CRANBROOK SCHOOL

Form VI/Year 12 MATHEMATICS - 2 Unit, 3 Unit(First Paper)

Term 3 1998 **Time : 3 hrs/BES,DMC**

- **All questions may be attempted.**
- **All questions are of equal value.**
- **All necessary working should be shown in every question.**
- **Full marks may not be awarded if work is careless or badly arranged.**
- **Standard integrals are provided at the end of the paper.**
- **Approved silent calculators may be used.**
- **Begin each question on a new page. Submit your work in three booklets :**

(i) qq. 1 - 4
(ii) qq. 5 - 7
(iii) qq. 8 - 10

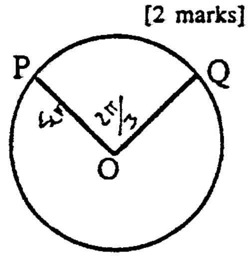
1.

(a) Find the exact value of $\dfrac{1}{5\times6+4}$ [1 mark]

(b) Simplify $(5x-3)-(7-4x)$. [2 marks]

(c) Write down the exact value of 225^0 in radians. [2 marks]

(d) In the diagram PQ is an arc of a circle with centre O. The radius OP = 5 cm and the angle POQ is $\dfrac{2\pi}{3}$.



Find the exact area of the sector.

[2 marks]

(e) Find $\int 2\tan 4x \sec 4x \; dx$ using the table of standard integrals. [2 marks]

(f) By rationalising the denominator express $\dfrac{4}{\sqrt{5}-2}$ in the form $a+\sqrt{b}$. [3 marks]

2. (new page please)

(a) The second derivative of a function is given by $\dfrac{d^2y}{dx^2}=\dfrac{1}{x^2}$ and the graph of the function passes through $(1,-1)$, with a slope of 2. Find the equation of the function. [5 marks]

(b) Consider the parabola $4y-x^2+4=0$.

(i) Find the vertex of the parabola.
(ii) Find the coordinates of the point(s) at which the curve cuts the x axis. [3 marks]

(c) Find the primitive of $\dfrac{\cos x}{\sin x+1}$. [1 mark]

(d) Find for what values of x the curve $y=xe^x$ is concave down. [3 marks]

3. (new page please)

(a) Given that a line AQ has the equation $2x + y - 4 = 0$ and the point P is $(1, -3)$

(i) Find the shortest distance between the line AQ and the point P.

(ii) Given that the line PQ is perpendicular to AQ show that the equation of the line PQ is $x - 2y - 7 = 0$.

(iii) Find the coordinates of Q.

[6 marks]

(b) Differentiate, simplifying your answers as far as is possible :

(i) $(1-5x)^7$ (ii) $x^2\left(1+\frac{1}{x}\right)$ (iii) $\log_e\left(\frac{x}{1-x}\right)$

[6 marks]

4. (new page please)

(a) Use Simpson's Rule with five function values (taken to 3 decimal places) to find an approximation for the area between the curve $y = e^{-x^2}$, the x axis and the ordinates $x = 0$ and $x = 1$ correct to 2 decimal places.

[3 marks]

(b) Prove that tan A sin A + cos A = sec A.

[2 marks]

(c) Sketch (on four separate diagrams) the regions of the plane for which each of the following equations is true, and the region for which all are true :

$y \geq x$, $x^2 + y^2 \leq 16$, $y < |x-2|$

[4 marks]

(d) Find the exact area formed between the y axis, the curve $y = \log_e(x-3)$ and the lines $y = 2$ and $y = 0$.

[3 marks]

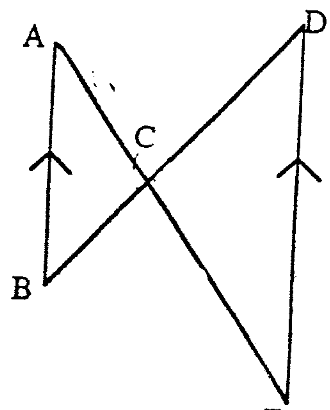
5. (new page please)

(a) In the diagram line AB is parallel to line DE.

(i) Prove that Δ ABC is similar to Δ EDC.

(ii) Sides AC = 6 cm, CD = 8.5 cm and CE = 7.5 cm.

Find the length of BC.



