

C.E.M. TUITION

Name : _____

Review Topic : Trapezoidal & Simpson's Rules

(HSC Course - Paper 3)

Year 12 - Mathematics

13.

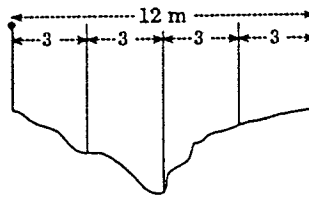
t	0	1	2	3	4
$v(t)$	0	20	50	30	10

(TREMAINING)

$$\begin{aligned}\int_0^4 v(t) dt &= \frac{h}{2} [\text{first} + \text{last} + 2(\text{remaining})] && (FLR) \\ &= \frac{1}{2} [f(0) + f(4) + 2[f(1) + f(2) + f(3)]] \\ &= \frac{1}{2} [0 + 10 + 2(20 + 50 + 30)] \\ &= \frac{1}{2} [210] \\ &= 105. && \therefore \int_0^4 v(t) dt = 105.\end{aligned}$$

$\int_0^4 v(t) dt$ represents the distance travelled by the particle in the first 4 seconds, that is, distance travelled by the particle in the first 4 seconds is 105 metres.

14. Illustrated is a ditch of width 12 metres with vertical banks. The depth (in metres) is measured at 3 metre intervals across a cross-section and the measurements recorded in this table.



Distance from A	0	3	6	9	12
Depth	0.95	1.25	1.70	1.05	0.85

- (a) Calculate the area of this cross-section using Simpson's Rule with 5 function values.
- (b) Assuming this cross-section is typical of the ditch, calculate the volume of fill required for a 50 m section of the ditch.



13.

t	0	1	2	3	4
$v(t)$	0	20	50	30	10

(TREMAINING)

$$\int_0^4 v(t) dt = \frac{h}{2} [\text{first} + \text{last} + 2(\text{remaining})] \quad (FLR)$$

$$= \frac{1}{2} [f(0) + f(4) + 2[f(1) + f(2) + f(3)]]$$

$$= \frac{1}{2} [0 + 10 + 2(20 + 50 + 30)]$$

$$= \frac{1}{2} [210]$$

$$= 105. \quad \therefore \int_0^4 v(t) dt = 105.$$

$\int_0^4 v(t) dt$ represents the distance travelled by the particle in the first 4 seconds, that is, distance travelled by the particle in the first 4 seconds is 105 metres.

common
end - point
↓

14.

d	0	3	6	9	12
D	0.95	1.25	1.70	1.05	0.85

← subinterval 1 → subinterval 2 →

(a) For first subinterval,

$$A_1 = \frac{1}{6}(b-a) \left[f(a) + f(b) + 4f\left(\frac{a+b}{2}\right) \right]$$

$$= \frac{1}{6}(6-0)[0.95 + 1.70 + 4(1.25)]$$

$$= 7.65.$$

$a = 0, b = 6, \frac{a+b}{2} = 3.$ $f(a) = 0.95,$ $f(b) = 1.70,$ $f\left(\frac{a+b}{2}\right) = 1.25.$

For 2nd subinterval,

$$A_2 = \frac{1}{6}(12-6)[f(6) + f(12) + 4f(9)]$$

$$= \frac{1}{6}(6)[1.70 + 0.85 + 4(1.05)]$$

$$= 6.75.$$

$a = 6, b = 12,$ $\frac{a+b}{2} = \frac{6+12}{2} = 9.$ $f(a) = f(6),$ $f(b) = f(12),$ $f\left(\frac{a+b}{2}\right) = f(9).$

Area = $7.65 + 6.75 = 14.4$.

Cross-sectional area is 14.4 m^2 .

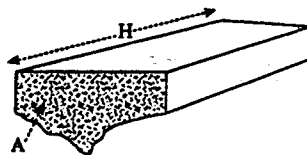
(b) $V = A \times H$

$$= 14.4 \times 50$$

$$= 720.$$

$A = 14.4$ $H = 50$

Volume of fill needed is 720 m^3 .



15. Four-function values allow for 3 subintervals with Trapezoidal Rule 1-2, 2-3, 3-4.

Put $I = \int_1^4 x \log_e x dx$.

For subinterval 1, then $I_1 = \frac{1}{2}(b-a)[f(a) + f(b)]$

$$= \frac{1}{2}(2-1)[f(1) + f(2)]$$

$$= \frac{1}{2}[0 + 1.39]$$

13.

t	0	1	2	3	4
$v(t)$	0	20	50	30	10

(TREMAINING)

$$\int_0^4 v(t) dt = \frac{h}{2} [\text{first} + \text{last} + 2(\text{remaining})] \quad (FLR)$$

$$= \frac{1}{2} [f(0) + f(4) + 2[f(1) + f(2) + f(3)]]$$

$$= \frac{1}{2} [0 + 10 + 2(20 + 50 + 30)]$$

$$= \frac{1}{2} [210]$$

$$= 105.$$

$$\therefore \int_0^4 v(t) dt = 105.$$

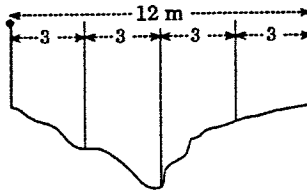
$\int_0^4 v(t) dt$ represents the distance travelled by the particle in the first 4 seconds, that is, distance travelled by the particle in the first 4 seconds is 105 metres.

