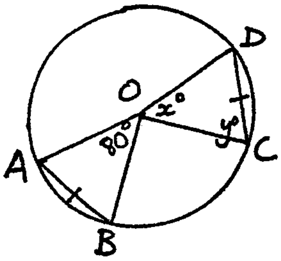
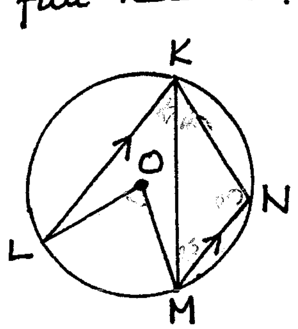
Stephanie Sim

CIRCLE GEOMETRY. O is the centre in all examples

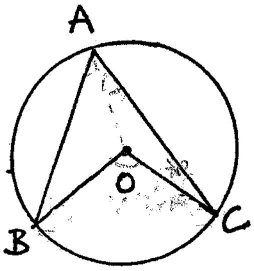
1. Find the value of x and y



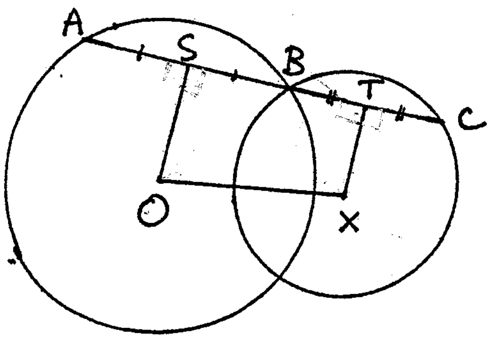
2. $KL \parallel NM$, $K\hat{N}M = 110°$, $N\hat{K}M = 45°$
Calculate the size of $L\hat{O}M$ giving full reasons.



3. If $\angle BOC = 104°$, $\angle ACB = 71°$; calculate the size of $\angle OAC$ giving reasons

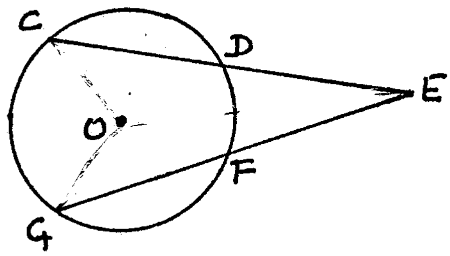


4. ABC is a straight line S and T are the midpoints of AB and BC. O and X are centres. Prove $\angle SOX + \angle OXT = 180°$

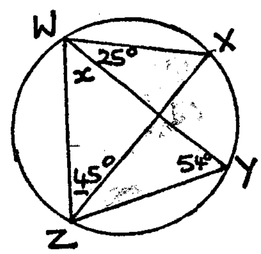


5. PAB is an isosceles Triangle inscribed in a circle of radius 26 mm. If $PA = PB$ and $AB = 48$ show that $AP = 12\sqrt{13}$ mm.

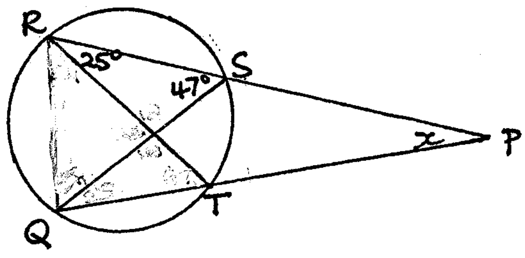
6. In the circle $CD = GF$. Prove that $DE = FE$; (O is the centre of the circle).



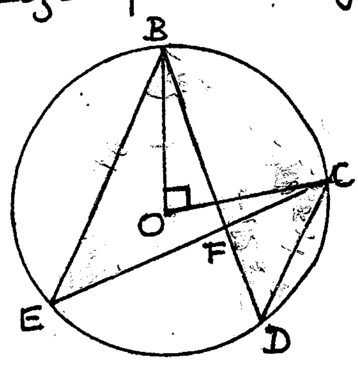
7. Find x.



8. Find the value of x giving reasons

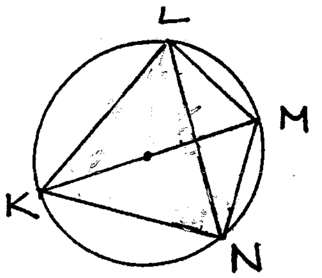


9. $BE \parallel CD$ and $BO \perp OC$. Calculate the size of $\angle CFD$ giving reason.

