CRANBROOK SCHOOL

Year 12 MATHEMATICS - 2 Unit/3 Unit Common Trial

Term 3 1999 Time:3 h /SKB,HRK,MJB,GJB,KMR,WMF.

All questions are of equal value.
All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.

Approved silent calculators may be used.

Submit your work in six bundles .

Q 1-3 : **8 page booklet**

Q 4-6 : **8 page booklet**

Q 7; Q 8; Q 9 ; Q 10. : each a 4 page booklet.

[Use an 8 page booklet for Questions 1 - 3]

1. (a) What is the value of π^2 correct to 2 significant figures ? [2 marks]

(b) Simplify $(10-2y)-(8y-4)$ [2 marks]

(c) A six-sided die is thrown twice. What is the probability that the same face is uppermost each time ? [2 marks]

(d) Find the exact value of $225°$ in radians ? [1 mark]

(e) Find the primitive of $\sin 3x$. [2 marks]

(f) If $3^x = \dfrac{1}{3}$ find the value of x . [1 mark]

(g) A Holden's value depreciates by 15 % p.a.
If it was worth $10 000 in 1998 what will it be worth in 2000 ? [2 marks]

2. (a) Differentiate the following functions with respect to x simplifying your answers where applicable :

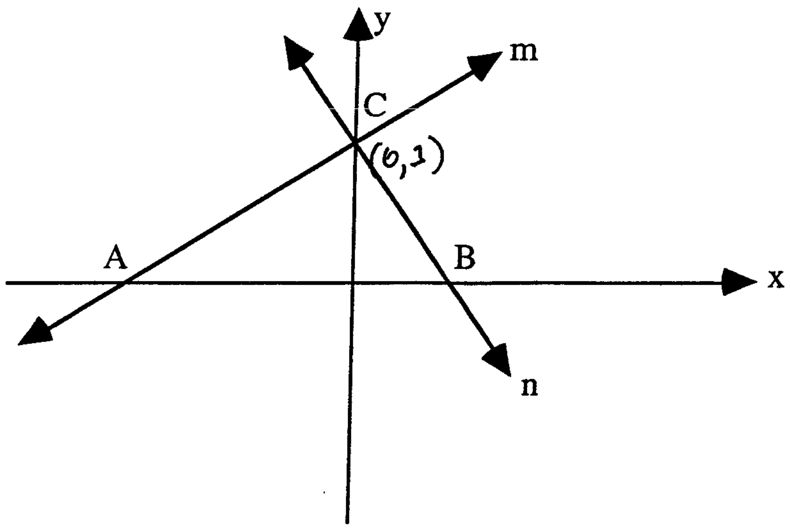
(i) $y = (3x^5 - 1)^7$ (ii) $y = \sin^2 5x$ (iii) $y = \dfrac{3}{\cos x}$ [6 marks]

(b) Find $\int \tan^2 x \, dx$ [2 marks]

(c) Evaluate $\int_0^1 e^{\frac{1}{2}x} \, dx$ [2 marks]

(d) Find $\int \dfrac{10x^2 - 1}{10x^3 - 3x} \, dx$ [2 marks]

3. On the diagram shown below the lines m and n meet at the point C.
The co-ordinates of C are (0 , 3).



(a) If the gradient of line m is $\frac{3}{4}$ what is the equation of line m ? [2 marks]

(b) If line m is perpendicular to line n what is the equation of line n ? [2 marks]

(c) What are the co-ordinates of A ? [1 mark]

(d) What are the co-ordinates of B? [1 mark]

(e) What is the area of triangle ABC ? [2 marks]

(f) Find the perpendicular distance from A to line n . [2 marks]

(g) Write down a pair of inequalities which define the region between lines m and n which also includes the origin. [2 marks]

[Use a new 8 page booklet for Questions 4 - 6]

4. (a) If α and β are the roots of the equation $5x^2 - 7x + 4 = 0$,
find the value of : $\alpha^2 + \beta^2$ [3 marks]

(b) The gradient of a curve is given by $\frac{dy}{dx} = 5x - 4$.
Find the equation of the curve if it passes through (-2, 5). [2 marks]

(c) The parabola S has equation $4x^2 = y + 2$.

