Complex Numbers Assessment 2002

Tommy Lim.

27/31

Question 1 (16 marks) - *Use a separate sheet of paper* — Marks

a) Evaluate i^{2002}. — 1

b) If $z = 2 + 2\sqrt{3}\,i$ find

i) $\dfrac{z}{1+i}$ in the form $a+ib$. — 1

ii) $\bar{z}$. — 1

iii) $|z|$ — 1

iv) $Arg(z)$ — 1

v) $\dfrac{1}{z^4}$ in modulus argument form. (use principle argument) — 2

c) On an Argand diagram sketch the locus specified by

$$Arg(z-1+i) = \frac{3\pi}{4}$$ — 2

d) z is a complex number where $|z|=1$ and $Arg(z)=A$. — 2

Find, in term of A the value of $Arg(z+1)$. Justify your answer.

Question 2

OABC is a rectangle with $OA = 3 \times OC$. — 3

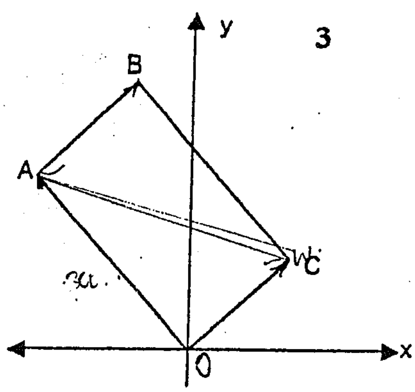
O is the origin.

If C represents the complex number w, find in terms of w the complex number represented by;

i) the point A

ii) the point B

iii) the vector from A to C



Question 3

a) On an Argand diagram sketch the locus specified by **2**

$|z-1-i| > |z|$

b) Find the modulus of $\dfrac{(2-i)^8}{(2+i)^6}$. **3**

c) i) Solve $z^5 = -1$ over the complex field. **2**

ii) If ω is the complex root of $z^5 = -1$ with smallest **3**

positive argument, show that the other complex roots equal

$-\omega^2$, ω^3 and $-\omega^4$.

iii) Using ω as in part ii) simplify $(1-\omega+\omega^2-\omega^3)^8$. **3**

d) Describe and sketch on an Argand diagram the locus **3**

of z given that $\text{Re}(z-\frac{1}{z}) = 0$ where $z \neq 0$.

SOLUTIONS

QUESTION 1

a. $i^{2002} = (i^2)^{1001}$

$= (-1)^{1001}$

$= -1$

b. i) $\dfrac{2+2\sqrt{3}i}{1+i}$

$= \dfrac{2+2\sqrt{3}i}{1+i} \times \dfrac{1-i}{1-i}$

$= \dfrac{2-2i+2\sqrt{3}i+2\sqrt{3}}{2}$

$= (1+\sqrt{3}) + i(\sqrt{3}-1)$

ii) $\bar{z} = 2-2\sqrt{3}i$

iii) $|z| = \sqrt{2^2+(2\sqrt{3})^2}$

$= 4$

iv) $\operatorname{Arg}(z) = \dfrac{\pi}{3}$

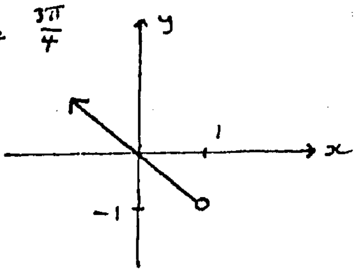
v) $\dfrac{1}{z^4} = z^{-4}$

$= \left[4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^{-4}$

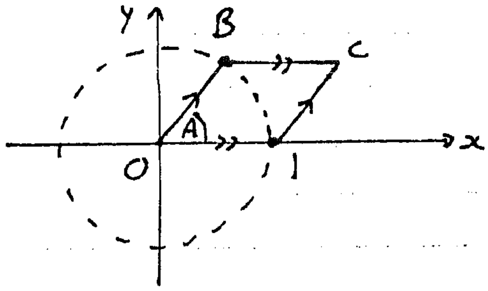
$= 4^{-4}\left(\cos -\frac{4\pi}{3} + i\sin -\frac{4\pi}{3}\right)$

$= \dfrac{1}{256}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

c. $\operatorname{Arg}(z-(1-i)) = \dfrac{3\pi}{4}$



d.



B represents z
C represents $z+1$
shape is a rhombus
diagonals bisect angles

$\therefore \operatorname{Arg}(z+1) = \dfrac{A}{2}$

QUESTION 2

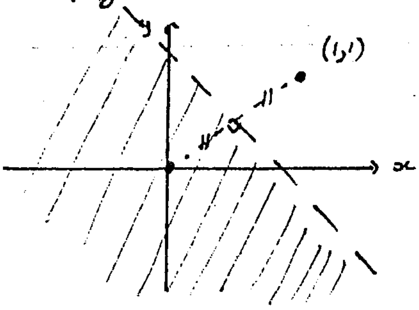
a i) $3iw$

ii) $3iw + w$
or $w(1+3i)$

iii) $w - 3iw$
or $w(1-3i)$

QUESTION 3

a. $|z-(1+i)| > |z|$



b. $|2-i| = \sqrt{5}$, $|2+i| = \sqrt{5}$

$\therefore \left|\dfrac{(2-i)^8}{(2+i)^6}\right| = \dfrac{\sqrt{5}^8}{\sqrt{5}^6}$

$= 5$

c. i) roots of $z^5 = -1$ are

$z_1 = \cos\frac{\pi}{5} + i\sin\frac{\pi}{5}$

$z_2 = \cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}$

$z_3 = -1$

$z_4 = \cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}$

$z_5 = \cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}$

ii) $\cos\frac{\pi}{5} + i\sin\frac{\pi}{5} = w$

$\therefore \cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5} =$

$= \left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)^3$

$= w^3$

$\therefore \cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}$

$= \left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)^7$

$= w^7$

$= w^5 . w^2 \qquad (w^5 = -1)$

$= -w^2$

$\therefore \cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}$

$= \left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)^9$

$= w^9$

$= w^5 . w^4$

$= -w^4$

iii) sum of roots of $z^5 + 1 = 0$ equals 0

$\therefore -1 + w - w^2 + w^3 - w^4 = 0$

or $1 - w + w^2 - w^3 = -w^4$

$\therefore (1 - w + w^2 - w^3)^8 = (-w^4)^8$

$= w^{32}$

$= w^{30} . w^2$

$= (w^5)^6 . w^2$

$= (-1)^6 . w^2$

$= w^2$

$= \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$

d. $\text{Re}\left(z - \frac{1}{z}\right) = 0 \qquad z \neq$

$\text{Re}\left(x + iy - \frac{1}{x+iy}\right) = 0$

$\text{Re}\left(x + iy - \frac{1}{x+iy} \times \frac{x-iy}{x-iy}\right) = 0$

$\text{Re}\left(x + iy - \frac{x-iy}{x^2+y^2}\right) = 0$

$\therefore x - \frac{x}{x^2+y^2} = 0$

$x(x^2+y^2) - x = 0$

$x(x^2+y^2-1) = 0$

$\therefore x = 0$ or $x^2 + y^2 = 1$

$(z \neq 0)$