

C.E.M. TUITION

Student Name : _____

**Review Topic : Rates of change
(HSC - Paper 1)**

Year 12 - 2 Unit


1995

1. When Nicolium crystals are placed in solution, they dissolve in such a way that their volume, $V \text{ mm}^3$, is given by

$$V = 20t - \frac{t^2}{4} + 2.$$

- (a) At what rate do the crystals dissolve at the end of 7 minutes?
- (b) What is the initial rate at which the crystals dissolve?
- (c) After how many minutes are the crystals dissolving at the rate of 10 cubic millimetres per minute?
- (d) How long does it take for the crystals to dissolve completely?
(Give your answer correct to the nearest minute.)

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2. Water was poured into a tank for 10 hours until it was full. At any time, t hours, the volume, V litres, of water in the tank was given by $V = 2(20t - t^2 + 100)$.
- (a) How much water was in the tank initially?
 - (b) How many litres of water were in the tank when it was full?
 - (c) At what rate was water poured into the tank at the end of 5 hours?
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3. Suppose that oil flows out of a pipe at the rate, R cubic metres per minute, where $R = 2t + 25$ and t is time in minutes.
- (a) Find the rate of flow of oil at the end of 10 minutes.
 - (b) Draw a sketch of R as a function of t .
 - (c) Calculate the total volume of oil that flows through the pipe in the first 10 minutes.
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4. A water tank had 5000 litres of water in it. Water is flowing into the tank at the rate R litres per minute, where $R = 1.4t$ and t is time in minutes.
- (a) Find the formula for the volume, V litres of water in the tank at any time t .
- (b) How much water is in the tank after 8 minutes?
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5. The rate, R grams per minute, at which a certain chemical compound is formed during a chemical reaction is given by the relationship

$$R = 20 + \frac{10}{2t + 2}, \text{ where } t \text{ is time in minutes.}$$

- (a) At what rate is the compound formed at the end of 4 minutes?
- (b) What is the value of R as t becomes very large?
- (c) Draw a sketch of R as a function of T .
- (d) How many grams of the compound are formed in the first 5 minutes of the chemical reaction?

6. A hazardous chemical leaks out from the bottom of a storage tank at the rate

$$\frac{dV}{dt} = 30e^{-0.006t},$$

where V is the volume of the chemical remaining in the tank in litres and t is time in hours.

- (a) At what rate is the chemical leaking out of the tank after 5 hours?
- (b) If $R = \frac{dV}{dt}$, draw a sketch of R as a function of t .
- (c) How much chemical leaks out of the tank in the first 20 hours?

1. (a) $V = 20t - \frac{t^2}{4} + 2$

Note $\frac{t^2}{4} = \frac{1}{4}t^2$

$= 20t - \frac{1}{4}t^2 + 2$

$\frac{dV}{dt} = 20 - \frac{1}{2}t$ (Rate = $\frac{dV}{dt}$)

when $t = 7$, $\frac{dV}{dt} = 20 - \frac{1}{2} \times 7 = 16.5$.

Therefore, the rate at which the crystals dissolve at the end of 7 minutes is 16.5 mm³ per minute.

(b) The initial rate \Rightarrow find $\frac{dV}{dt}$, when $t = 0$.

$t = 0$, $\frac{dV}{dt} = 20 - \frac{1}{2}t = 20 - \frac{1}{2} \times 0 = 20$.

Therefore, the initial rate is 20 mm³ per minute.

(c) In this question we are asked to find t , given $\frac{dV}{dt} = 10$.

$\frac{dV}{dt} = 20 - \frac{1}{2}t$, when $\frac{dV}{dt} = 10$
 $20 - \frac{1}{2}t = 10$

(Multiply both sides by 2.)
 $40 - t = 20$
 $t = 20$.

Therefore, after 20 minutes the crystals dissolve at the rate of 10 mm³ per minute?

(d) The crystals dissolve completely when $V = 0$.

