ASSIGNMENT 13: CIRCULAR FUNCTIONS I + II

1 AB is a chord of length six centimetres in a circle (centre O) of radius five centimetres. The point X lies on the minor arc AB. Using your own sketch, find:

(a) the size of angle AOB, in degrees and minutes

(b) the area of ΔAOB

(c) the area of the minor sector AOB

(d) the length of the minor arc AXB

(e) the area of the major segment formed by the chord AB.

2 Express, in terms of π, the size of each interior angle of a regular octagon.

3 (a) What is the period and amplitude of the curve $y = 2\sin\frac{x}{3}$?

(b) Show, by substitution, that the point $\left(\frac{\pi}{2}, 1\right)$ lies on the graphs of:

$$y = 2\sin\frac{x}{3}$$

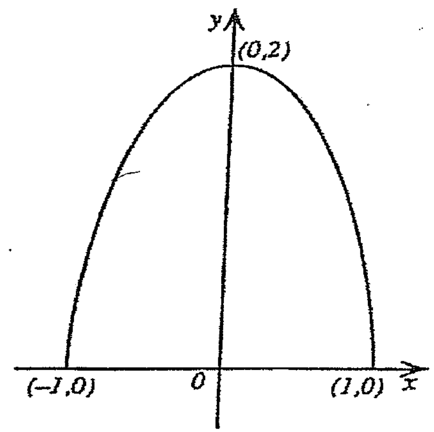
$$y = \frac{2x}{\pi}$$

(c) Using the domain $0 \le x \le \pi$ sketch the two functions $y = 2\sin\frac{x}{3}$ and $y = \frac{2x}{\pi}$ on the same axes. Find the area enclosed by these curves.

4 Differentiate $e^{\sin x}$ with respect to x and hence find $\int_0^{\frac{\pi}{2}} \cos x\, e^{\sin x} dx$.

5 The curve $y = \tan x$ for $0 \le x \le \frac{\pi}{3}$ is rotated about the x-axis. Find the volume of the solid of revolution thus generated.

6



An ornamental arch window 2 metres wide and 2 metres high is to be made in the shape of an arc of either a cosine curve or a parabola, as illustrated on axes above.

(a) If the arch is made in the shape of the curve:

$$y = 2\cos\frac{\pi}{2}x,$$

find the area of the window. (Your answer may be left in terms of π.)

(b) If the arch is made in the shape of an arc of a parabola, find the equation of the parabola and the area of the window.

7 Evaluate $\lim\limits_{x \to 0} \frac{\sin 5x}{x}$

8 Find the slope of the tangent to $y = \frac{1}{\sin x + \cos x}$ at $x = 0$.

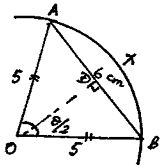
ASSIGNMENT 13

(1) (a) $DB = 3\text{ cm}$

$\therefore \sin\frac{\theta}{2} = \frac{3}{5}$

$\frac{\theta}{2} = 36^\circ 52'$

$\therefore \theta = 73^\circ 44' = \angle AOB$



(b) Area of $\triangle AOB = \frac{1}{2}\times 5\times 5 \sin 73^\circ 44'$

$= 12\text{ cm}^2$

(c) Area of minor sector $AOB = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 5^2 \times \frac{73^\circ 44'}{180^\circ}\times \pi^c$

$= 16.1\text{ cm}^2$ (to 1 d.p.)

(d) Length of minor arc $AXB = r\theta$

$= 5\times\frac{73^\circ 44'}{180^\circ}\times\pi^c$

$= 6.4$ cm (to 1 d.p.)

(e) Area of major sector

= Area of circle − Area of minor segment

$= \pi(5^2) - 16.086 - 12$

$= 50.5\text{ cm}^2$ (to 1 d.p.)

