COMPLEX NUMBERS (I)

• A complex number takes the form $x + iy$ where x and y are real and $i = \sqrt{-1}$
• If $z = x + iy$ then x is referred to as the real part of z and y is referred to as the imaginary part of z.
• The conjugate of $z = x + iy$ is given by $\overline{z} = x - iy$.
• $i^2 = -1$
• complex numbers may be plotted in the Argand diagram.

166. Express the following in the form $a + ib$ where a and b are real.

a) $(3+i)+(1+2i)$ b) $(5-3i)+(4+3i)$ c) $(2-3i)-(1+2i)$
d) $(1+i)-(1-i)$ e) $(2+3i)(4+5i)$ f) $(2-i)(3+2i)$

167. Simplify

a) i^3 b) i^4 c) i^5 d) i^6 e) $\frac{1}{i^2}$ f) $\frac{1}{i}$ g) $\frac{1}{i^3}$

168. a) If $z = 1 + i$, find $z\overline{z}$.   b) If $z = 5 - 4i$, find $z\overline{z}$.

169. Express the following in the form $a + ib$ where a and b are real.

a) $\frac{1}{2-3i}$ b) $\frac{3i-2}{1+2i}$

170. Simplify

a) $(4-5i)^2$ b) $(1-i)^3$ c) $\frac{1}{(1+i)^3}$

171. Plot the following points on an Argand diagram

a) $\sqrt{3}+i$ b) $-1+i$ c) $2-2i$ d) $-1-i$

172. Find the conjugate $\overline{z}$ of each of the complex numbers z in the previous question.

173. Let $z = 3 + 2i$.

(a) Express z^2 and $\frac{1}{z}$ in $a + ib$ form.

(b) Plot z, z^2 and $\frac{1}{z}$ on the same Argand diagram.

COMPLEX NUMBERS (II)

If $z = x + iy$
- the modulus $|z|$ of z is given by $|z| = \sqrt{x^2 + y^2}$
- the argument $\theta = \arg(z)$ of z satisfies $\tan(\theta) = \dfrac{y}{x}$

174. Find the modulus and argument of each of the following complex numbers:

a) $\sqrt{3} + i$ b) $-1 + i$ c) $2 - 2i$ d) $-1 - i$

175. (a) Express the following complex numbers in $a + ib$ form :

(i) $\dfrac{1}{3 - 2i}$ (ii) $\dfrac{1}{(1 - i)^2}$

(b) Find the modulus and argument of each of the complex numbers $z = 1 + 2i$ and $w = 2 - i$ and represent z and w clearly by points A and B on an Argand diagram. Find also the sum and product of z and w and mark the corresponding points C and D in your diagram.

176. Given that $z = 3 + i$ and $w = 1 + 3i$ express in the form $a + ib$, where a and b are real, the complex numbers

(a) zw (b) $\dfrac{z}{w}$ (c) $z^2 - w^2$

and find their moduli and arguments in degrees, correct to the nearest degree.

177. Solve the quadratic equations

a) $z^2 - 4z + 29 = 0$ b) $4z^2 + 7 = 0$
c) $2z^2 + 3z + 5 = 0$ d) $4z^2 + 4z + 5 = 0$

178. Find **all** the solutions to the equation $(x^2 + 4)(5x^2 - 7) = 0$.

SOLUTIONS

Complex numbers (I)

166. a) $4+3i$ b) 9 c) $1-5i$ d) $2i$ e) $-7+22i$ f) $8+i$

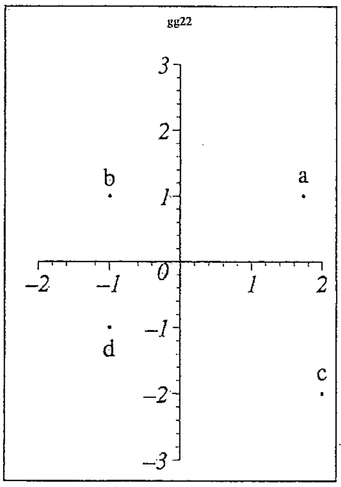
167. a) $-i$ b) 1 c) i d) -1 e) -1 f) $-i$ g) i

168. a) 2 b) 41

169. a) $\frac{2}{13}+\frac{3}{13}i$ b) $\frac{4}{5}+\frac{7}{5}i$

170. a) $-9-40i$ b) $-2-2i$ c) $-\frac{1}{4}(1+i)$

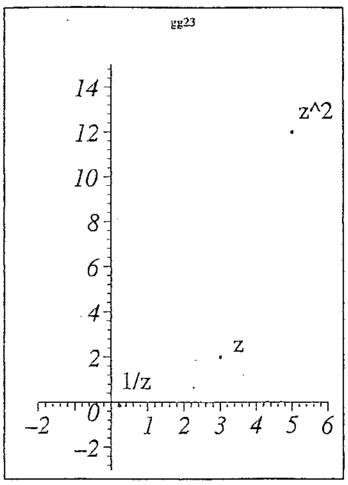
171.



172. a) $\sqrt{3}-i$ b) $-1-i$ c) $2+2i$ d) $-1+i$

173. (a) $5+12i$, $\frac{3}{13}-\frac{2}{13}i$

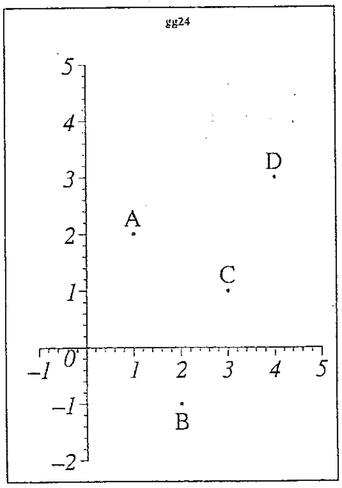
(b)



Complex Numbers (II)

174. a) $2; 30°$ b) $\sqrt{2}; 135°$ c) $2\sqrt{2}; -45°$ d) $\sqrt{2}; -135°$

175. a) $\dfrac{3+2i}{13}$, $\dfrac{i}{2}$ b) $\sqrt{5}$, $63.4°$, $\sqrt{5}$, $-26.6°$, $3+i$, $4+3i$.



176. a) (i) $10i$, 10, $90°$ ii) $\dfrac{3-4i}{5}$, 1, $-53°$ b) 16, 16, 0.

177. a) $2 \pm 5i$ b) $\pm\dfrac{1}{2}\sqrt{7}i$ c) $\dfrac{-3 \pm \sqrt{31}i}{4}$ d) $\dfrac{1}{2}(-1 \pm 2i)$

178. $\pm\sqrt{\dfrac{7}{5}}$, $\pm 2i$.