13. Utilise the given table to evaluate $\int_0^4 v(t) dt$, using the Trapezoidal Rule.

t	0	1	2	3	4
$v(t)$	0	20	50	30	10

If t represents time in seconds and $v(t)$ velocity in ms^{-1} after t seconds, describe a possible physical interpretation of $\int_0^4 v(t) dt$.

14. Illustrated is a ditch of width 12 metres with vertical banks. The depth (in metres) is measured at 3 metre intervals across a cross-section and the measurements recorded in this table.



Distance from A	0	3	6	9	12
Depth	0.95	1.25	1.70	1.05	0.85

- (a) Calculate the area of this cross-section using Simpson's Rule with 5 function values.
- (b) Assuming this cross-section is typical of the ditch, calculate the volume of fill required for a 50 m section of the ditch.

15. This is a table of values for $f(x) = x \log_e x$. Use the Trapezoidal Rule with all 4 function values given to calculate the approximate value of

x	1	2	3	4
$f(x)$	0	1.39	3.30	5.55

$$\int_1^4 x \log_e x \, dx.$$

13.

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 (TREMAINING)

$$\int_0^4 v(t) dt = \frac{h}{2} [\text{first} + \text{last} + 2(\text{remaining})] \quad (FLR)$$

$$= \frac{1}{2} [f(0) + f(4) + 2[f(1) + f(2) + f(3)]]$$

$$= \frac{1}{2} [0 + 10 + 2(20 + 50 + 30)]$$

$$= \frac{1}{2} [210]$$

$$= 105. \quad \therefore \int_0^4 v(t) dt = 105.$$

$\int_0^4 v(t) dt$ represents the distance travelled by the particle in the first 4 seconds, that is, distance travelled by the particle in the first 4 seconds is 105 metres.

common
end - point
↓

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D	0.95	1.25	1.70	1.05	0.85

← subinterval 1 → ← subinterval 2 →

(a) For first subinterval,

$$A_1 = \frac{1}{6}(b-a) \left[f(a) + f(b) + 4f\left(\frac{a+b}{2}\right) \right]$$

$$= \frac{1}{6}(6-0) [0.95 + 1.70 + 4(1.25)]$$

$$= 7.65.$$

$$a = 0, b = 6, \frac{a+b}{2} = 3.$$

$$f(a) = 0.95,$$

$$f(b) = 1.70,$$

$$f\left(\frac{a+b}{2}\right) = 1.25.$$

For 2nd subinterval,

$$A_2 = \frac{1}{6}(12-6) [f(6) + f(12) + 4f(9)]$$

$$= \frac{1}{6}(6) [1.70 + 0.85 + 4(1.05)]$$

$$= 6.75.$$

$$a = 6, b = 12,$$

$$\frac{a+b}{2} = \frac{6+12}{2} = 9.$$

$$f(a) = f(6),$$

$$f(b) = f(12),$$

$$f\left(\frac{a+b}{2}\right) = f(9).$$

Area = $7.65 + 6.75 = 14.4$.

Cross-sectional area is 14.4 m^2 .

(b) $V = A \times H$

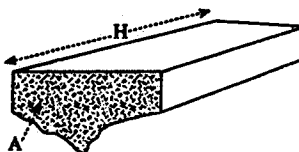
$$= 14.4 \times 50$$

$$= 720.$$

$$A = 14.4$$

$$H = 50$$

Volume of fill needed is 720 m^3 .



15. Four-function values allow for 3 subintervals with Trapezoidal Rule 1-2, 2-3, 3-4.

Put $I = \int_1^4 x \log_e x dx$.

For subinterval 1, then $I_1 = \frac{1}{2}(b-a)[f(a) + f(b)]$

$$= \frac{1}{2}(2-1)[f(1) + f(2)]$$

$$= \frac{1}{2}[0 + 1.39]$$

$$= 0.695.$$

$$I_2 = \frac{1}{2}(3-2)[f(2) + f(3)] = \frac{1}{2}[1.39 + 3.30] = 2.345.$$

$$I_3 = \frac{1}{2}(4-3)[f(3) + f(4)] = \frac{1}{2}[3.30 + 5.55] = 4.425.$$

$$\text{Then } I = 0.695 + 2.345 + 4.425 = 7.465.$$

$$\int_0^4 x \log_e x \, dx \approx 7.465.$$

OR alternate method.

(TREMAINING)

x	1	2	3	4
$f(x)$	0	1.39	3.30	5.55

$$h = 2 - 1 = 1$$

\uparrow \uparrow \uparrow \uparrow
F *REMAINING* *L* (*FLR*)

$$\begin{aligned} \int_1^4 f(x) \, dx &= \frac{h}{2} [FIRST + LAST + 2(REMAINING)] \\ &= \frac{1}{2} [f(1) + f(4) + 2(f(2) + f(3))] \\ &= \frac{1}{2} [0 + 5.55 + 2(1.39 + 3.30)] \\ &= 7.465. \end{aligned}$$

$$\therefore \int_1^4 x \log_e x \, dx = 7.465.$$