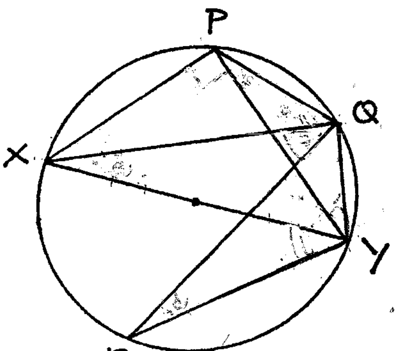


Page 2 Circle Geometry:

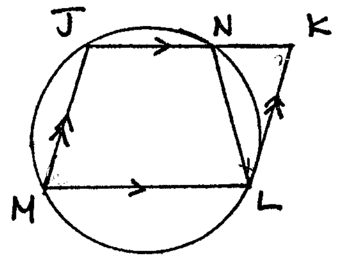
10. KM is a diameter
If $\angle KLN = 48°$ $KL = LN$
Calculate the size of $\angle LNM$



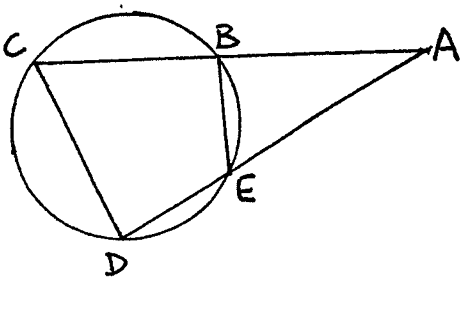
11. XY is a diameter of the circle
Prove $\angle XQP + \angle PYQ + \angle QRY = 90°$



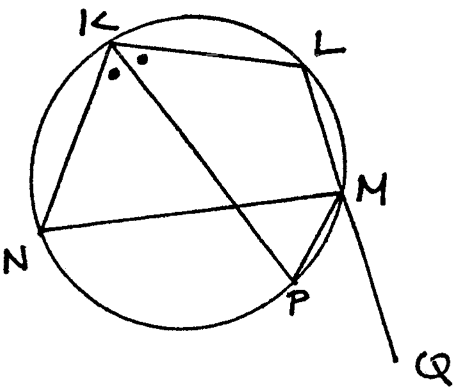
12.
JKLM is a parallelogram
Prove $NL = LK$



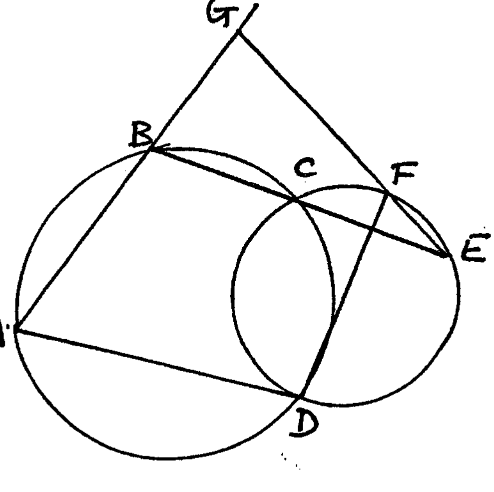
13.
Prove $\triangle ABE \;|||\; \triangle ADC$



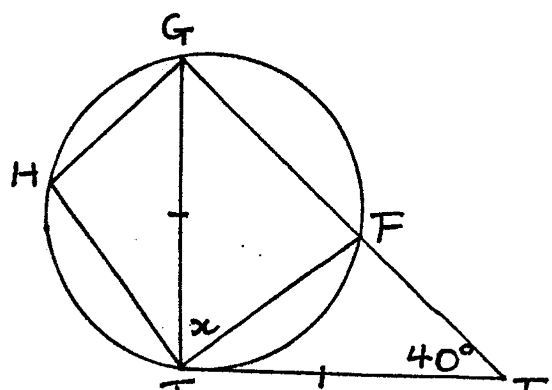
14. KLMN is a cyclic quadrilateral in which P is a point on the circumference such that $\angle NKP = \angle PKL$
LM is produced to Q
Prove PM bisects $\angle NMQ$



15. Prove GFDA is a cyclic quadrilateral.



16 $TJ = JG$ and $\angle GTJ = 40°$
Find the value of x giving reasons.