(i) What are the co-ordinates of the vertex of S ? [1 mark]

(ii) What are the co-ordinates of the focus of S ? [2 marks]

(d) Find the equation of the tangent to the curve $y = \log_e 2x$ at the point where $x = 3$.
Write this equation in general form. [4 marks]

5. (a) If the area under the curve $y = x^2 - 9$ from $x = 1$ to $x = 3$ is rotated about the y-axis, calculate the volume of the solid of revolution formed. [3 marks]

(b) (i) Find an expression in terms of a, r, and n, for the difference between the Limiting Sum and the sum to n terms of a G. P. [2 marks]

(ii) Hence or otherwise find the difference between the limiting sum and the sum of the first five terms of the sequence $\frac{2}{3}, \frac{1}{2}, \frac{3}{8}, \ldots$ [3 marks]

(c) Use Simpson's Rule with five function values to find an approximate value for π using the formula below. (Give your answer correct to 5 significant figures.)

$$\pi = 4\int_0^1 \frac{dx}{x^2+1}$$ [4 marks]

6. (a) Solve $\log_e(x^2 - x) = \log_e 2 + \log_e(3x+4)$ [2 marks]

(b) A can is to be made in the form of a cylinder of volume $512\,cm^3$. The can is to have no top and the radius, r, of the base is restricted to $2 \le r \le 8$.

(i) If S represents the area of metal required to make the cylinder, show that

$$S = \pi r^2 + \frac{1024}{r}$$ [3 marks]

(ii) The formula in (i) has one turning point in the given Domain. Find the coordinates of this turning point. [3 marks]

(iii) Sketch the graph of the area, S, against the radius, r, in the given Domain, showing clearly the turning point and the end points. [3 marks]

(iv) Find the value of r for which the area of metal is greatest. [1 mark]

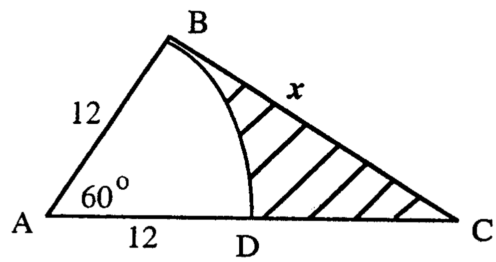
[Use a new 4 page booklet for Question 7]

7. (a) In the diagram ABD is a sector of a circle of a circle, centre A.

AB = AD = 12cm

$B\hat{A}C = 60^\circ$, $A\hat{B}C = 90^\circ$

BC = x cm



(i) Copy the diagram into your examination book and then calculate the exact value of x. [2 marks]

(ii) Convert angle A to radian measure and hence calculate the shaded area. [4 marks]

(b) If $f(x) = ax^2 + bx + c$ and $a > 0$, show that the minimum value of the function is $\frac{4ac - b^2}{4a}$. [3 marks]

(c) For what values of k are the roots of $(k+1)x^2 - 4kx + 3k = 0$ real and different? [3 marks]

[Use a new 4 page booklet for Question 8]

8. (a) Two straight roads AB and AC are inclined to each other at $62°35'$. Two bike riders start simultaneously from A and travel along the roads at 15 km/h and 25 km/h respectively.