[4 marks]

(b) Sketch the parabola P which has focus (2, 3) and directrix $y = -1$. Mark clearly the coordinates of the vertex and the equation of the axis of the parabola.

[3 marks]

(c) Solve the equation $1 + \cos\phi = 2\sin^2\phi$ for all values of ϕ between 0° and 360°. [3 marks]

(d) A radar scanner rotates at a speed of 30 rev/min. Express this angular velocity in rad/sec.

[2 marks]

6. (new page please)

(a) Given the function $y = x^3 - 6x^2$

(i) Find the intercepts of the function.

(ii) Find, and distinguish between, the stationary points of the function.

(iii) Find all points of inflexion.

(iv) Using your results, and any others needed, sketch the graph of the function, showing all its essential features. [8 marks]

(b) Find the minimum possible product of two numbers that differ by 10. [4 marks]

7. (new page please)

(a) If the first term of an arithmetic progression is 4 and the nineteenth term is 31 find the thirtieth term. [3 marks]

(b) p, q and 32 are the first three terms of a geometric series and q, 4, p are the first three terms of another geometric series. Find p and q. [3 marks]

(c) A mould is observed over a period of time. Its initial diameter is 40 mm. It grows 9 mm during the first week of observation. Each succeeding week the growth is 70% of the previous week's growth. Assuming this pattern of growth continues, calculate the mould's ultimate diameter. [3 marks]

(d) If $y = \frac{1}{2}(e^x + e^{-x})$ prove that $1 + \left(\frac{dy}{dx}\right)^2 = y^2$. [3 marks]

8. (new page please)

(a) A room in a house is in the shape of a trapezium. The distance between the parallel sides is 3.9 metres. The floor is to be tiled with black and white triangular tiles each of perpendicular height 30 cm. The first row contains 13 tiles with a black one on each end, and each succeeding row contains two more tiles than the previous one (each of the rows has a black tile on each end).

(i) How many rows of tiles will there be?

(ii) How many tiles will be in the last row?

(iii) How many tiles will there be altogether?

(iv) How many tiles will be black? [6 marks]

(b) The tangent to the curve $y = (x+2)^3$ at the point A (-3, -1) meets the curve again at B. Find

(i) the equation of the tangent.

(ii) the coordinates of B.

(iii) the angle the tangent at B makes with the positive x-axis (nearest degree). [6 marks]

9. (new page please)

(a) An urn contains 5 white, 3 black and 2 blue balls. A ball is drawn and replaced, a second is drawn and replaced, and a third is drawn. What is the probability that :

(i) 3 white balls are drawn?
(ii) a black ball is drawn in the first two drawings, but not in the third?
(iii) white, black, blue balls are drawn in that order?
(iv) not more than two white balls are drawn?
(v) a white or a black ball is drawn in each drawing? [5 marks]

(b) The mass in grams of a radioactive substance after t days is given by $M = M_o e^{-0.005t}$.

(i) If the initial mass was 125 g, find the mass after 30 days (nearest gram).
(ii) After how many days will the mass be reduced to 75 grams (nearest day)? [4 marks]

(c) Graph the solution of $|x^2 - 1| \leq 4$ on a number line. [3 marks]

10. (new page please)

A(-4, 0), B(6, 0) and C(2, 4) are the vertices of a triangle.

(a) Find the equation of the perpendicular bisector of AB. [2 marks]

(b) Find the equation of the perpendicular bisector of BC. [3 marks]

(c) Find the coordinates of the point S where the perpendicular bisectors meet. [2 marks]

(d) Prove that S is equidistant from each of the vertices. [3 marks]

(e) Show that, if M is the midpoint of AC, then SM is perpendicular to AC. [2 marks]

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} \quad (n \neq -1;\ x \neq 0 \text{ if } n < 0)$$

$$\int \frac{1}{x} \, dx = \log_e x \quad (x > 0)$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax \quad (a \neq 0)$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax \quad (a \neq 0)$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} \quad (a \neq 0)$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax \quad (a \neq 0)$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax \quad (a \neq 0)$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \quad (a \neq 0)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} \quad (a > 0,\ -a < x < a)$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \log_e \left\{ x + \sqrt{x^2 - a^2} \right\} \quad (|x| > |a|)$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \log_e \left\{ x + \sqrt{x^2 + a^2} \right\}$$

