

C.E.M. TUITION

Name : _____

Review Topic : Trapezoidal & Simpson's Rules

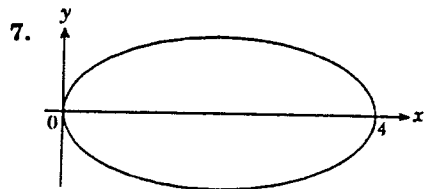
(HSC Course - Paper 2)

Year 12 - Mathematics

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The loop of the curve
 $y^2 = x(4-x)^2$ has an
 area of A units².
 (Note symmetry about x axis.)

- (a) Complete the table of values
 for $y = (4-x)\sqrt{x}$, $0 \leq x \leq 4$.

x	0	1	2	3	4
y					

- (b) Use the table of values and Simpson's Rule with 5 function

values to evaluate $\int_0^4 (4-x)\sqrt{x} \, dx$.

- (c) Calculate A correct to 2 decimal places.



8. A bike rider racing around an oval track has her speed, v , in km/h recorded at 3 minute intervals as recorded in the following table:

time (t) mins	0	3	6	9	12
speed (v) km/h	0	14	18	16	15

- (a) Explain why the distance x , in km travelled by the rider over the 12 minutes, is given by $x = \int_0^{12} v \, dt$.
- (b) Calculate x using the Trapezoidal Rule with four subintervals.

9. Complete this table of values

for $y = \frac{e^x + e^{-x}}{e^x}$.

x	0	0.5	1
y			

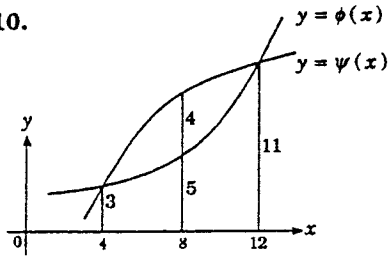
(a) Using Simpson's Rule with three function values, estimate the

value of $\int_0^1 \frac{e^x + e^{-x}}{e^x} dx$ correct to 2 decimal places.

(b) Show that $\frac{e^x + e^{-x}}{e^x} = 1 + e^{-2x}$.

(c) Calculate the area under the curve $y = \frac{e^x + e^{-x}}{e^x}$ between $x = 0$ and $x = 1$ by integration. Find the difference between the actual area and the approximate area correct to two decimal places.

10.



This diagram shows two unknown curves, $y = \phi(x)$ and $y = \psi(x)$, drawn for $4 \leq x \leq 12$. The ordinates are marked on the diagram.

(a) Complete these tables of value for $y = \phi(x)$ and $y = \psi(x)$.

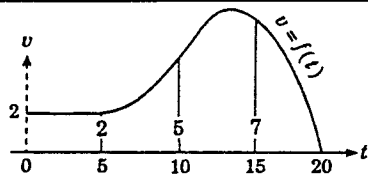
x	4	8	12
$\phi(x)$			

x	4	8	12
$\psi(x)$			

(b) Using the Trapezoidal Rule and 3 function values, calculate the area contained between the curves $y = \phi(x)$ and $y = \psi(x)$.



11.



(a) Use Simpson's Rule with five function values to calculate the area under the curve $v = f(t)$ between $t = 0$ and $t = 20$.

(b) If the graph represents the velocity of a particle moving in a straight line, such that at time t the velocity is $v \text{ ms}^{-1}$, calculate the distance travelled in the first 20 seconds.

12. Use the Trapezoidal Rule with three function values to evaluate

$$\int_0^1 \sin \pi x \, dx.$$

Compare this answer with the actual value of the integral.



7. (a)

x	0	1	2	3	4
y	0	3	2.828	1.732	0

 $h = 1 - 0 = 1$ (TODD Simpson)

(b) $\int_0^5 (4-x)\sqrt{x} dx = \frac{h}{3} [\text{first} + \text{last} + 2 \text{ odd} + 4 \text{ even}]$ (FLOE)
 $= \frac{1}{3} [f(0) + f(4) + 2f(2) + 4[f(1) + f(3)]]$
 $= \frac{1}{3} [0 + 0 + 2(2.828) + 4(3 + 1.732)]$
 $= 8.772\ 016\ 9.$

(c) Total $A = 2 \times \int_0^4 (4-x)\sqrt{x} dx$
 $= 2 \times 8.772\ 016\ 9$
 $= 17.544\ 034.$
 Area is 17.54 units².

$\int_0^4 (4-x)\sqrt{x} dx$
 represents the area
 above the x axis and
 below the curve.

8.

t	0	3	6	9	12
v	0	14	18	16	15

 $v = f(t)$

(a) $x = \int_0^{\frac{1}{5}} v dt$ 12 mins = $\frac{1}{5}$ hour

Velocity is $\frac{dx}{dt}$ (rate of change of displacement),

hence distance will be $\int v dt$.

This integral is measuring the distance covered over $\frac{1}{5}$ of an hour as velocity has been measured in km/hour.

(b) $x = \frac{1}{2}(3)[f(0) + 2f(3) + 2f(6) + 2f(9) + f(12)]$

$= 1.5[0 + 2(14) + 2(18) + 2(16) + 15]$
 $= 166.5.$

Four strips, 0 - 3,
 3 - 6, 6 - 9, and 9 - 12.
 Strip width = 3

Rider has ridden 166.5 km.