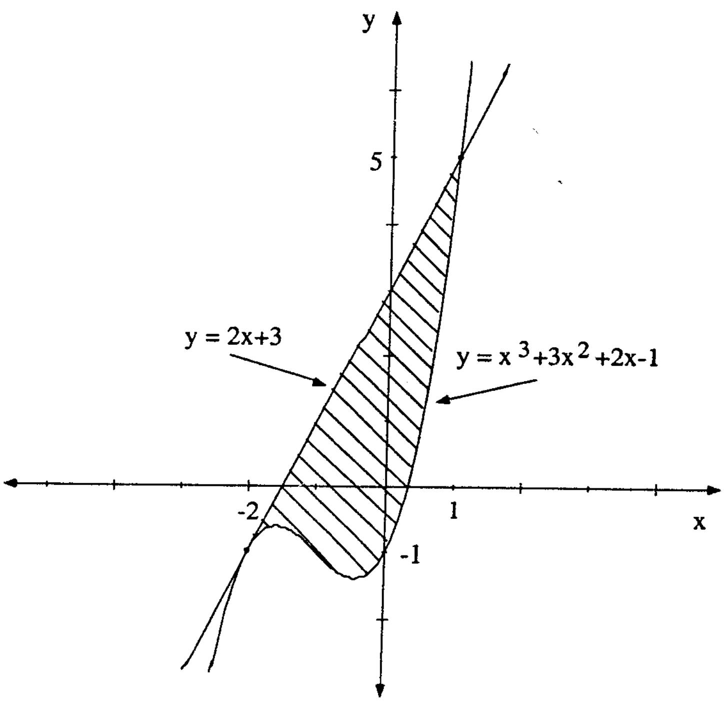
How long is it before they are $90\,km$ apart?
(Give your answer correct to the nearest minute.) [5 marks]

(b) Show that the curve $y = e^{-x^2}$ has one maximum turning point and two points of inflexion. Find the co-ordinates of these points and sketch the curve. [7 marks]

[Use a new 4 page booklet for Question 9]

9. (a) As shown in the diagram below, $y = 2x + 3$ is a tangent to the curve $y = x^3 + 3x^2 + 2x - 1$ at the point (-2, -1) and cuts the curve again at (1, 5).

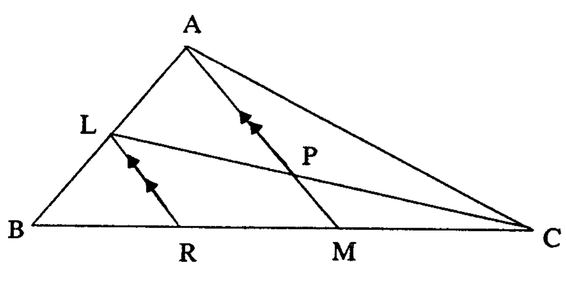
Find the area enclosed by these two curves.



[4 marks]

(b) In the diagram below,

LR || AM, BM : MC = 10 : 9 and BL : LA = 3 : 2.



(i) Prove that Δ BLR is similar to Δ BAM. [2 marks]

(ii) What is the ratio of BR : RM. State the geometric fact(s) used. [2 marks]

(ii) Stating all geometric facts used, find the ratio of MP : RL. [4 marks]

[Use a new 4 page booklet for Question 10]

10. (a) A crab fisherman nets 25 crabs of which 5 are female which, at this time of year, should be returned to the water. His mate takes 15 crabs, 4 of them female, and both bring their catches ashore in hessian sacks. The fishing inspector is awaiting them, tosses a coin to decide whose catch he will examine, and then randomly pulls a crab out of the sack. If it is a female he will fine the fisherman and also take one crab out of the other's sack to check it as well. By considering the possible outcomes, or otherwise,

(i) What is the probability that neither of the fishermen is fined?

(ii) What is the probability that both of the fishermen are fined? [6 marks]

(b) A couple takes a mortgage of \$150 000 to buy a house.

The loan is to be repaid in equal weekly installments over a 15 year period at an annual rate of interest of 6·24%.

Using 1 year = 52 weeks, it is calculated that the weekly repayments would be \$296·25 over the 15 year period. However the couple decides to pay weekly installments of \$320.

(i) How long will it take them to repay the loan?

(ii) How much will they save compared with the 15 year period? [6 marks]

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2UNIT TRIAL July 1999.

1. (a) $\pi^2 = 9.869604401\ldots$

$= 9.9$ (2 sig. figs) (2m)

(b) $(10-2y)-(8y-4)$

$= 14-10y$ (2m)

(c) P(same face) $= 6\times\frac{1}{6}\times\frac{1}{6}$

$= \frac{1}{6}$ (2m)

(d) $225^\circ = 225^\circ \times \frac{\pi}{180^\circ}$

$= \frac{5\pi}{4}$ (1m)

(e) let $\frac{dy}{dx} = \sin 3x$

$\therefore y = -\frac{1}{3}\cos 3x + c$ (2m)

