C.E.M.TUITION

Name : ______________________________

Review Topic : Kinematics

(HSC Course - Paper 3)

Year 12 - 2 Unit

11. A particle moves so that its displacement x metres from an origin after t seconds is given by $x = t^3 - 6t^2 + 9t + 1$.

(a) Show that the particle will move forward for one second, and then return passing its starting point after three seconds.

(b) Find the total distance travelled in the first three seconds.

(c) Show that the particle reaches it greatest velocity after 2 seconds and determine where this occurs.

12. For a particle moving in a straight line, its acceleration at time t seconds is given as $a = 6t$. Its velocity is 10 ms^{-1} after 2 seconds at a point 1 metre to the right of a fixed point 0.

(a) Find an expression for the velocity v, in terms of t.

(b) Find an expression for the displacement x, in terms of t.

(c) Show that the particle must pass through 0 during the second second.

13. A particle moves along a straight line so that its distance x, in metres, from a fixed point 0, is given by $x = 1 - \cos \pi t$, where the time t is measured in seconds from $t = 0$.

(a) Where is the particle initially?

(b) Sketch the graph of x as a function of t, where $0 \leq t \leq 3$.

(c) Where and when is the particle next at rest?

14. The acceleration a metres per second per second of a moving object is given at time t seconds, by $a = 4\sin 2t$. At time $t = 0$, the object is at a point $x = 0$ and travelling with velocity $v = -1$ metres per second.

(a) Find the velocity v and the displacement x as a function of t, for $t \geq 0$.

(b) Find, for $t \leq \pi$, the values of t for which the object is stationary.

(c) Find, for $t \leq \pi$, the largest value of x.

15. A particle moving in a straight line is distant x metres from the origin 0 at time t seconds, where $x = t(t-2)^2$.

(a) Find when the particle is at the origin.

(b) Show that $v = (t-2)(3t-2)$.

(c) On a number plane, graph velocity v as a function of time t.

(d) Find where the particle is stationary, and its acceleration at those points.

11. (a) $x = t^3 - 6t^2 + 9t + 1$

$\therefore\ v = \dfrac{dx}{dt} = 3t^2 - 12t + 9$

Let $v = 0$

$\therefore\ 3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$\therefore\ (t-3)(t-1) = 0$

$t = 3, 1$

$\therefore$ particle stops after one second and three seconds.

Now, check position of particle:

subs. $t = 0$ in x $\therefore x = 1$

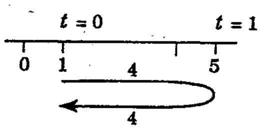
subs. $t = 1$ in x

$\therefore\ x = 1 - 6 + 9 + 1$
$= 5$

subs. $t = 3$ in x

$\therefore\ x = (3)^3 - 6(3)^2 + 9(3) + 1$
$= 27 - 54 + 27 + 1$
$= 1$

$\therefore$ particle begins at one unit to right of origin, is five units to right of origin after one second and returns to one unit to right of origin after 3 seconds.



(b) From the diagram in (a), particle travels 8 metres in first 3 seconds.

(c) $v = \dfrac{dx}{dt} = 3t^2 - 12t + 9$

Greatest velocity when acceleration is zero.

$$\left(\frac{dv}{dt} = 0\right)$$

$\therefore\ a = \dfrac{dv}{dt} = 6t - 12 = 0$

$\therefore\ 6t = 12$

$t = 2.$

Subs. $t = 2$ in x

$\therefore\ x = (2)^3 - 6(2)^2 + 9(2) + 1$
$= 8 - 24 + 18 + 1$
$= 3$

$\therefore$ greatest velocity at a position 3 metres to the right of origin.

12. (a) $a = 6t$

$v = \int 6t\ dt$
$= 3t^2 + c$

Now, $t = 2$, $v = 10$

$\therefore\ 10 = 3(2)^2 + c$

$10 = 12 + c$

$\therefore\ c = -2$

$\therefore\ v = 3t^2 - 2.$

(b) $x = \int v\ dt$
$= \int 3t^2 - 2\ dt$
$= t^3 - 2t + k$

Now, $t = 2, x = 1$

$\therefore\ 1 = (2)^3 - 2(2) + k$

$1 = 8 - 4 + k$

$1 = 4 + k$

$\therefore\ k = -3$

$\therefore\ x = t^3 - 2t - 3.$

(c) Subs. $t = 1$ in x

$\therefore\ x = (1)^3 - 2(1) - 3$
$= 1 - 2 - 3$
$= -4.$

Subs. $t = 2$ in x

$\therefore\ x = (2)^3 - 2(2) - 3$
$= 8 - 4 - 3$
$= 1$

$\therefore$ particle is 4 m to left of 0 after 1 second and 1 m to right of 0 after 2 seconds,

$\therefore$ passes through 0 in second second.

13. (a) $x = 1 - \cos \pi t$

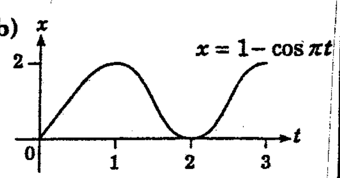
Subs. $t = 0$ in x

$\therefore\ x = 1 - \cos 0$
$= 1 - 1$

$\therefore\ x = 0$

$\therefore$ particle is at origin 0.

