C.E.M. TUITION

Student Name : ______________________________

Review Topic : Exponential Growth & Decay

(HSC Course - Paper 1)

Year 12 - 2 Unit

1996

1. Solve for x (giving answers correct to 3 decimal places):

(a) $e^x = 1{\cdot}02$ **(b)** $3e^x = 2$ **(c)** $4e^{-2x} = 7$

2. If $T = 405e^{-0.2t}$, find:

(a) T when $t = 12$ (Give answer correct to the nearest whole number.)

(b) t when $T = 260$ (Give answer correct to two decimal places.)

3. If $M = 4500e^{0\cdot 2t}$, find:

(a) M when $t = 7\cdot 5$ (Give answer correct to the nearest whole number.)

(b) t when $M = 9000$ (Give answer correct to one decimal place.)

4. The population of a town can be estimated by using the formula $P = Po\,e^{kt}$, where t is the time in years, and Po and k are constants. If the population of a town at the beginning of 1960 was 2300, and at the beginning of 1974 it was 3050,

(a) find the annual growth rate (i.e. find the value of k), correct to three significant figures;

(b) in what year will the population of the town be double that of 1960?

5. Suppose that the number of bacteria in a colony was $1\cdot15 \times 10^5$ four hours ago and that it increases continuously at the rate proportional to the existing number of bacteria in the colony. If the present number of bacteria in the colony is $2\cdot75 \times 10^5$, what will the number of bacteria be after a further five hours?

1. (a) $e^x = 1\cdot02$

$x = \ln(1\cdot02)$

$\boxed{e^x = a \Rightarrow x = \ln a}$

$\therefore \quad x \approx 0\cdot020.$

(b) $3e^x = 2$

(Divide both sides by 3)

$e^x = \dfrac{2}{3}$

$\boxed{e^x = a \Rightarrow x = \ln a}$

$x = \ln\left(\dfrac{2}{3}\right)$

$\therefore \quad x \approx -0\cdot405.$

(c) $4e^{-2x} = 7$

(Divide both sides by 4)

$e^{-2x} = \dfrac{7}{4}$

$\boxed{e^x = a \Rightarrow x = \ln a}$

$-2x = \ln\left(\dfrac{7}{4}\right)$

(Divide both sides by -2)

$x = \dfrac{\ln\left(\frac{7}{4}\right)}{-2}$

$\therefore \quad x \approx -0\cdot280.$

2. (a) $T = 405e^{-0\cdot2t}$

When $t = 12$,

$T = 405e^{-0\cdot2\times12}$

$\therefore \quad T \approx 37.$

(b) $T = 405e^{-0\cdot2t}$

When $T = 260$,

$260 = 405e^{-0\cdot2t}$

$\dfrac{260}{405} = e^{-0\cdot2t}$

$-0\cdot2t = \ln\left(\dfrac{260}{405}\right)$

$t = \dfrac{\ln\left(\frac{260}{405}\right)}{-2}$

$\therefore \quad t \approx 0\cdot22.$

3. (a) $M = 4500e^{0\cdot2t}$

When $t = 7\cdot5$,

$M = 4500e^{0\cdot2\times7\cdot5}$

$\therefore \quad M \approx 20\,168.$

(b) When $M = 9000$,

$9000 = 4500e^{0\cdot2t}$

$\dfrac{9000}{4500} = e^{0\cdot2t}$

$2 = e^{0\cdot2t}$

$0\cdot2t = \ln 2$

$t = \dfrac{\ln 2}{0\cdot2}$

$\therefore \quad t \approx 3\cdot5.$

4. (a) When $t = 0$, (beginning 1960), $P = 2300$,

$P = Po\,e^{kt}.$

When $t = 0$, $P = 2300$,

$2300 = Po\,e^{k\times0}$

$2300 = Po\,e^{0}$

$\boxed{e^0 = 1}$

$\therefore \quad Po = 2300$

$\therefore \quad P = 2300e^{kt}.$

When $t = 14$, (beginning 1974), $P = 3050$,

$\therefore \quad 3050 = 2300e^{k\times14}$

$\dfrac{3050}{2300} = e^{14k}$

$\therefore \quad 14k = \ln\left(\dfrac{3050}{2300}\right)$

$k = \dfrac{\ln\left(\frac{3050}{2300}\right)}{14}$

$\therefore \quad k \approx 0\cdot0202.$

Therefore, the annual growth rate, k, is 0.0202.

(b) $Po = 2300$, $k = 0\cdot0202$

$\therefore \quad P = 2300e^{0\cdot0202t}.$

The question is asking us to find t when $P = 2\times2300 = 4600.$

When $P = 4600$,

$\therefore \quad 4600 = 2300e^{0\cdot0202t}$

$\dfrac{4600}{2300} = e^{0\cdot0202t}$

$2 = e^{0\cdot0202t}$

$0\cdot0202t = \ln 2$

$k = \dfrac{\ln 2}{0\cdot0202}$

$\approx 34\cdot3.$

Note In the year 1960, $t = 0$

Therefore, the year in which the population will double that of 1960 is 1994.

1960 + 34·3 years is the year 1994.

5. (a) The number of bacteria increases continuously at the rate proportional to the existing number of bacteria $\Rightarrow$ Exponential Growth,

i.e. $\dfrac{dN}{dt} = kN$

where N is the number of bacteria present and k is a constant.

Therefore, we can use the formula $N = No\,e^{kt}$ where No is the initial number of bacteria present.

When $t = 0$, $N = 1\cdot15\times10^5$

$\Rightarrow \quad No = 1\cdot15\times10^5$

Remember No is the value of N when $t = 0$.

$\therefore \quad N = 1\cdot15\times10^5\,e^{kt}.$

When $t = 4$,

$N = 2\cdot75\times10^5$

$2\cdot75\times10^5 = 1\cdot5\times10^5\,e^{k\times4}$

$e^{4k} = \dfrac{2\cdot75\times10^5}{1\cdot15\times10^5}$

$4k = \ln\left(\dfrac{2\cdot75}{1\cdot15}\right)$

$k = \dfrac{\ln\left(\frac{2\cdot75}{1\cdot15}\right)}{4}$

$\approx 0\cdot218$ (3 sig. figs.)

$\therefore N = 1\cdot15\times10^5\,e^{0\cdot218t}$

The question is asking us to find N, when t = further 5 hours $= (4+5)$ hours $= 9$ hours.

When $t = 9$,

$N = 1\cdot15\times10^5\,e^{0\cdot218\times9}$

$\approx 8\cdot18\times10^5$

$\therefore$ After a further 5 hours there will be $8\cdot18\times10^5$ bacteria present.