(f) $3^x = \frac{1}{3}$

$\therefore 3^x = 3^{-1}$

$\therefore x = -1$ (1m)

(g) $D = P\left(1-\frac{r}{100}\right)^n$

$\therefore D = 10000\left(1-\frac{15}{100}\right)^2$

$= 7225$

$\therefore$ Holden is worth \$7225 in 2000. (2m)

2. (a) (i) let $y = (3x^5-1)^7$

$\therefore \frac{dy}{dx} = 7(3x^5-1)^6 . 15x^4$

$= 105x^4(3x^5-1)^6$ (2m)

(ii) let $y = \sin^2 5x$

$\therefore y = (\sin 5x)^2$

$\therefore \frac{dy}{dx} = 2\sin 5x . 5\cos 5x$

$= 10\sin 5x\cos 5x$ (2m)

(iii) let $y = \frac{3}{\cos x}$

$\therefore y = 3\sec x$

$\therefore \frac{dy}{dx} = 3\sec x\tan x$ (2m)

(b) $\int \tan^2 x\, dx$

$= \int \sec^2 x - 1\, dx$

$= \tan x - x + c$ (2m)

(c) $\int_0^1 e^{\frac{1}{2}x}\, dx$

$= \left[2e^{\frac{1}{2}x}\right]_0^1$

$= \left[2e^{\frac{1}{2}} - 2e^0\right]$

$= 2\left[e^{\frac{1}{2}} - 1\right]$ (2m)

(d) $\int \frac{10x^2-1}{10x^3-3x}\, dx$

$= \frac{1}{3}\int \frac{30x^2-3}{10x^3-3x}\, dx$

$= \frac{1}{3}\ln|10x^3-3x| + c$ (2m)

3. (i) eq'n of line m is:

$y - 3 = \frac{3}{4}(x-0)$

$\therefore 4y - 12 = 3x$

$\therefore 3x - 4y + 12 = 0$ (2m)

(ii) gradient of line n is $-\frac{4}{3}$

$\therefore$ eq'n of line n is:

$y - 3 = -\frac{4}{3}(x-0)$

$\therefore 3y - 9 = -4x$

$\therefore 4x + 3y - 9 = 0$ (2m)

II
2m

(iii) As m meets x-axis at A sub $y=0$ into m's eq'n.

$\therefore\ 3x+12=0$

$\therefore\ x=-4$

$\Rightarrow$ A is : $(-4,0)$. (1m)

(iv) Similarly sub. $y=0$ into n's eq'n.

$\therefore\ 4x-9=0$

$\therefore\ x=+2\frac{1}{4}$

$\Rightarrow$ B is : $(2\frac{1}{4},0)$. (1m)

(v) Area of Δ ABC $=\frac{1}{2}bh$

$=\frac{1}{2}\times 6\frac{1}{4}\times 3$

$=9\frac{3}{8}$ units^2

(2m)

(vi) As $m \perp n$ the perp. distance from A to line n is the same as the distance between A and C.

$\therefore\ d=\sqrt{(-4-0)^2+(0-3)^2}$

$=\sqrt{16+9}$

$=5$

$\therefore$ perp. distance is 5 units. (2m)

(vii) Test $(0,0)$ in $3x-4y+12>0$

$\therefore\ 12>0$ true

$\therefore$ m's inequality is: $3x-4y+12>0$

Test $(0,0)$ in $4x+3y-9<0$

$\therefore\ -9<0$ true

$\therefore$ n's inequality is $4x+3y-9<0$

$\therefore$ required region is defined by $3x-4y+12>0$ and $4x+3y-9<$

(2m

4 a) $\alpha+\beta = \frac{7}{5}$

$\alpha\beta = \frac{4}{5}$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \frac{9}{25}$

b) $y = \frac{5x^2}{2} - 4x + c$

$\left.\begin{matrix} x=2 \\ y=5 \end{matrix}\right\}$ $C = -13$

c) $x^2 = \frac{1}{4}(y+2)$

VERTEX $(0,-2)$

$a = \frac{1}{16}$

FOCUS $\left(0, -1\frac{15}{16}\right)$

d) $y = \log_e 2x$

$\left.x=3\right\}$ $y = \log_e 6$

$\frac{dy}{dx} = \frac{1}{2x} \times 2$

$= \frac{1}{x}$

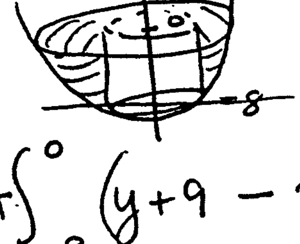
$\therefore$ at $x=3$, $m = \frac{1}{3}$