(1) (a) $\frac{1}{34}$

(b) $9x - 10$

(c) $\frac{5\pi}{4}$

(d) $\frac{25\pi}{3}$ units^2

(e) $\frac{1}{2}\sec 4x + c$

(f) $8 + 4\sqrt{5}$

(2) (a) $y = 3x - \ln x - 4$

(b)(i) $V(0,-1)$

(ii) $(-2,0),(2,0)$

(c) $\ln(\sin x + 1) +$

(d) $x < -2$

(3)(a) (i) $\sqrt{5}$ units

(ii) Proof

(iii) $Q(3,-2)$

(b)(i) $-35(1-5x)^6$

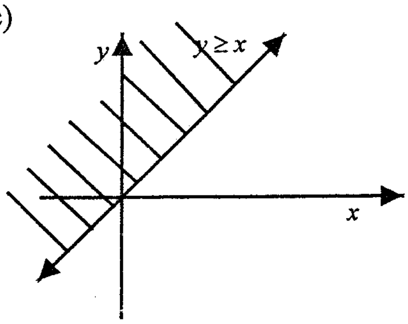
(ii) $2x+1$

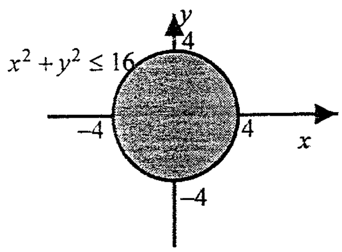
(iii) $\frac{1}{x(1-x)}$

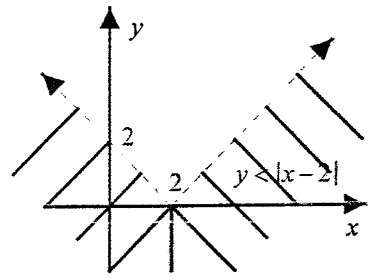
(4) (a) 0.75 (to 2d.p)

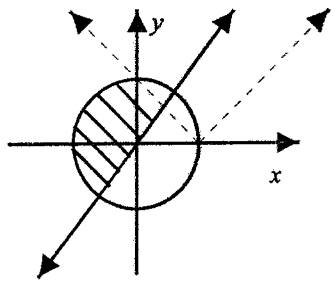
(b) Proof

(c)







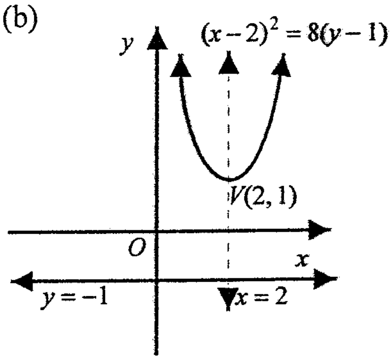


The region required for the intersection of all three functions.

(d) $e^2 + 5$ units^2

(5)(a)(i) Proof (ii) 6.8 cm

(b)



(c) $60^0, 180^0, 300^0$

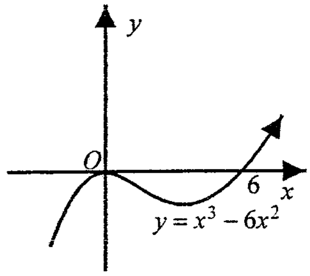
(d) π rad/sec

(6)(a)(i) $(0,0),(6,0)$

(ii) $(0,0)$ max; $(4,-32)$ mi

(iii) $(2,-16)$ Pt. of inflexio

(b)



(7) (a) $T_{13} = 22$

(b) $p = 2, q = 8$

(c) 70 mm

(d) Proof

(8)(a)(i) 13 rows

(ii) 37 tiles

(iii) 325 tiles

(iv) 26 black tile

(b)(i) $3x - y + 8 = 0$

(ii) $B(0,8)$

(iii) 85^0 (to nearest deg)

(9) (a) (i) $\frac{1}{8}$ (ii) 0.063 (iii) 0.03

(iv) 0.75 (v) $\frac{64}{125}$

(b)(i) 108 g (ii) 103 days

(c) (i) $-\sqrt{5} \le x \le \sqrt{5}$

(10) (a) $x = 1$ (b) $x - y - 2 = 0$

(c) $S(1,-1)$ (d) Proof

(e) $M(-1,2)$

Show that $m_{SM} \times m_{AC} = -1$