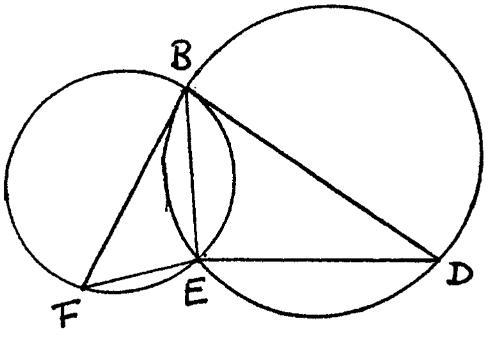


ge 3. Circle Geometry.

7. FB and DB are tangents to the respective circles

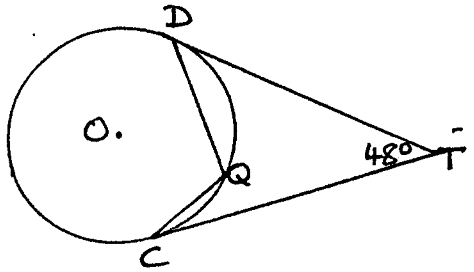
a) Prove BE bisects FED

b) If $\angle FBD = 70°$ calculate $\angle BED$ giving reasons



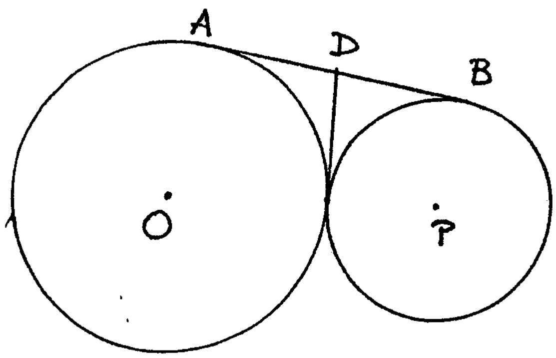
18.

. TD and TC are tangents
calculate $\angle DQC$

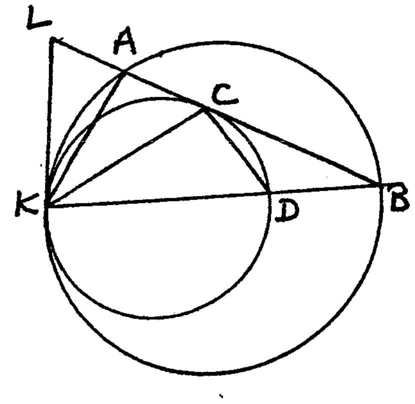


19.

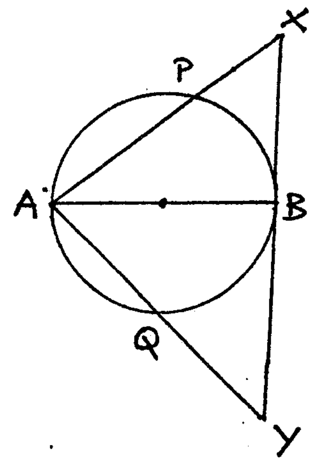
AB is a common tangent
The common tangent at C meets AB at D.
Prove $\angle ODP = 90°$



20. LK is a tangent to both circles
LC is a tangent to the small circle. Prove $\angle AKC = \angle CKB$.



21 AB is a diameter of the circle APQB. XBY is a tangent
Prove $AP.AX = AQ.AY = AB^2$



22. AB, AC are two equal chords of a circle; AP is another chord of the circle which cuts BC at Q.

Prove $AP.AQ = AB^2$

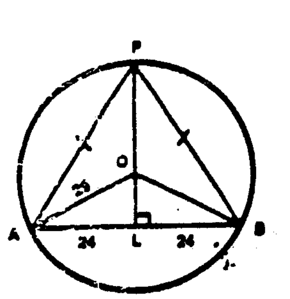
1. $x = 80$ ($AB = CD$, equal chords subtend equal angles at the centre)
$OD = OC$ (radii of a circle)
$\therefore \triangle ODC$ is isosceles.
$\therefore \widehat{ODC} = \widehat{OCD} = y°$ (angles opposite equal sides of isosceles triangle)
$\therefore y = 50$ (angle sum of $\triangle ODC$ is $180°$)

2 $\widehat{KMN} = 25°$ (angle sum $\triangle KMN$ is $180°$)
$\widehat{LKM} = \widehat{KMN} = 25°$ (alternate angles, $KL \| NM$)
$\therefore \widehat{LOM} = 2\widehat{LKM} = 50°$ (angle at centre twice angle at circumference).