$y - \log_e 6 = \frac{1}{3}(x-3)$

$x - 3y - 3 + 3\log_e 6 = 0$

5

a)



$V = \pi\int_{-8}^{0} (y+9)\,dy - \pi.1^2.8$

$= \pi\left(\frac{y^2}{2} + 9y\right)_{-8}^{0} - 8\pi$

$= 40\pi - 8\pi$

$= 32\pi\ u^3$

b)

$S_\infty - S_n$

$= \frac{a}{1-r} - \frac{a-ar^n}{1-r}$

$= \frac{ar^n}{1-r}$

(ii) $a = \frac{2}{3}$ $r = \frac{3}{4}$

$S_\infty - S_n = \frac{81}{128}$

c) $\pi \doteqdot 4\int_0^1 \frac{dx}{x^2+1}$

$= 4\left(\frac{\frac{1}{4}}{3}\right)\left[1 + \frac{1}{2} + 4\left(\frac{16}{17} + \frac{16}{25}\right) + 2\left(\frac{4}{5}\right)\right]$

$= 3.1416$ (5 sig figs)

(ii) $\frac{dS}{dr} = 2\pi r - \frac{1024}{r^2}$

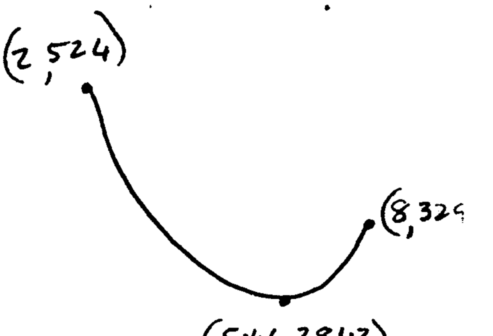
$2\pi r - \frac{1024}{r^2} = 0$

$r = \sqrt[3]{\frac{512}{\pi}} = \frac{8}{\sqrt[3]{\pi}} \doteqdot 5.46$

Then $S \doteqdot 281.2$

When $r = 2$ $S \doteqdot 524$

$r = 8$ $S \doteqdot 329$



(iv) $r = 2$.

6(a) $\log(x^2-x) = \log 2(3x+4)$

$x^2 - x = 6x + 8$

$x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$

$x = 8, -1$

b) $V = 512 = \pi r^2 h$

$h = \frac{512}{\pi r^2}$

Area of base $= \pi R^2$

Curved area $= 2\pi R h$

$= 2\pi r \frac{512}{\pi r^2}$

$= \frac{1024}{r}$

$\therefore$ TSA $= \pi r^2 + \frac{1024}{r}$

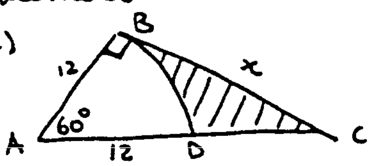
IV

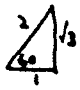
2u

2U Trial Paper

Question 7.

(a)





✓

(i) $\tan 60 = \frac{x}{12}$ non exact

$x = 12 \tan 60$ units

$= 12\sqrt{3}$ cm ✓

(ii) $180^\circ = \pi^c$

$60^\circ = \frac{\pi^c}{3}$ ✓ 1.047

Area A $= \frac{12 . 12\sqrt{3}}{2}$ ✓

$= 72\sqrt{3}$ 124.7

Area sector $= \frac{1}{2} 12^2 . \frac{\pi}{3}$ 2 ✓

$= 24\pi$ 75.4

Shaded area $= 72\sqrt{3} - 24\pi$ ✓

$= 24(3\sqrt{3} - \pi)$ cm^2 49.3

(b) $f(x) = ax^2 + bx + c$

Quadratic => axis of symmetry

$x = -\frac{b}{2a}$ ✓

Min value $y = a\left(-\frac{b}{2a}\right)^2 + b . -\frac{b}{2a} + c$ ✓

So solve for T.P.

