Centre Number | 1 | 2 | 5 |

Student Number

CRANBROOK SCHOOL

2015

Preliminary Examination

Assessment Task 5

Mathematics Extension 1

Reading time	5 minutes
Writing time	100 minutes
Total Marks	51
Task weighting	50%

General Instructions

- Write using blue or black pen
- A Board-approved calculator may be used
- All relevant working should be shown for each question

Additional Materials Needed

- Multiple Choice Answer Sheet
- 3 writing booklets

Structure & Suggested Time Spent

- **Section I** (Multiple Choice) 5 Marks
  Attempt all questions
  Allow about 10 minutes
- **Section II** (Extended Response) 46 Marks
  Attempt all questions
  Start a new booklet for each question
  Allow about 90 minutes

This paper must not be removed from the examination room

STANDARD INTEGRALS

$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1; x \neq 0, \text{if } n<0$

$\int \frac{1}{x} dx = \ln x,\ x>0$

$\int e^{ax} dx = \frac{1}{a}e^{ax},\ a \neq 0$

$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ a \neq 0$

$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ a \neq 0$

$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,\ a \neq 0$

$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ a \neq 0$

$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$

$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a},\ a>0,\ -a<x<a$

$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x>a>0$

$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE: $\ln x = \log_e x,\ x>0$

Section I

5 marks
Attempt Questions 1 - 5
Allow about 10 minutes for this section
Use the multiple-choice answer sheet for Questions 1-5
Marked by GHW

1 What is the solution to the equation $|x-2| = 2x+1$?

(A) $x = -3$

(B) $x = -\frac{1}{3}$

(C) $x = \frac{1}{3}$

(D) $x = 3$

2 What are the coordinates of the point that divides the interval joining the points $A(2,-4)$ and $B(3,-3)$ externally in the ratio 2:3?

(A) $(0,-6)$

(B) $(1,3.4)$

(C) $(2.4,-3.6)$

(D) $(5,-17)$

3 Given $t = \tan\frac{\theta}{2}$, $\frac{\sin\theta}{\cos\theta+2}$ can be expressed as:

(A) $\frac{2t}{1+t^2}$

(B) $\frac{2t}{3+t^2}$

(C) $\frac{2t}{1-t^2}$

(D) $\tan\theta+2$

4 A parabola has the parametric equations $x = 6t$ and $y = 3t^2$.
What is the gradient of the tangent to the parabola at $t = 3$?

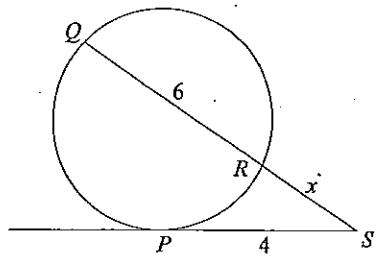
(A) $m = 3$

(B) $m = 3t$

(C) $m = \frac{3}{2}$

(D) $m = \frac{-3}{2}$

5 Line SP is a tangent to the circle at P and SQ is a secant meeting the circle at Q and R.



Given that $SP = 4$, $QR = 6$ and $SR = x$, what is the value of x?

(A) -8

(B) 2

(C) 3

(D) 4

End of Section I

Section II

46 marks
Attempt Questions 6 – 8
Allow about 90 minutes for this section

Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (16 marks) **START NEW BOOKLET** **Marks**

Marked by JJA

(a) The point P divides the interval AB joining $A(-6,2)$ and $B(2,10)$ internally in the ratio 3:5. What are the coordinates of P? **2**

(b) Solve the inequality $\frac{2x+3}{x-2} \leq 1$ **3**

(c) The polynomial $P(x) = x^3 - 6x^2 + 4x + 2$ has roots α, β and γ.

(i) Find the value of $\alpha + \beta + \gamma$. **1**

(ii) Find the value of $\alpha\beta\gamma$. **1**

(iii) Find the value of $(\alpha - 3)(\beta - 3)(\gamma - 3)$. **1**

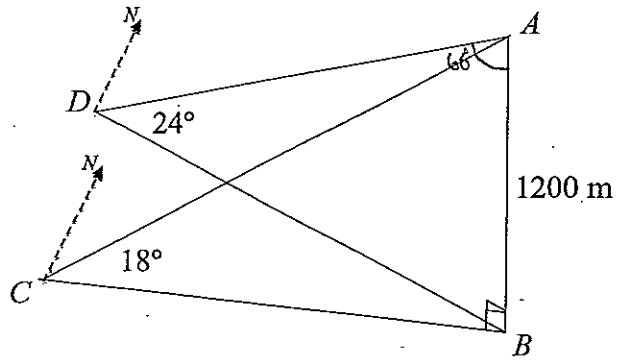
(d) Find the acute angle between the lines $y = 2x - 1$ and $x - 3y + 3 = 0$. **2**
Answer to the nearest degree.

