PAST EXAMINATION QUESTIONS:

HSC 07

(4)

(b) (i) Show that $\sin 3\theta = 3\sin\theta\cos^2\theta - \sin^3\theta$. **2**

(ii) Show that $4\sin\theta\sin\left(\theta+\frac{\pi}{3}\right)\sin\left(\theta+\frac{2\pi}{3}\right) = \sin 3\theta$. **2**

(iii) Write down the maximum value of $\sin\theta\sin\left(\theta+\frac{\pi}{3}\right)\sin\left(\theta+\frac{2\pi}{3}\right)$. **1**

$\boxed{\frac{1}{4}}$

(5)

(a) A bag contains 12 red marbles and 12 yellow marbles. Six marbles are selected at random without replacement.

(i) Calculate the probability that exactly three of the selected marbles are red. Give your answer correct to two decimal places. **1**

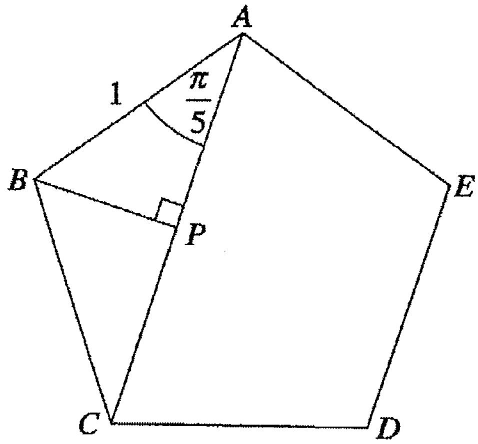
0.36 (to 2 d.p)

(ii) Hence, or otherwise, calculate the probability that more than three of the selected marbles are red. Give your answer correct to two decimal places. **2**

0.32 (to 2 d.p)

(5)

(d)



In the diagram, $ABCDE$ is a regular pentagon with sides of length 1. The perpendicular to AC through B meets AC at P.

Copy or trace this diagram into your writing booklet.

(i) Let $u = \cos\frac{\pi}{5}$. **2**

Use the cosine rule in ΔACD to show that $8u^3 - 8u^2 + 1 = 0$.

(ii) One root of $8x^3 - 8x^2 + 1 = 0$ is $\frac{1}{2}$. **2**

Find the other roots of $8x^3 - 8x^2 + 1 = 0$ and hence find the exact value of $\cos\frac{\pi}{5}$.

$\boxed{\frac{1+\sqrt{5}}{4}}$

(6)

(a) (i) Use the binomial theorem **1**

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \cdots + b^n$$

to show that, for $n \geq 2$,

$$2^n > \binom{n}{2}.$$

(ii) Hence show that, for $n \geq 2$, **2**

$$\frac{n+2}{2^{n-1}} < \frac{4n+8}{n(n-1)}.$$

(iii) Prove by induction that, for integers $n \geq 1$, **3**

$$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + \cdots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}.$$