$V = 20t - \frac{t^2}{4} + 2$

When $V = 0$,

$20t - \frac{t^2}{4} + 2 = 0$

(Multiply both sides by 4.)

$80t - t^2 + 8 = 0$

(Multiply both sides by -1.)

$t^2 - 80t - 8 = 0$

(Use Quadratic Formula to solve)

$a = 1$, $b = -80$, $c = -8$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-80) \pm \sqrt{(-80)^2 - 4(1)(-8)}}{2 \times 1}$
 $= \frac{-(-80) \pm \sqrt{6432}}{2}$
 $= \frac{80 + \sqrt{6432}}{2}$ or $\frac{80 - \sqrt{6432}}{2}$

$= 80.0998$ or -0.0998
 ≈ 80.1 or -0.1 (1dp)
 $t = -0.1$ is meaningless, and discarded. Therefore the crystals dissolve completely after 80 minutes.

2. $V = 2(20t - t^2 + 100)$
 $V = 40t - 2t^2 + 200$

(a) Initially (i.e. $t = 0$),
 $V = 40 \times 0 - 2 \times (0^2) + 200 = 200$

Therefore, initially there were 200 litres of water in the tank.

(b) The tank is completely full after 10 hours.

When $t = 10$,
 $V = 40t - 2t^2 + 200$
 $V = (40 \times 10) - 2 \times (10^2) + 200 = 400$.

Therefore, there were 400 litres of water when the tank was full.

(c) $V = 40t - 2t^2 + 200$

$\frac{dV}{dt} = 40 - 4t$ [$\frac{dV}{dt} = \text{rate}$]

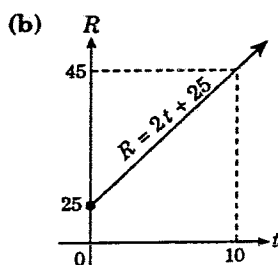
When $t = 5$,

$\frac{dV}{dt} = 40 - 4 \times 5 = 20$.

\therefore After 5 hours water poured in the tank at the rate of 20 litres/h.

3. (a) $R = 2t + 25$
 When $t = 10$, $R = 2 \times 10 + 25 = 45$.

Therefore, the rate of flow of oil at the end of 10 minutes is 45 m³ per minute.



(c) R represents the rate of flow, i.e. $\frac{dV}{dt} = R = 2t + 25$

Therefore, the total volume of oil that flows through the pipe in the first 10 minutes is given by:

$\int_0^{10} R dt = \int_0^{10} (2t + 25) dt$
 $= [t^2 + 25t]_0^{10}$

$= [10^2 + 25(10)] - [0 + 0]$
 $= (100 + 250) - (0)$
 $= 350$.

Therefore, 350 litres of oil flows through the pipe in the first 10 minutes.

4. (a) $R = 1.4t$

$\frac{dV}{dt} = 1.4t$

Note $R = \frac{dV}{dt}$

and when $t = 0$, $V = 5000$.

$\frac{dV}{dt} = 1.4t$

$V = \int 1.4t dt$

$V = 0.7t^2 + c$,

when $t = 0$, $V = 5000$,
 $5000 = 0 + c \Rightarrow c = 5000$
 $\therefore V = 0.7t^2 + 5000$.

(b) When $t = 8$,

$V = 0.7t^2 + 5000$
 $= 0.7 \times 8^2 + 5000$
 $= 5044.8$.

Therefore, after 8 minutes the tank contains approximately 5045 litres of water.

5. (a) $R = 20 + \frac{10}{2t + 2}$

when $t = 4$, $R = 20 + \frac{10}{8 + 2}$
 $= 20 + \frac{10}{10}$
 $= 20 + 1$
 $= 21$.

Therefore, at the end of 4 minutes the compound is formed at the rate of 21 grams per minute.