3 Join OA.
$\triangle BOC$ is isosceles (OB, OC are equal radii)
$\therefore \widehat{OBC} = \widehat{OCB}$ (base angles of isosceles triangle)
$\therefore 2\widehat{OBC} + 104° = 180°$ (angle sum of $\triangle OBC$)
$\therefore \widehat{OBC} = 38°$
and $\widehat{OCA} + 38° = 71°$ ($\widehat{ACB} = 71°$)
$\therefore \widehat{OCA} = 33°$
$\therefore \widehat{OAC} = 33°$ (base angles of isosceles triangle OCA; OC, OA are equal radii).

4 $\widehat{OSB} = 90°$ (line drawn from centre of circle to midpoint of chord $\perp$ chord)
Similarly, $\widehat{XTB} = 90°$
$\therefore \widehat{OST} + \widehat{STX} = 180°$
But $\widehat{OST} + \widehat{STX} + \widehat{TXO} + \widehat{XOS} = 360°$ (angle sum of quadrilateral $STXO$)
$\therefore \widehat{TXO} + \widehat{XOS} = 180°$
$\therefore \widehat{SOX}$ is the supplement of $\widehat{OXT}$.

5



Let L be the midpoint of AB. Join PL.
$\therefore OL \perp AB$ (line joining centre of circle to midpoint of chord $\perp$ chord)
and $PL \perp AB$ (line joining midpoint of base of isosceles to opposite vertex $\perp$ base)
$\therefore P, O, L$ are collinear
$AB = 48$ mm so $AL = 24$ mm.
In $\triangle AOL$, $OL^2 = 26^2 - 24^2$ (Pythagoras' Theorem)
$\therefore OL = 10$ mm
$\therefore PL = 36$ mm
In $\triangle PLA$, $PA^2 = 24^2 + 36^2$ (Pythagoras' Theorem)
$\therefore PA^2 = 1872$
$PA = 12\sqrt{13} \doteq 43$ mm.

6 Let A be the midpoint of CD and B the midpoint of GF.
Join OA, OB, OE.
$\widehat{OAD} = 90°$ (line joining centre of circle to midpoint of chord $\perp$ chord).
Similarly $\widehat{OBF} = 90°$
In $\triangle$s OAE and OBE
$\widehat{OAE} = \widehat{OBE} = 90°$ (proved above)
OE is a common side.
$OA = OB$ (equal chords, CD, GF, are equidistant from the centre)
$\therefore \triangle OAE \equiv \triangle OBE$ (R.H.S.)
$\therefore AE = BE$ (corresponding sides of congruent triangles)
and $AD = BF$ (halves of equal chords)
Now $DE = AE - AD$
$FE = BE - BF$
$\therefore DE = FE$.

7 $\widehat{WXZ} = 54°$ (angles in the same segment)
$m° + 25° + 45° + 54° = 180°$ (angle sum of $\triangle WXZ$)
$m = 56$

8 $\widehat{RTQ} = 47°$ (angles in the same segment)
$\therefore \widehat{RTP} = 133°$ (supplement of RTQ, Q, T, P collinear)
$\therefore x = 22$ (angle sum of $\triangle RTP$ is $180°$)

9. $B\widehat{O}C = 90^\circ$ ($BO \perp OC$)
$\therefore B\widehat{E}C = 45^\circ$ (angle at centre is twice angle at circumference)
Similarly, $B\widehat{D}C = 45^\circ$
and $D\widehat{C}E = C\widehat{E}B = 45^\circ$ (alternate angles, $CD \parallel BE$)
$\therefore C\widehat{F}D = 90^\circ$ (angle sum of $\triangle FDC$ is 180°).

