= 4^3 - 9(4)^2 + 24(4)$
$= 64 - 144 + 96$
$= 16.$

Also, subs. $t = 2$ in

$x = t^3 - 9t^2 + 24t$
$= 2^3 - 9(2)^2 + 24(2)$
$= 8 - 36 + 48$
$= 20.$

$\therefore$ particle comes to rest after 2 seconds at 20 m to right of origin and after 4 seconds at 16 m to the right of origin.

(b) acceleration zero,

$\therefore a = 0$

$\therefore 6t - 18 = 0$

$6t = 18$

$t = 3$

Subs. $t = 3$ in

$x = t^3 - 9t^2 + 24t$
$= (3)^3 - 9(3)^2 + 24(3)$
$= 27 - 81 + 72$
$= 18.$

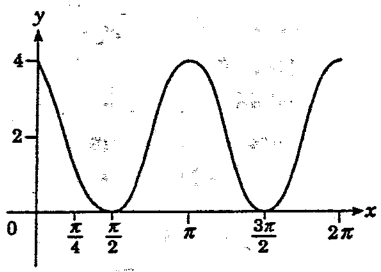
Also, subs. $t = 3$ in

$v = 3t^2 - 18t + 24$

$\therefore v = 3(3)^3 - 18(3) + 24$
$= 27 - 54 + 24$
$= -3.$

$\therefore$ the object's acceleration is zero, at position 18 m to the right of the origin where the velocity is 3 ms^{-1} towards the origin.

5. (a) $x = 2 + 2\cos 2t$



(b) $v = -4\sin 2t$

(c) Let $v = 0$

$\therefore -4\sin 2t = 0$

$\therefore \sin 2t = 0$

$\therefore 2t = 0, \pi, 2\pi, 3\pi, 4\pi$

$t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$\therefore$ particle stationary if $t = 0$, $\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.

(d) $a = \frac{dv}{dt} = -8\cos 2t$

Now, subs. $x = 0$ in

$x = 2 + 2\cos 2t$

$0 = 2 + 2\cos 2t$

$2\cos 2t = -2$

$\cos 2t = -1$

$\therefore 2t = \pi, 3\pi$

$\therefore t = \frac{\pi}{2}, \frac{3\pi}{2}$.

Now subs. $t = \frac{\pi}{2}$ in

$a = -8\cos 2t$

$= -8\cos\pi$

$= -8 \times -1$

$= 8.$

Similarly for $t = \frac{3\pi}{2}$

$\therefore$ acceleration at the origin is 8 ms^{-2}.