(2) Sum of angles of an octagon

$= (8-2)\times 180^\circ = 6\times 2\pi$

$\therefore$ Each interior angle

$= \frac{12\pi}{8} = \frac{3\pi}{2}$

(3) (a) Period $= T = \frac{2\pi}{n} = \frac{2\pi}{\frac{1}{3}} = 6\pi$ seconds.

Amplitude = 2

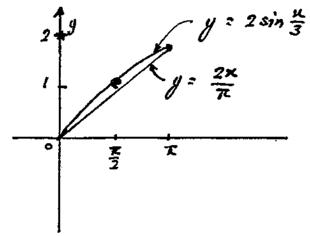
(b) (i) Sub. $x = \frac{\pi}{2}$ into $y = 2\sin\frac{\pi}{6}$

$= 2\times\frac{1}{2} = 1$ $\therefore$ point $(\frac{\pi}{2}, 1)$ lies on graph

(ii) Sub. $x = \frac{\pi}{2}$ into $y = \frac{2x}{\pi} = \frac{2\pi}{\pi\times 2} = 1$ $\therefore$ " " " " "

Assign 13

(3) (c)



(4) $y = e^{\sin x} \Rightarrow \frac{dy}{dx} = \cos x\, e^{\sin x}$

If $\frac{d}{dx}\left[e^{\sin x}\right] = \cos x\, e^{\sin x}$

then $\int_0^{\frac{\pi}{2}} \cos x . e^{\sin x}\,dx = \left[e^{\sin x}\right]_0^{\pi/2}$

$= e^1 - e^0$

$= e - 1$

(5) Volume required $= \pi\int y^2\,dx$

$= \pi\int_0^{\frac{\pi}{3}} \tan^2 x\,dx = \pi\int_0^{\frac{\pi}{3}} \sec^2 x - 1\,dx$

$= \pi\left[\tan x - x\right]_0^{\frac{\pi}{3}}$

$= \pi\left[\left(\tan\frac{\pi}{3} - \frac{\pi}{3}\right) - 0\right]$

$= \pi\left(\sqrt{3} - \frac{\pi}{3}\right)$

$= \frac{\pi}{3}(3\sqrt{3} - \pi)$ cu. units

(6) (a) Area $= \int_{-1}^{1} 2\cos\frac{\pi}{2}x\,dx$

$= 2\int_0^1 2\cos\frac{\pi x}{2}\,dx$

$= 4\left[\frac{\sin\frac{\pi x}{2}}{\frac{\pi}{2}}\right]_0^1$

$= \frac{8}{\pi}\sin\frac{\pi}{2}$

$= \frac{8}{\pi}$ sq. units.

Assign. B

(6) (b) To find the equation of the parabola

$$y = k(x-1)(x+1)$$

Sub $(0,2)$ to find k

$$2 = k(-1)(+1)$$

$$k = -2$$

$$\therefore\ y = -2(x^2-1) = -2x^2+2$$

$$\therefore\ \text{area required} = \int_{-1}^{1} 2-2x^2\,dx$$

$$= 2\int_0^1 2(1-x^2)\,dx$$

$$= 4\left[x-\frac{x^3}{3}\right]_0^1$$

$$= 4\left[1-\frac{1}{3}-0\right]$$

$$= 4\times\frac{2}{3} = \frac{8}{3} = \underline{2\frac{2}{3}\text{ sq. units}}$$

(7) $$\lim_{x\to 0} 5\times\frac{\sin 5x}{5x} = 5\underbrace{\lim_{x\to 0}\frac{\sin 5x}{5x}}$$

$$= 5\times 1$$

$$= \underline{5}$$

(8) $$y = \frac{1}{(\sin x+\cos x)} = (\sin x+\cos x)^{-1}$$

$$\therefore\ \frac{dy}{dx} = -1(\sin x+\cos x)^{-2}\times(\cos x-\sin x)$$

$$= \frac{-1(\cos x-\sin x)}{(\sin x+\cos x)^2}$$

At $x=0$

$$\frac{dy}{dx} = \frac{-1(1-0)}{(0+1)^2}$$

$$= -1$$