Question 6 continues on next page

(e) Solve the following equation for θ for $0° \leq \theta \leq 360°$, by letting $t = \tan\frac{\theta}{2}$: **3**

$$\sin\theta + \cos\theta = -1$$

(f) A mountain AB is 1200 metres above sea level. Daniel is on a boat with a bearing of 140° to the mountain and angle of elevation to the top of the mountain of 24°. Chloe is on a boat with a bearing of 110° to the mountain and angle of elevation to the top of the mountain of 18°.



What is the distance between Daniel and Chloe? **3**
Answer to the nearest metre.

End of Question 6

Question 7 (15 marks) **START NEW BOOKLET** **Marks**

Marked by HRK

(a) A polynomial $P(x)=x^3+ax^2+bx+6$ is divisible by both $(x+3)$ and $(x+1)$

(i) Find the values of a and b. **2**

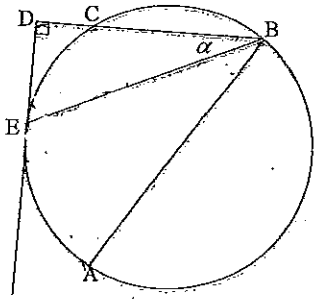
(ii) Write the polynomial $P(x)$ as a product of its factors. **2**

(b) (i) Show that $\dfrac{1+\cos 2x}{\sin 2x}=\cot x$ **2**

(ii) Hence show the exact value of $\cot 15^\circ$ is $2+\sqrt{3}$. **1**

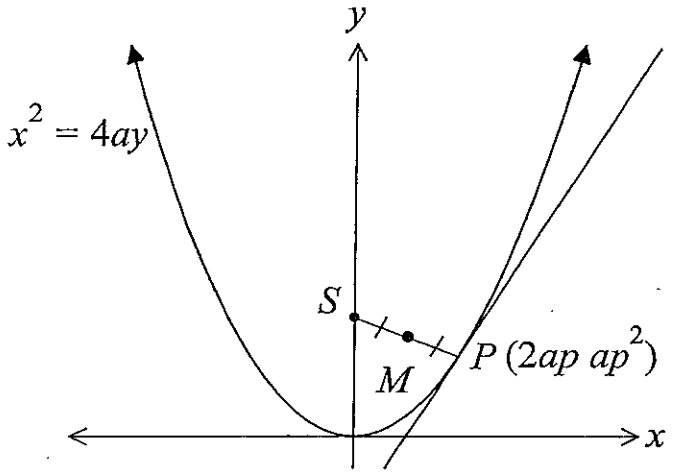
(c) AB is a diameter and BC is a chord of a circle. A tangent at E meets BC produced to D such that DE is perpendicular to BD. **3**

By letting $\angle CBE=\alpha$, prove that BE bisects $\angle ABC$.



Question 7 continues on next page.

(d) $P(2ap,ap^2)$ is a variable point on the parabola $x^2=4ay$ with focus $S(0,a)$. M is the midpoint of SP.



(i) Show that the tangent to the parabola at P has equation $y=px-ap^2$. **2**

(ii) Where does the tangent to the parabola at P cut the x-axis? **1**

(iii) Find M, the midpoint of the interval SP. **1**

(iv) Find the cartesian equation of the locus of M. **1**

End of Question 7

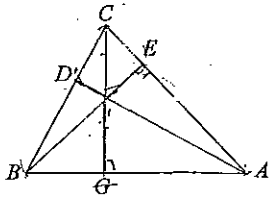
Question 8 (15 marks) **START NEW BOOKLET** **Marks**

Marked by RDS

(a) (i) Express $2\sin x - \cos x$ in the form $A\sin(x-\alpha)$ for $0^\circ \le \alpha \le 90^\circ$. Leave α to the nearest minute. **2**

(ii) Hence solve the equation $2\sin x - \cos x = 1$ for $0^\circ \le x \le 360^\circ$. **2**

(b) In ΔABC, AD and BE are perpendicular to CB and AC respectively. The lines AD and BE intersect at F and CF is produced to meet AB at G.



