

1. A committee of three is to be chosen from a group of four males and five females. The committee must include at least one male and at least one female. How many different committees can be chosen?
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There is a row of five coloured lights, each with its own switch. When switched on, each light is equally likely to show green, red or amber.

2. How many different colour patterns are possible?
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What is the probability that

3. The first three lights from the left are green, but not the fourth?
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4. The first three lights from the left are the same colour, but not the fourth?
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5. Exactly three of the lights are green?
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6. If all the lights are switched on five times, find, as a decimal correct to three places, the probability that exactly three lights will be the same colour on two or three occasions.

Combinatorics

$$\begin{aligned} \textcircled{1} \quad & {}^4C_1 \times {}^5C_1 \times {}^7C_1 \checkmark \\ & = 4 \times 5 \times 7 \\ & = 140 \end{aligned}$$

$$\textcircled{2} \quad \boxed{3 \quad 3 \quad 3 \quad 3 \quad 3} \checkmark$$

$$\begin{aligned} \text{permutations} &= 3^5 \\ &= 243 \checkmark \end{aligned}$$

$$\textcircled{3} \quad \boxed{\frac{1}{8} \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} \times 1} \checkmark$$

$$P(E) = \frac{2}{81} \checkmark$$

$$\begin{aligned} \textcircled{4} \quad P(E) &= P(G) + P(R) + P(A) \\ &= P(\text{Gas lights}) + P(\text{R lights}) + P(\text{A lights}) \\ &= 3 \times \frac{2}{81} \checkmark \end{aligned}$$

$$= \frac{2}{27} \checkmark$$

$$\textcircled{5} \quad \boxed{\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{2}{3} \quad \frac{2}{3}} \checkmark$$

$$P(E) = \frac{4}{243} \checkmark$$

$$\textcircled{6} \quad \text{let } p = \frac{1}{3} \quad q = \frac{2}{3} \quad (p+q)^5$$

$$\begin{aligned} P(E) &= {}^5C_3 p^2 q^3 + {}^5C_2 p^3 q^2 \\ &= {}^5C_3 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^3 + {}^5C_2 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 \\ &= \frac{80}{243} + \frac{40}{243} \quad \begin{array}{l} -x > 1 \\ -x < 1 \end{array} \\ &= \frac{120}{243} = \frac{40}{81} \end{aligned}$$