C.E.M.TUITION

Name :_____

Topic : Further Trigonometry

(Specimen Paper)

Year 12 - Extension 1 Maths

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(1) Given that $\sin A = \dfrac{8}{17}$ and $\cos B = \dfrac{3}{5}$ where both A and B are acute angles. Find the exact values of

(a) $\sin 2A$ [2]

(b) $\cos (A - B)$ [2]

(2) Express the term $\tan(x - 45°)$ in terms of $\tan x$.

Hence solve the equation
$$\tan(x - 45°) = 6\tan x,$$

giving all solutions between $0°$ and $360°$ [6]

(3) Prove the identity

$$\frac{\sin(\theta+60)-\sin(\theta+30)}{\cos(\theta+60)+\cos(\theta+30)} = \tan 15$$

[4]

(4) Use the formulae for $\cos(A + B)$ and $\cos(A - B)$ to prove that

(a) $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

(b) $\sin^2 A = \dfrac{1}{2}(1 - \cos 2A)$ [4]

Solutions:

(1)
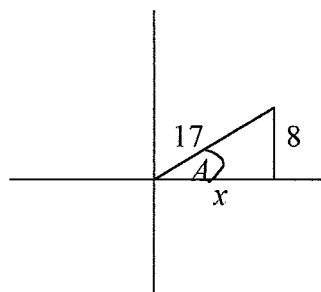
By Pythagoras' theorem
$$17^2 = 8^2 + x^2$$
$$\Rightarrow \qquad x^2 = 225$$
$$\Rightarrow \qquad x = 15$$
$$\Rightarrow \qquad \cos A = \frac{15}{17}$$

$$\sin B = \frac{4}{5}$$
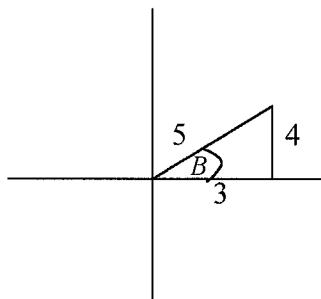
(a)
$$\sin 2A = 2 \sin A \cos A$$
$$= 2 \times \frac{8}{17} \times \frac{15}{17}$$
$$\sin 2A = \frac{240}{289}$$

(b)
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$= \frac{15}{17} \times \frac{3}{5} + \frac{8}{17} \times \frac{4}{5}$$
$$\cos (A - B) = \frac{77}{85}$$

(2)

Using

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(x - 45°) = \frac{\tan x - \tan 45°}{1 + \tan x \tan 45°}$$

$$\tan(x - 45°) = \frac{\tan x - 1}{1 + \tan x}$$

$$\tan(x - 45°) = 6 \tan x$$

$\Rightarrow \qquad \dfrac{\tan x - 1}{1 + \tan x} = 6 \tan x$

$\times(1 + \tan x) \qquad \tan x - 1 = 6 \tan x(1 + \tan x)$

$\qquad \qquad \tan x - 1 = 6 \tan x + 6 \tan^2 x$

$\Rightarrow \qquad 6 \tan^2 x + 5 \tan x + 1 = 0$

$\Rightarrow \qquad (3 \tan x + 1)(2 \tan x + 1) = 0$

$\Rightarrow \qquad 3 \tan x + 1 = 0 \quad$ or $\quad 2 \tan x + 1 = 0$

$$\tan x = -\frac{1}{3}, \qquad \tan x = -\frac{1}{2}$$

$$\tan^{-1}\frac{1}{3} = 18.4° \qquad \tan^{-1}\frac{1}{2} = 26.6°$$

tangent is negative in 2nd and 4th quadrants

$x = 161.1°, 341.6°$

$x = 153.4°, 333.4°$
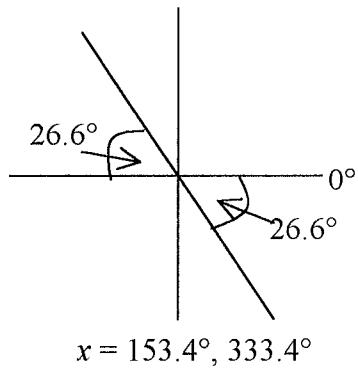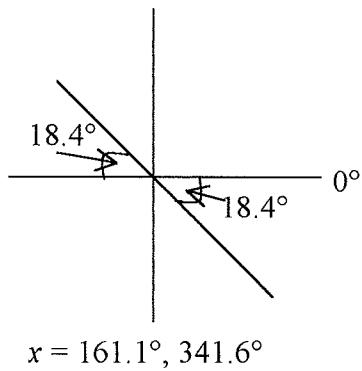
(3) Using the factor formula for the difference of two sines and the sum of two cosines,

$$\frac{\sin(\theta+60)-\sin(\theta+30)}{\cos(\theta+60)+\cos(\theta+30)} =$$

$$\frac{\cancel{2}\sin\frac{1}{2}\left[(\theta+60)-(\theta+30)\right]\cos\frac{1}{2}\left[(\theta+60)+(\theta+30)\right]}{\cancel{2}\cos\frac{1}{2}\left[(\theta+60)+(\theta+30)\right]\cos\frac{1}{2}\left[(\theta+60)-(\theta+30)\right]}$$

$$= \frac{\sin\frac{1}{2}(30)\cos\frac{1}{2}(2\theta+90)}{\cos\frac{1}{2}(2\theta+90)\cos\frac{1}{2}(30)}$$

$$= \frac{\sin15\,\cancel{\cos(\theta+45)}}{\cancel{\cos(\theta+45)}\,\cos15}$$

$$= \frac{\sin15}{\cos15}$$

$$\Rightarrow \qquad \frac{\sin(\theta+60)-\sin(\theta+30)}{\cos(\theta+60)+\cos(\theta+30)} = \tan 15$$

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(4) (a)

$$\cos(A-B) = \cos A \cos B + \sin A \sin B \qquad -\{1\}$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B \qquad -\{2\}$$

$\{1\} - \{2\}$

$$\cos(A-B) - \cos(A+B) = 2\sin A \sin B$$

(b)　Using solution from (a)

Let $B = A$

$$\Rightarrow \qquad 2\sin A \sin A = \cos(A-A) - \cos(A+A)$$
$$2\sin^2 A = 1 - \cos 2A$$
$$\sin^2 A = \frac{1}{2}(1-\cos 2A)$$

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