Chapter 6

SERIES AND APPLICATIONS

Facts and Formulas

EXERCISE 1

Circle the correct answer:

1. The n^{th} term of an arithmetic sequence is given by:

   (A) $T_n = a + (n-1)d$ (B) $T_n = \frac{n}{2}[2a+(n-1)d]$

   (C) $T_n = a + nd$ (D) $T_n = a(n-1)d$

2. The sum of n terms of an arithmetic series, given a and d.

   (A) $S_n = \frac{n}{2}[a+l]$ (B) $S_n = \frac{n}{2}[a+(n-1)d]$

   (C) $S_n = \frac{n}{2}[2a+(n-1)d]$ (D) $S_n = 2n[2a+(n-1)d]$

3. The sum of n terms of an arithmetic series, given a and l.

   (A) $S_n = \frac{n}{2}[a+2l]$ (B) $S_n = \frac{n}{2}[a+l]$

   (C) $S_n = n[a+l]$ (D) $S_n = \frac{n}{2}[2a+l]$

4. The n^{th} term of a geometric progression is given by:

   (A) $T_n = ar^n$ (B) $T_n = a + r^n$

   (C) $T_n = ar^{n-1}$ (D) $T_n = a + r^{n-1}$

5. The sum of n terms of a geometric series is given by:

   (A) $S_n = \frac{a(r^n-1)}{(r-1)}$ (B) $S_n = \frac{a(r-1)^n}{(r+1)}$

   (C) $S_n = \frac{a^n(r-1)}{(r-1)}$ (D) $S_n = \frac{a(r^n-1)}{(1-r)}$

6. The limiting sum of a geometric series is given by:

   (A) $S_\infty = \frac{a}{r-1}$ (B) $S_\infty = \frac{a}{r+1}$

   (C) $S_\infty = \frac{a}{1-r}$ (D) $S_\infty = \frac{a-1}{r}$

7. The condition for an infinite geometric series to have a limiting sum is:

   (A) $r > 1$ (B) $r < 1$

   (C) $r > -1$ (D) $-1 < r < 1$

8. Formula for Compound Interest

   (A) $A = P(1 - \frac{r}{100})^n$ (B) $A = P(1 + \frac{r}{100})^n$

EXERCISE 2

1. Which of the following are sequences and which are series?:

(a) 3, 5, 7, 9, 11, ...

(b) $2 + 4 + 8 + 16 + \ldots\ldots$

(c) $-7 - 5 - 3 - 1 \ldots\ldots$

(d) ½, ¼, ⅛, ...

2. Find the next two terms in each sequence:

(a) 4, 7, 10, 13, ...

(b) 2, 6, 18, 54, ...

(c) 1, 8, 27, 64, ...

(d) 0, 3, 8, 15, 24, ...

(e) 1, 1, 2, 3, 5, 8, ...

Arithmetic Progressions

EXERCISE 3

1. For each of the following, state the common difference:

(a) 6, 9·4, 12·8, ...

(b) 8, −1, −10, −19, . . .

2. The n^{th} term of an arithmetic sequence is given by

$$T_n = \frac{5n+4}{2}$$

Evaluate the common difference d.

3. Find the value of x which makes the sequence $19, x, 81$ arithmetic.

4. For an A.P. $a = 4$ and $d = 9$.

Find (a) T_{24}

(b) S_{24}

5.* For the A.P. : $\sqrt{5}, \sqrt{45}, \sqrt{125}, \sqrt{245}, \ldots\ldots$

Find d and T_{10}

6. Given the sequence 4, 7, 10, 13, ...

(a) Find T_{20}

(b) Find S_{20}

7. Find S_{15} for the A.P. 8, 12, 16, 20, . . .

Given the sequence 20, 14, 8, 2, ...

(a) Find T_{18}

(b) Find S_{18}

9. $x + 3$, $2x - 5$, $4x + 6$ are three consecutive terms of an A.P.

(a) Evaluate x.

(b) State the three terms.

(c) State the common difference.

10. $x - 8$, $3x + 4$, $6x - 10$ are three consecutive terms of an A.P.

(a) Evaluate x.

(b) State the three terms.

(c) State the common difference.

11. $y + 3$, $2y + 7$, $3y + 11$, $4y + 15$ are four consecutive terms of an A.P.

(a) If $y = 5$ evaluate the terms and state the common difference.

(b) If $y = -10$ evaluate the terms and state the common difference.

12. Which term of 2, 9, 16, 23, ... has a value of 254?

13. How many terms of 2, 9, 16, 23, ... are needed to give a sum of 16082?

14.* Which term of $-3, 1, 5, 9, 13, \ldots\ldots$ is the first term to have a value greater than 200?

15. How many terms of $8, 5, 2, -1, -4, \ldots\ldots$ are needed to give a sum of -4495?

16. Which term of $546, 543, 540, \ldots\ldots$ is the first term to be negative?

17. Evaluate $\sum_{n=1}^{20}(9n-4)$.

18. Evaluate $\sum_{n=1}^{18}(23-8n)$.

19. Evaluate $\sum_{n=1}^{12}(1-4n)$.

20. Evaluate $\sum_{n=5}^{30}(6n-10)$.

21. Evaluate k if $\sum_{n=1}^{k}(5n+7) = 1800$.

22.* Find the sum of all integers from 1 to 70 that are not divisible by 9.

23. The 50^{th} term of an A.P. is 337. The 72^{nd} term of this A.P. is 491. Find:

(a) d

(b) a

(c) T_{23}

(d) S_{23}

24.* A bell rings at 6:32 am and then every 3 minutes until it last rings at 10:14 am. Using arithmetic sequences calculate the number of times the bell rings.

25. An A.P. with $a = 13$ has $S_{30} = 3435$. Find the value of d.