(b) As t becomes very large, (i.e. $t \rightarrow \infty$)

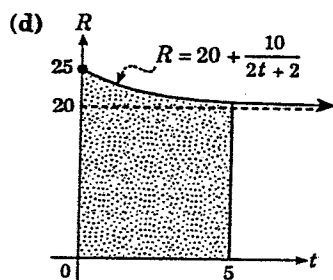
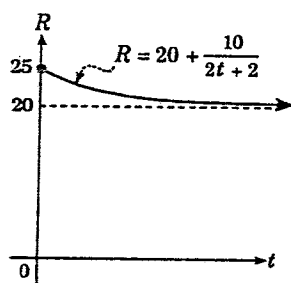
$R \rightarrow 20 + 0 \rightarrow 20$

Note $\lim_{t \rightarrow \infty} \frac{10}{2t + 2} = 0$

Therefore, as t becomes very large, R approaches 20 grams/minute.

(c) To sketch $R = 20 + \frac{10}{2t+2}$

Note When $t = 0$
 $R = 20 + \frac{10}{0+2}$
 $= 20 + 5 = 25$
 as $t \rightarrow \infty$, $R \rightarrow 20$
 $t \geq 0$, $R \geq 20$.



The amount of compound formed is given by the shaded area under the curve,

$$\begin{aligned} &= \int_0^5 R dt \\ &= \int_0^5 \left(20 + \frac{10}{2t+2} \right) dt \\ &= [20t + 5 \ln(2t+2)]_0^5 \\ &= (100 + 5 \ln 12) - (0 + 5 \ln 2) \\ &= 100 + 5 \ln 12 - 5 \ln 2 \\ &= 100 + 5(\ln 12 - \ln 2) \\ &= 100 + 5 \ln \left(\frac{12}{2} \right) \end{aligned}$$

Note $\log \frac{a}{b} = \log a - \log b$

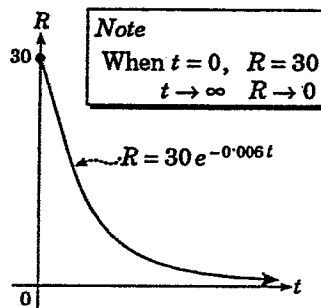
$= 100 + 5 \ln 6$.
 Therefore, the amount of compound formed in the first 5 minutes of the reaction is $100 + 5 \ln 6 \approx 109$ grams.

6. (a) $\frac{dV}{dt} = 30e^{-0.006t}$
 When $t = 5$, $\frac{dV}{dt} = 30e^{-0.006 \times 5}$
 $= 29.1$.

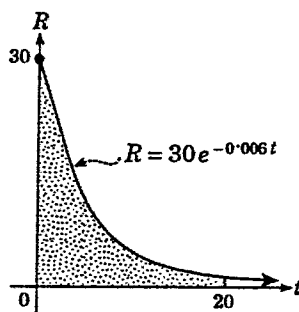
Therefore, after 5 hours, the chemical is leaking out of the tank at the rate of 29.1 litres per hour.

(b) To sketch

$$R = \frac{dV}{dt} = 30e^{-0.006t}$$



(c) The amount of chemical that leaks out of the tank in the first 20 hours is represented by the area under the curve $R = f(t)$ between $t = 0$ and $t = 20$.



Area under curve

$$\begin{aligned} &= \int_0^{20} R dt \\ &= \int_0^{20} 30e^{-0.006t} dt \\ &= \left[\frac{30e^{-0.006t}}{-0.006} \right]_0^{20} \\ &= [-5000e^{-0.006t}]_0^{20} \\ &= -5000[e^{-0.006t}]_0^{20} \\ &= -5000[e^{-0.006 \times 20} - e^{-0.006 \times 0}] \\ &= -5000[e^{-0.12} - e^0] \\ &= -5000[e^{-0.12} - 1] \end{aligned}$$

Note $e^0 = 1$
 $= 565.39782$
 ≈ 565 (nearest whole number).

Therefore 565 litres of the compound leaks out of the tank in the first 20 hours.