10. $K\widehat{N}M = 90^\circ$ (angle in a semicircle, KM diameter)
$\triangle LKN$ is isosceles ($LK = LN$, given)
$\therefore L\widehat{K}N = L\widehat{N}K$ (base angles of isosceles triangle)
$\therefore 2L\widehat{N}K + 48^\circ = 180^\circ$ (angle sum of $\triangle LKN$)
$\therefore L\widehat{N}K = 66^\circ$
$\therefore L\widehat{N}M = 90^\circ - 66^\circ$ ($K\widehat{N}M - L\widehat{N}K$)
$= 24^\circ$.

11. Let $Q\widehat{P}Y = x^\circ$, $Q\widehat{X}P = y^\circ$.
$P\widehat{Y}Q = y^\circ$ (angles in the same segment)
Similarly $Q\widehat{R}Y = x^\circ$
and $Q\widehat{X}Y = x^\circ$
$X\widehat{Q}Y = 90^\circ$ (angle in a semicircle, XY diameter)
$\therefore x^\circ + 90^\circ + (y^\circ + P\widehat{Y}X) = 180^\circ$ (angle sum of $\triangle XQY$)
$\therefore P\widehat{Y}X = 90^\circ - (x + y)^\circ$
$\therefore P\widehat{Q}X = 90^\circ - (x + y)^\circ$ (angles in the same segment)
$\therefore X\widehat{Q}P + P\widehat{Y}Q + Q\widehat{R}Y = 90^\circ - (x + y)^\circ + y^\circ + x^\circ$
$= 90^\circ$.

12. Let $J\widehat{K}L = x^\circ$
$\therefore J\widehat{M}L = x^\circ$ (opposite angles of parallelogram are equal)
$\therefore J\widehat{N}L = (180 - x)^\circ$ (opposite angles of cyclic quadrilateral $JNLM$ are supplementary)
$\therefore L\widehat{N}K = x^\circ$ ($J\widehat{N}K = 180^\circ$)
$\therefore L\widehat{N}K = L\widehat{K}N$ (both x°)
$\therefore LK = LN$ (sides opposite equal angles of a triangle).

13. Let $A\widehat{B}E = x^\circ$.
$\therefore C\widehat{B}E = (180 - x)^\circ$ (CBA a straight line)
$\therefore C\widehat{D}E = x^\circ$ (opposite angles of cyclic quadrilateral are supplementary)
In $\triangle$s ABE and ADC
$A\widehat{B}E = A\widehat{D}C$ (both x°)
$B\widehat{A}E = D\widehat{A}C$ (same angle)
$\therefore \triangle ABE \parallel\!\mid \triangle ADC$.

14. Let $N\widehat{K}P = x^\circ$
$\therefore P\widehat{K}L = x^\circ$
$N\widehat{K}P = N\widehat{M}P = x^\circ$ (angles in the same segment)
$P\widehat{M}Q = P\widehat{K}L = x^\circ$ (exterior angle of cyclic quadrilateral $KLMP$ equals interior opposite angle)
$\therefore N\widehat{M}P = P\widehat{M}Q$ (both x°)
$\therefore PM$ bisects $N\widehat{M}Q$.

15. Join CD and let $B\widehat{A}D = x^\circ$.
$\therefore D\widehat{C}E = B\widehat{A}D = x^\circ$ (exterior angle of cyclic quadrilateral $BADC$ equals interior opposite angle)
$D\widehat{F}E = D\widehat{C}E = x^\circ$ (angles in the same segment)
$\therefore D\widehat{F}G = (180 - x)^\circ$ (GFE a straight line)
$\therefore G\widehat{A}D + D\widehat{F}G = x^\circ + (180 - x)^\circ$
$= 180^\circ$
$\therefore GADF$ is a quadrilateral with a pair of opposite angles supplementary
$\therefore GADF$ is a cyclic quadrilateral.

6 $\triangle GJT$ is isosceles ($GJ = JT$)
$\therefore J\widehat{G}T = 40^\circ$ (base angles of isosceles triangle)
$T\widehat{J}F = J\widehat{G}T = 40^\circ$ (angle in the alternate segment)
$\therefore (x + 40) + 40 + 40 = 180$ (angle sum of $\triangle GJT$)
$\therefore x = 60$.