(b)



(c) $x = 1 - \cos \pi t$

$v = \dfrac{dx}{dt} = \pi \sin \pi t = 0$

$\therefore\ \sin \pi t = 0$

$\pi t = 0, \pi, 2\pi, 3\pi$

$\therefore\ t = 0, 1, 2, 3.$

Particle is first at rest when $t = 0$,

$\therefore$ next at rest at $t = 1$.

Now, subs $t = 1$ in x

$\therefore\ x = 1 - \cos \pi(1)$
$= 1 - \cos 1$
$= 0$

$\therefore$ next at rest after 1 second at origin 0.

14. (a) $a = 4 \sin 2t$

$v = \int a\ dt = \int 4 \sin 2t\ dt$
$= -2 \cos 2t + c.$

Now, $t = 0$, $v = -1$

$\therefore\ -1 = -2\cos 0 + c$

$-1 = -2 + c$

$\therefore\ c = 1$

$v = 1 - 2\cos 2t$

$\therefore$ velocity is $v = 1 - 2\cos 2t$

$x = \int v\ dt$
$= \int 1 - 2\cos 2t\ dt$
$= t - \sin 2t + k.$

Now, $t = 0$, $x = 0$

$\therefore\ 0 = 0 - \sin 0 + k$

$\therefore\ k = 0$

$\therefore\ x = t - \sin 2t$

(b) $v = 0$ $\therefore\ 1 - 2\cos 2t = 0$

$2\cos 2t = 1$

$\cos 2t = \dfrac{1}{2}$

$\therefore\ 2t = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

$\therefore\ t = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

$\therefore$ object is stationary after $\frac{\pi}{6}$ seconds and $\frac{5\pi}{6}$ seconds.

(c) Largest value of x occurs when $v = 0$ or at end-points of domain (i.e. $t = 0, t = \pi$).

Subs. $t = 0$ in x

$\therefore\ x = 0 - \sin 0$
$= 0$

Subs. $t = \pi$ in x

$\therefore\ x = \pi - \sin \pi$
$= \pi$

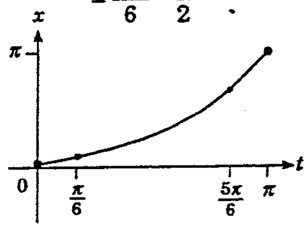
Subs. $t = \dfrac{\pi}{6}$ in x

$\therefore\ x = \dfrac{\pi}{6} - \sin\dfrac{\pi}{6}$
$= \dfrac{\pi}{6} - \dfrac{1}{2}$

Subs. $t = \dfrac{5\pi}{6}$ in x

$\therefore \quad x = \frac{5\pi}{6} - \sin\frac{5\pi}{6}$

$= \frac{5\pi}{6} - \frac{1}{2}$



$\therefore$ greatest value of x is π when $t = \pi$.

15. (a) $x = t(t-2)^2$

$x = 0 \quad \therefore 0 = t(t-2)^2$

$\therefore \quad t(t-2)^2 = 0$

$\therefore \quad t = 0, 2$

$\therefore$ at origin initially and after 2 seconds.

(b) $v = \frac{dx}{dt}$ [Using Product Rule]

$= (t-2)^2.1 + t.2(t-2).1$

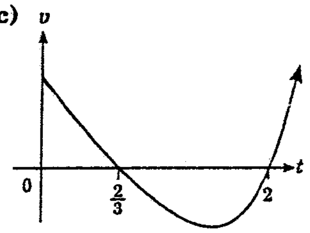
$= (t-2)^2 + 2t(t-2)$

$= (t-2)[t-2+2t]$

$= (t-2)(3t-2)$

$\therefore \; v = (t-2)(3t-2)$

(c)



(d) Stationary when $v = 0$

$\therefore \; (t-2)(3t-2) = 0$

$t = 2, \frac{2}{3}$. [When velocity function cuts t axis.]

Subs. $t = 2$ in x

$\therefore \quad x = 2(2-2)^2$

$= 0$

Subs. $t = \frac{2}{3}$ in x

$\therefore \quad x = \frac{2}{3}(\frac{2}{3} - 2)^2$

$= \frac{2}{3}\left(\frac{16}{9}\right) = \frac{32}{27}$

$= 1\frac{5}{27}$.

Now, $a = \frac{dv}{dt}$

$= (3t-2)1 + (t-2)3$

[or $v = 3t^2 - 8t + 4$, $\frac{dv}{dt} = 6t - 8$]

$= 3t - 2 + 3t - 6$

$= 6t - 8$.

Subs. $t = 2$ in a

$\therefore \quad a = 6(2) - 8$

$= 4$

Subs. $t = \frac{2}{3}$ in a

$\therefore \quad a = 6\left(\frac{2}{3}\right) - 8$

$= -4$

$\therefore$ particle is stationary after $\frac{2}{3}$ second at $1\frac{5}{27}$ m to right of 0 with acceleration of -4 ms^{-2}, and stationary after 2 seconds at origin 0 with acceleration of 4 ms^{-2}.