Exercise 5.4

Find the points of intersection of the following lines and curves, stating clearly those cases where the lines and curves touch each other.

1. $y = 3x - 7$; $y = x^2 - 4x + 3$
2. $y = 3x + 5$; $y = 2x^2 + 5x - 7$
3. $y = 5x + 4$; $y = x^3 + x^2 - 4x - 5$
4. $y = x^2 + 3x + 2$; $y = x^3 + 4x^2 - 3x - 6$
5. $y = 2x + 5$; $y = x^3 - x + 3$
6. $y = 2x^2 + 3x - 5$; $y = x^3 + x^2 - 5x + 7$

Find the value of k such that the given line touches the given curve in each of the following cases.

7. $y = x + 3$; $y^2 = kx$
8. $y = kx - 4$; $xy + 8 = 0$
9. $y = 3x + k$; $3x^2 + 5y^2 = 15$
10. $y = 2x + k$; $x^2 + y^2 + 4x - 10y - 16 = 0$

Find the coordinates of the points of intersection of the given lines and the given curves with parametric equations in each of the following cases.

11. $y = x - 6$; $x = 2t^2$, $y = 4t$
12. $2x + y = 6$; $x = 4t^2$, $y = 8t$
13. $x - 2y = 1$; $x = t^2 - 2$, $y = 2t + 1$
14. $2x + 3y = 15$; $x = 3t$, $y = \frac{3}{t}$

Without finding the coordinates of the points of intersection, P and Q, between the following lines and curves, find the coordinates of the mid-point of PQ.

15. $y = 3x + 2$; $y^2 = 6x + 8$
16. $y = 2x + 1$; $3x^2 + 4y^2 = 12$
17. $3x + 2y + 5 = 0$; $xy = 4$

Exercise 5.4

1. (2, –1), (5, 8)
2. (–3, –4), (2, 11)
3. (–1, –1), (3, 19), (–3, –11)
4. (–4, 6), (–1, 0), (2, 12)
5. (2, 9), touch at (–1, 3)
6. (–3, 4), touch at (2, 9)
7. 12
8. $\frac{1}{2}$
9. $\pm 4\sqrt{3}$
10. –6, 24
11. (2, –4), (18, 12)
12. (1, 4), (9, –12)
13. (–1, –1), (23, 11)
14. (3, 3), $(\frac{9}{2}, 2)$
15. $(-\frac{1}{3}, 1)$
16. $(-\frac{8}{19}, \frac{3}{19})$
17. $(-\frac{5}{6}, -\frac{5}{4})$