C.E.M. TUITION

Student Name : ______________________________

Review Topic : Rates of change
(HSC - Paper 2)

Year 12 - 2 Unit

1995

7. The atmospheric pressure, P kilopascals, at a height h kilometres above sea-level, is given by the formula $P = 100e^{-0 \cdot 13h}$.
   (a) Find the atmospheric pressure at sea-level.
   (b) At what rate is the atmospheric pressure decreasing 2 kilometres above sea-level?

8. The rate of sales, R in dollars per day, of a certain Australian company is given by $R = 2t + 120$, where t is the time in days.

(a) What is the rate of sales at the end of 10 days?

(b) Draw a sketch of R as a function of t.

(c) Calculate the total amount of sales in the first 8 days.

(d) On what day will the accumulated sales exceed $10 000?

9. Water flows from a full tank at the rate given by $\frac{dh}{dt} = -1{\cdot}5\sqrt{t}$ where h is the depth of water in metres at any time, t minutes. The height of the tank is 20 metres.

(a) Find an equation for h in terms of t.

(b) How long does it take for the tank to empty?

10. The rate of water flow into a tank which initially contains 900 litres of water is given by the relationship

$$\frac{dV}{dt} = 6(6 - t),$$

where V litres is the volume of water in the tank after t minutes.

(a) Find the volume of water in the tank after 10 minutes.

(b) How long does it take for the tank to empty? (Give your answer to the nearest minute.)

7. $P=100e^{-0\cdot13h}$

(a) At sea-level, $h=0$.
When $h=0$,

$P=100e^{-0\cdot13\times0}$
$=100e^{0}$ **Note $e^0=1$**
$=100.$

Therefore the atmospheric pressure at sea-level is 100 kilopascals.

(b) $P=100e^{-0\cdot13h}$

$\frac{dP}{dh}=100e^{-0\cdot13h}\times-0\cdot13$
$=-13e^{-0\cdot13h}.$

When $h=2$,

$\frac{dP}{dh}=-13e^{-0\cdot13\times2}$
$=-10\cdot023\,671$
≈-10 (nearest whole no.).

> The negative value indicates that as height (h) increases, the pressure (P) decreases,
> i.e. the rate is –ve.

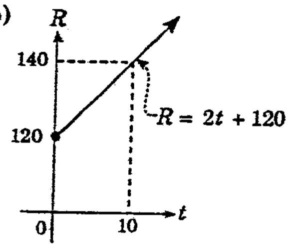
Therefore, at a height of 2 kilometres above sea-level the atmospheric pressure decreases at the rate of approximately 10 kilopascals per kilometre.

8. (a) $R=2t+120$

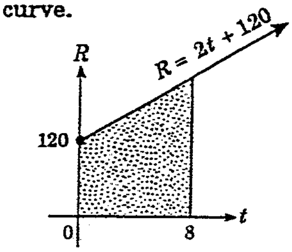
When $t=10$, $R=2\times10+120$
$=20+120$
$=140.$

Therefore, after 10 days the rate of sales is $140 per day.

(b)



(c) The total amount of sales in the first 8 days is represented by the shaded area under the curve.



Therefore, accumulated sales for the first 8 days is given by:

$\int_0^8 R\,dt$
$=\int_0^8 (2t+120)\,dt$
$=\left[t^2+120t\right]_0^8$
$=\left[8^2+120(8)\right]-[0+0]$
$=1024.$

Therefore, the total amount of sales in the fist 8 days is equal to $1024.

(d) To find the day which the accumulated sales exceed $10 000 we need to find the value of k such that

$\int_0^k (2t+120)\,dt>10\,000$

where k is the required day.

$\int_0^k (2t+120)\,dt>10\,000$
$\left[t^2+120t\right]_0^k>10\,000$
$\left(k^2+120k\right)-(0+0)>10\,000$
$k^2+120k>10\,000$
$k^2+120k-10\,000>0.$

> Solve $k^2+120k-10\,000=0$ by using the Quadratic Formula:
>
> $k=\frac{-120\pm\sqrt{120^2-4(1)(-10\,000)}}{2\times1}$
> $=\frac{-120\pm\sqrt{54\,400}}{2}$
> $=\frac{-120+\sqrt{54\,400}}{2}$ or $\frac{-120-\sqrt{54\,400}}{2}$
> $\approx56\cdot6$ or $-176\cdot6$.

Neglect the negative value since $k>0$ (since it represents time).

Therefore $k=56\cdot6$. But k is a required day, therefore it must have an integral value ($k=57$).

Therefore, on the 57th day the accumulated sales exceed $10 000?

9. (a) Need to find h where $\frac{dh}{dt}=-1\cdot5\sqrt{t}$, and when $t=0$, $h=20$.

$h=\int-1\cdot5\sqrt{t}\,dt$
$=\int-1\cdot5t^{\frac{1}{2}}\,dt$ **Note $\sqrt{t}=t^{\frac{1}{2}}$**
$=\frac{-1\cdot5t^{\frac{3}{2}}}{\frac{3}{2}}+C$
$=-t^{\frac{3}{2}}+C$

i.e. $h=-\sqrt{t^3}+C$. **Note. $t^{\frac{3}{2}}=\sqrt{t^3}$**

But when $t=0$, $h=20$,

$\therefore\ 20=-\sqrt{0^3}+C$
$20=0+C\Rightarrow C=20$
$\therefore\ h=-\sqrt{t^3}+20$
i.e. $h=20-\sqrt{t^3}$.

(b) The tank is empty when $h=0$, $h=20-\sqrt{t^3}$.
When $h=0$,

$0=20-\sqrt{t^3}$
$\sqrt{t^3}=20$
(square both sides)
$t^3=400$
(take cube root on both sides)
$t=\sqrt[3]{400}$
$\approx7\cdot4$ (1dp).

Therefore, it takes approximately 7·4 hours for the tank to empty.

10.(a) First we need to find V, where $\frac{dV}{dt}=6(6-t)$, and when $t=0$, $V=900$.

$\frac{dV}{dt}=6(6-t)$ expand first

$=36-6t$

$V=\int(36-6t)\,dt$

$V=36t-3t^2+C$

when $t=0,\ V=900$

$\therefore\ 900=0-0+C \Rightarrow C=900$

$\therefore\quad V=36t-3t^2+900$

V represents the volume of water in the tank at any time *t*.

When $t=10$, and

$V=36t-3t^2+900$

$=36(10)-3(10)^2+900$

$=360-300+900$

$=960.$

Therefore, after 10 minutes the tank will contain 960 litres of water.

(b) The tank is empty when $V=0$ (i.e. there is no water in the tank).

$V=36t-3t^2+900$

When $V=0$,

$0=36t-3t^2+900$

$36t-3t^2+900=0$

(divide both sides by -3)

$t^2-12t-300=0$

(quadratic equation) Solve it by using the Quadratic formula.

$t=\frac{12\pm\sqrt{(-12)^2-4(1)(-300)}}{2}$

$=\frac{12\pm\sqrt{1344}}{2}$

$=\frac{12+\sqrt{1344}}{2}$ or $\frac{12-\sqrt{1344}}{2}$

$=24{\cdot}330\,303$ or $-12{\cdot}330\,303$

≈ 24 or -12 (nearest whole no.)

Neglect the negative value, since $t\geq 0$, i.e. $t=24$ (nearest whole number). Therefore, the tank is empty after 24 minutes.