(i) Why is $ABDE$ a cyclic quadrilateral? Give a reason. **1**

(ii) Why is $EFDC$ a cyclic quadrilateral? Give a reason. **1**

(iii) Prove that $\angle FCD = \angle DAB$. **1**

(c) The parabolas $y = x^2$ and $y = (x-4)^2$ intersect at the point A.

(i) Find the coordinates of A. **1**

(ii) Find the size of the obtuse angle between the tangents to curves at A? Answer to the nearest minute. **2**

(d) Consider the equation: $y = \dfrac{x}{2x^2 - x - 15}$

(i) Find the coordinates of the *y-intercept.* **1**

(ii) Find the equation of any vertical asymptotes. **1**

(iii) Find the $\lim_{x \to \infty}$ **1**

(iv) Hence or otherwise, sketch the curve $y = \dfrac{x}{2x^2 - x - 15} + 1$ **2**

End of paper

Multiple Choice

1. $|x-2| = 2x+1$

$x-2 = 2x+1$

$x = -3$

$|-3-2| = 2(-3)+1$

$5 \neq -5$ ∴ not soln.

$-x+2 = 2x+1$

$3x = 1$

$x = \frac{1}{3}$

$|\frac{1}{3}-2| = 2(\frac{1}{3})+1$

$\frac{5}{3} = \frac{5}{3}$ ∴ soln.

∴ $x = \frac{1}{3}$ (C)

2. $(2,-4)$ $(3,-3)$

-2×3

$x = 3\times2 + -2\times3$, $y = 3\times-4 + 3\times2$

$(0,-6)$ (A)

3. $\frac{\sin\theta}{\cos\theta+2} = \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}+2}$

$= \frac{2t}{1-t^2+2+2t^2}$

$= \frac{2t}{t^2+3}$ (B)

4. $x = 6t$ $y = 3t^2$

$t = \frac{x}{6}$ $y = 3(\frac{x}{6})^2$

$= \frac{3x^2}{36}$

$= \frac{x^2}{12}$

$\frac{dy}{dx} = \frac{2x}{12}$

$= \frac{x}{6}$

@ $t=3$, $x = 6\times3 = 18$

∴ $y' = \frac{18}{6} = 3$. (A)

5. $x(x+6) = 16$

$x^2+6x = 16$

$x^2+6x-16 = 0$.

$(x+8)(x-2) = 0$.

as $x>0$

$x = 2$ (B)

Section 2

Question 5

a) $(-6,2)$ $(2,10)$

3:5

$x = \frac{5\times-6+3\times2}{8}$ $y = \frac{5\times2+3\times10}{8}$ ✓

$P(-3,5)$ ✓

need to learn correct formula.

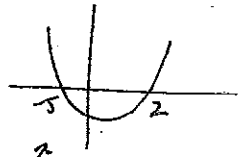
b) $\frac{2x+3}{x-2} \leq 1$ $x \neq 2$

$(x-2)(2x+3) \leq (x-2)^2$ ✓

$(x-2)(2x+3) - (x-2)^2 \leq 0$

$(x-2)(2x+3-(x-2)) \leq 0$ ← many left out brackets so had wrong signs.

$(x-2)(x+5) \leq 0$ ✓



Some drew incorrect parabola for their info so got inequality wrong

∴ $-5 \leq x < 2$ ✓

must remember $x \neq 2$

c) i) $\alpha+\beta+\gamma = -\frac{b}{a} = 6$ ✓

ii) $\alpha\beta\gamma = -\frac{d}{a} = -2$ ✓

need to learn correct formula

iii) $(\alpha-3)(\beta-3)(\gamma-3) = \alpha\gamma\beta - 3\alpha\beta - 3\alpha\gamma - 3\beta\gamma + 9\gamma + 9\alpha + 9\beta - 27$

$= \alpha\beta\gamma - 3(\gamma\alpha+\beta\gamma+\alpha\beta) + 9(\alpha+\beta+\gamma) - 27$

many couldn't do the algebra.