$= +\frac{ab^2}{4a^2} - \frac{b^2}{2a} + c$

$= \frac{+b^2 - 2b^2 + 4ac}{4a}$

$= \frac{4ac - b^2}{4a}$ ✓

(c) Real and different

$b^2 > 4ac$ $(-4k)^2 =$

$\Rightarrow (-4k)^2 > 4(k+1)3k$ ✓

$16k^2 > 12k^2 + 12k$ +12

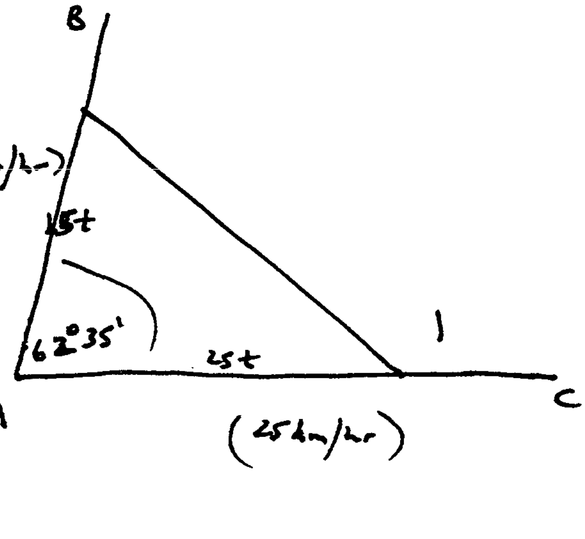
$4k^2 - 12k > 0$

$4k(k-3) > 0$ ✓

$0 > k$ or $k > 3$ ✓ inequality

(b) find $f'(x)$ then not find value of x.

8 (a) Let t hrs be the required time



Dist. travelled along AB $= 15t$ km

" " " AC $= 25t$ km 1

If distance between $= 90$ km

$$8100 = (15t)^2 + (25t)^2 - 2(15t)(25t)\cos 62^\circ 35'$$ 1

$$= t^2\left(225 + 625 - 750\cos 62^\circ 35'\right)$$

$$\therefore t^2 = \frac{8100}{850 - 750\cos 62^\circ 35'}$$ 1

$$= 16.0505$$

$$\therefore t = 4.006$$

i.e. Time = 4 hrs 0 mins 1 (nearest minute)

(b) $y = e^{-x^2}$

$$\frac{dy}{dx} = -2x\,e^{-x^2}$$ 1

$$\frac{d^2y}{dx^2} = -2\left[x.(-2x)e^{-x^2} + e^{-x^2}.1\right]$$

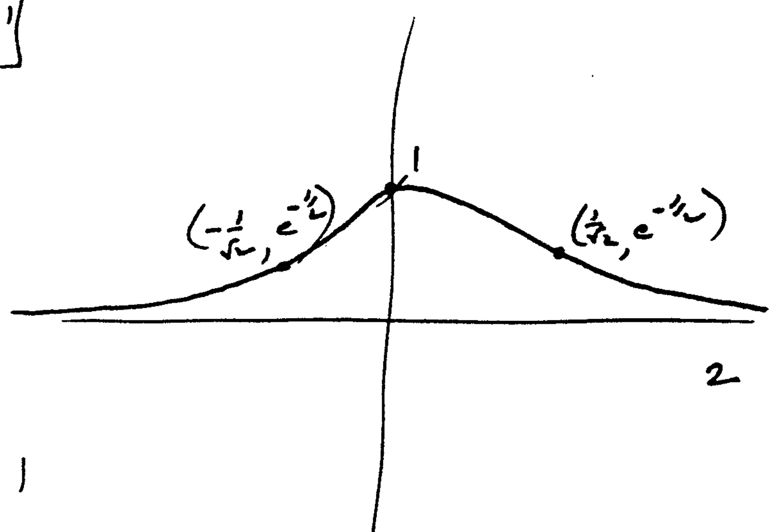
$$= -2\,e^{-x^2}\left(1 - 2x^2\right)$$ 1