17 (a) Let $F\widehat{B}E = x^\circ$, $D\widehat{B}E = y^\circ$.
$\therefore B\widehat{D}E = x^\circ$ (angle in the alternate segment).
Similarly $B\widehat{F}E = y^\circ$
$\therefore F\widehat{E}B = 180 - (x + y)^\circ$ (angle sum of $\triangle FEB$)
and $D\widehat{E}B = 180 - (x + y)^\circ$ (angle sum of $\triangle DEB$)
$\therefore F\widehat{E}B = D\widehat{E}B$
$\therefore BE$ bisects $F\widehat{E}D$.

(b) $x + y = 70$
$\therefore B\widehat{E}D = 180^\circ - (70)^\circ$
$= 110^\circ$.

18 Join OD, OC
$O\widehat{D}T = O\widehat{C}T = 90^\circ$ (tangent $\perp$ radius drawn to point of contact)
$\therefore D\widehat{O}C = 132^\circ$ (angle sum of $DOCT$ is 360°)
$\therefore$ Reflex $D\widehat{O}C = 228^\circ$
$\therefore D\widehat{Q}C = 114^\circ$ (angle at centre is twice angle at circumference).

19 Join AO, OC, CP, PB.
OCP is a straight line (line of centres passes through point of contact)
$O\widehat{A}D = 90^\circ$ (radius $\perp$ tangent)
and $O\widehat{C}D = 90^\circ$ (radius $\perp$ tangent)
$\therefore ADCO$ is a cyclic quadrilateral (pair of opposite angles supplementary)
Also $P\widehat{B}D = 90^\circ$ (radius $\perp$ tangent)
$P\widehat{C}D = 90^\circ$ (supplement of $O\widehat{C}D$)
$\therefore DBPC$ is a cyclic quadrilateral (pair of opposite angles supplementary).

20 Let $L\widehat{K}A = x^\circ$, $A\widehat{K}C = y^\circ$.
$\therefore K\widehat{B}A = L\widehat{K}A = x^\circ$ (angle in the alternate segment)
and $K\widehat{D}C = L\widehat{K}C = (x + y)^\circ$ (angle in the alternate segment).
but $K\widehat{D}C = D\widehat{C}B + C\widehat{B}D$ (exterior angle of $\triangle CDB$)
$\therefore D\widehat{C}B = y^\circ$
$D\widehat{K}C = D\widehat{C}B = y^\circ$ (angle in the alternate segment)
$\therefore A\widehat{K}C = C\widehat{K}B$ (both equal to y°).

21 $AB^2 = AX^2 - XB^2$ (Pythagoras $\triangle AXB$)

But $XB^2 = AX.XP$ (   )

$\therefore AB^2 = AX^2 - AX.XP$

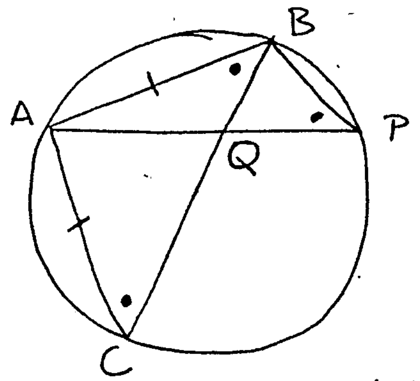
$= AX(AX - XP)$

$\therefore AB^2 = AX.AP$

Similarly

$AB^2 = AQ.AY$

22



$\triangle ABC$ is isosceles ($AB = AC$ given)

$\therefore \angle ABC = \angle ACB$ (base $\angle$s isosceles $\triangle$)

$\angle C = \angle P$ (angles in same segment)

$\therefore \angle ABC = \angle BPA$. — (1)

In $\triangle$s ABQ and ABP

1. $\angle BAQ$ is common

2 $\angle ABQ = \angle BPA$ ((1) above)

3. $\angle AQB = \angle \cancel{A}ABP$ (3rd $\angle$ of $\triangle$s)

$\therefore \triangle ABQ \,|||\, \triangle ABP$

$\therefore \dfrac{AB}{AQ} = \dfrac{A\cancel{BP}}{AB}$ (corresponding sides similar $\triangle$s)

$\therefore AB^2 = AQAP$