9. $y = \frac{e^x + e^{-x}}{e^x}$

x	0	0.5	1
y	2	1.3679	1.1353

 $a = 0, b = 1$
 $\frac{a+b}{2} = 0.5$

(a) $\int_0^1 \frac{e^x + e^{-x}}{e^x} dx = \frac{1}{6}(1-0)[f(0) + f(1) + 4f(0.5)]$
 $= \frac{1}{6}[2 + 1.1353 + 4(1.3679)]$
 $= 1.434\ 483\ 3 \approx 1.43$ (2 decimal places).

(b) $\frac{e^x + e^{-x}}{e^x} = \frac{e^x}{e^x} + \frac{e^{-x}}{e^x} = 1 + e^{-2x}$

(c) Approximate area under the curve, using Simpson's Rule, is 1.43 units² [from (a)].

$$\begin{aligned} \int_0^1 \frac{e^x + e^{-x}}{e^x} dx &= \int_0^1 1 + e^{-2x} dx \\ &= \left[x - \frac{1}{2} e^{-2x} \right]_0^1 = \left[1 - \frac{1}{2} e^{-2} \right] - \left[0 - \frac{1}{2} e^0 \right] \\ &= \left[1 - 0.067\ 667\ 6 \right] + \frac{1}{2} \\ &= 1.432\ 332\ 4 \quad = 1.43 \text{ (2 dec. places)} \end{aligned}$$

Difference is zero when considering areas correct to 2 decimal places. However, from (a), approximate area is 1.4344833, while by integration the area is 1.4323324. Difference is 0.00215094, which is negligible.

10. (a)

x	4	8	12
$\phi(x)$	3	5	11

x	4	8	12
$\psi(x)$	3	9	11

(b) Area contained between the curves is the area under higher curve, $y = \psi(x)$, less the area under the lower curve, $y = \phi(x)$.

$$A = \int_4^{12} \psi(x) dx - \int_4^{12} \phi(x) dx,$$

using $A = \frac{h}{2} [\text{first} + \text{last} + 2(\text{remaining})]$

$$\int_4^{12} \psi(x) dx = \frac{4}{2} [\psi(4) + \psi(12) + 2\psi(8)] = 2[3 + 11 + 2(9)] = 64.$$

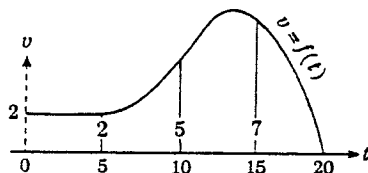
$$\int_4^{12} \phi(x) dx = \frac{4}{2} [\phi(4) + \phi(12) + 2\phi(8)] = 2[3 + 11 + 2(5)] = 48.$$

Area = 64 - 48 = 16. Area between curves is 16 units².

11. (a)

t	0	5	10	15	20
$f(t)$	2	2	5	7	0

(TODD Simpson)



$$A = \frac{h}{3} [\text{first} + \text{last} + 2 \text{ odd} + 4 \text{ even}] \quad (\text{FLOE})$$

$$= \frac{h}{3} [f(0) + f(20) + 2f(10) + 4(f(5) + f(15))]$$

$h = 5 - 0$ $= 5$

$$= \frac{5}{3} [2 + 0 + 2(5) + 4(2 + 7)]$$

$$= \frac{5}{3} [48]$$

$$= 80.$$

Area under the curve is 80 units².

(b) This area represents the distance travelled by the particle as

$$x = \int_0^{20} v dt. \text{ Distance travelled in the first 20 seconds is}$$

$$\int_0^{20} v dt = \int_0^{20} f(t) dt = 80.$$

Distance travelled in the first 20 seconds is 80 metres.

12. Put $I = \int_0^1 \sin \pi x \, dx$, where $f(x) = \sin \pi x$.
 Two subintervals are $0 - 0.5$, $0.5 - 1$.

Now $I = \frac{1}{2}(b-a)[f(a) + f(b)]$

x	0	0.5	1
$f(x)$	0	1	0

$$I_1 = \frac{1}{2}(0.5-0)[f(0) + f(0.5)] = 0.25[0 + 1] = 0.25$$

$$I_2 = \frac{1}{2}(1-0.5)[f(0.5) + f(1)] = 0.25[1 + 0] = 0.25$$

$\therefore I = I_1 + I_2 = 0.5 \int_0^1 \sin \pi x \, dx = 0.5$.

For actual value, $\int_0^1 \sin \pi x \, dx = \left[-\frac{1}{\pi} \cos \pi x \right]_0^1$
 $= -\left[\frac{1}{\pi} \cos \pi - \frac{1}{\pi} \cos 0 \right]$
 $= -\left[\frac{1}{\pi}(-1) - \frac{1}{\pi}(1) \right]$
 $= -\left[\frac{-2}{\pi} \right]$
 $= \frac{2}{\pi}$
 $= 0.636\,619\,7$
 ≈ 0.637 (3 dec. places).

Actual value is 0.637, which is 0.137 greater than the value found using the Trapezoidal Rule.