Stat pt — one, viz $(0, 1)$ 1

Since $\frac{d^2y}{dx^2} = -2 < 0$

$(0, 1)$ is a maximum 1

Inflexions — two, viz $\left(\frac{1}{\sqrt{2}}, e^{-1/2}\right)$ and $\left(-\frac{1}{\sqrt{2}}, e^{-1/2}\right)$ 1

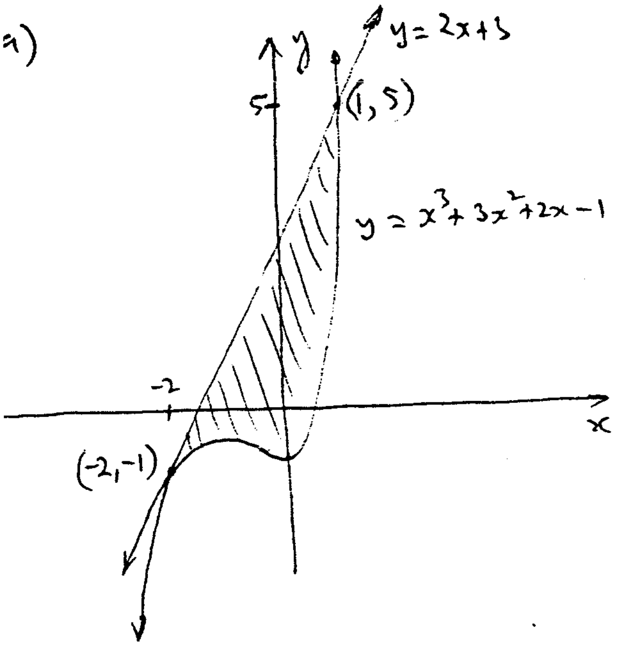


$x \to \pm\infty \quad y \to 0$

$y > 0 \quad \forall x$

1999 2U HSC TRIAL – QUESTION 9.

a)



The Shaded area is given by

$$= \int_{-2}^{1}\left[(2x+3)-(x^3+3x^2+2x-1)\right]dx$$

$$= \int_{-2}^{1}(2x+3-x^3-3x^2-2x+1)\,dx$$

$$= \int_{-2}^{1}(-x^3-3x^2+4)\,dx \quad \boxed{2}$$

$$= \left[\frac{-x^4}{4}-x^3+4x\right]_{-2}^{1} \quad \boxed{1}$$

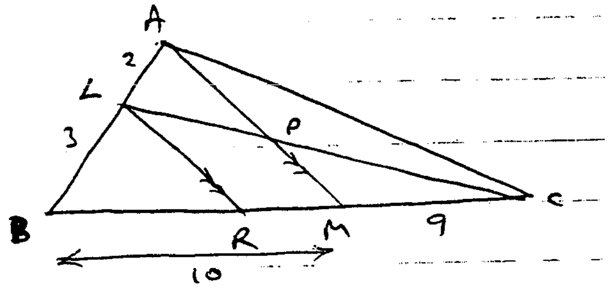
$$= \left(-\frac{1}{4}-1+4\right)-\left(-\frac{(-2)^4}{4}-(-2)^3+4(-2)\right)$$

$$= -\frac{1}{4}-1+4+4-8+8$$

$$= 6\tfrac{3}{4}$$

∴ the area is $6\frac{3}{4}$ units² $\boxed{1}$

(b)



(i) RTP $\triangle BLR \,|||\, \triangle BAM$

Proof: In $\triangle BLR$ & $\triangle BAM$

$\angle LBR = \angle ABM$ (same angle)

$\angle BLR = \angle BAM$, $\angle BRL = \angle BMA$ (Corresponding angles are equal: $LR \parallel AM$)

∴ $\triangle BLR \,|||\, \triangle BAM$ (Equiangul

$\boxed{2}$ (Deduct errors.)

(ii) $BR : RM = BL : LA = 3:2$ $\boxed{1}$

(A line (LR) parallel to one side (AM) o a triangle cuts of intercepts on the other sides in the same ratio) $\boxed{1}$

(iii) $BR : RM = 3:2$

∴ $BR : BM = 3:5$ but $BM = 10$

∴ $\frac{BR}{10} = \frac{3}{5}$

∴ $BR = 6$

∴ $RM = 4$

and so $CM : CR = 9:13$. $\boxed{2}$

Now in $\triangle CPM$ and CLR.

$\angle PCM = \angle LCR$ (same angle)

$\angle CPM = \angle CLR$, $\angle CMP = \angle CRL$ (Corresponding angles are equal: $PM \parallel LR$) $\boxed{1}$

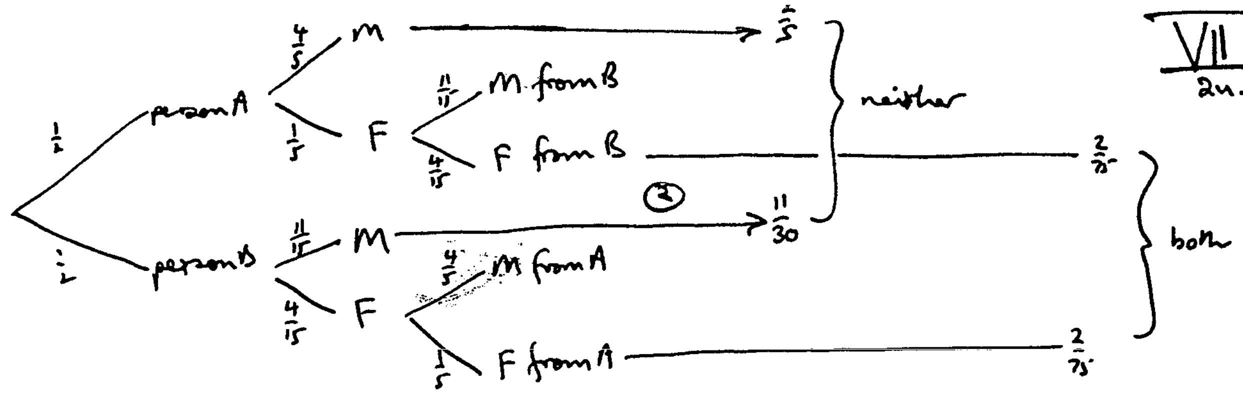
∴ $\triangle CPM \,|||\, \triangle CLR$.

∴ $MP : RL = CM : MR$ (Corresponding s of similar tri are in the same

$= 9:13$. $\boxed{1}$

VII
2u. 2.

(a)



(i) Neither: $\frac{2}{5} + \frac{11}{30} = \frac{23}{30}$ (2)

(ii) Both: $\frac{2}{75} + \frac{2}{75} = \frac{4}{75}$ (2)

(b) Interest $= \frac{6.24}{52} = 0.12\%$ per week

Owing after 1 week $= 150000(1.0012) - 320$ (1)

" " 2 " $= (150000(1.0012)-320)(1.0012) - 320$

$= 150000(1.0012)^2 - 320(1+1.0012)$

" " n weeks $= 150000(1.0012)^n - 320(1+1.0012+\dots+1.0012^{n-1})$ (1)

$= 0$ if repaid

$150000(1.0012)^n = 320 S_n$ (1) ——— (1)

where S_n is sum geom series $a = 1$, $r = 1.0012$

ie $\frac{1(1.0012^n - 1)}{.0012}$ (1)

(1) becomes $150000(1.0012)^n = \frac{320}{.0012}(1.0012^n - 1)$

$.5625 \times 1.0012^n = 1.0012^n - 1$

$(1.0012)^n(1 - .5625) = 1$

$1.0012^n = \frac{1}{0.4375}$

$n \log(1.0012) = \log\left(\frac{1}{0.4375}\right)$

$n = 689.31\ldots$ (1)

$= 690$ weeks (nearest)

Total paid $= n \times 320 = \$220579.86$

$= \$220580$ near enough

Original total $= 15 \times 52 \times \$296.25 = 231075$

$\therefore$ Savings $= \$10495$ (1)