$= 2\times2 - 3\times4 + 9\times6 - 27$

$= 13$ ✓

d) $m_1 = 2 \quad m_2 = \frac{1}{3}$ ✓

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

need to learn correct formula

$$= \left|\frac{2 - \frac{1}{3}}{1 + 2 \times \frac{1}{3}}\right|$$

$$= 45^\circ \quad ✓$$

e) $\sin\theta + \cos\theta = -1$

$$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = -1$$

$$2t + 1 - t^2 = -1 - t^2$$

$$2t = -2$$

$$t = -1 \quad ✓$$

$$\tan\frac{\theta}{2} = -1 \qquad 0^\circ \leq \frac{\theta}{2} \leq 180^\circ$$

need to adjust the domain

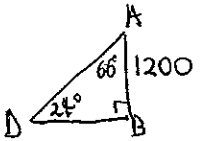
$$\frac{\theta}{2} = 135^\circ$$

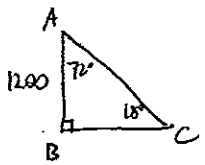
$$\theta = 270^\circ \quad ✓$$

check 180° $\quad \sin(180^\circ) + \cos(180^\circ) = -1$

$\theta = 180^\circ$ ✓

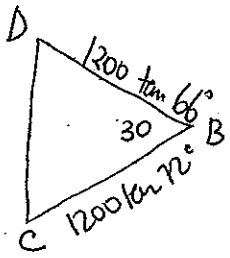
f)





$$\tan 66^\circ = \frac{DB}{1200} \qquad \tan 72^\circ = \frac{BC}{1200}$$

$$DB = 1200\tan 66^\circ \qquad BC = 1200\tan 72^\circ \quad ✓$$



$$DC^2 = (1200\tan 66)^2 + (1200\tan 72)^2 - 2\times(1200\tan 66)(1200\tan 72^\circ)\times\cos 30^\circ \quad ✓$$

$$= 3663157.471$$

$$\therefore DC = 1914\text{m} \ (0\text{dp}) \quad ✓$$

this mark is for 30°.

question asks to nearest metre. Students need to read requirements - not penalised!

(a)

(i) $P(x) = x^3 + ax^2 + bx + 6$

$P(3) = -27 + 9a - 3b + 6 = 0$

$9a - 3b = 21$

$3a - b = 7$ ①

Be Careful with signs!

$-1 + a - b + 6 = 0$

$a - b = -5$ ② ✓

① − ②

$2a = 12$

$a = 6$

$b = 11$ ✓

$P(x) = x^3 + 6x^2 + 11x + 6$

$= (x+3)(x^2+3x+2)$ ✓✓ Easiest by inspection

$= (x+3)(x+1)(x+2)$

b) i)

$LHS = \dfrac{1+\cos 2x}{\sin 2x}$ ✓

$= \dfrac{1 + 2\cos^2 x - 1}{2\sin x \cos x}$ ✓

$= \cot x$ ✓

RTQ HENCE MEANS MUST USES (i)

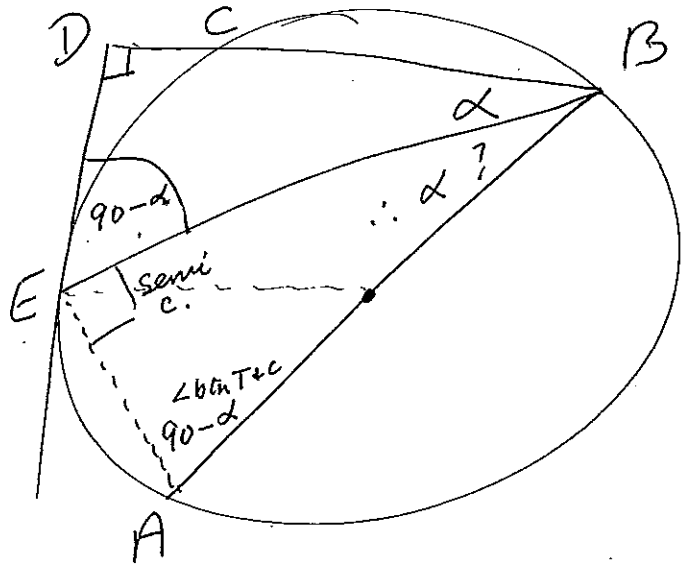
ii) $\cot 15^\circ = \dfrac{1+\cos 30^\circ}{\sin 30^\circ}$

$= \dfrac{1 + \frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= 2\left(\dfrac{2+\sqrt{3}}{2}\right)$

$= 2 + \sqrt{3}$

(c)



JOIN (Construct) EA

$\angle DEB = 90 - \alpha$ ✓ ($\angle$ sum of $\triangle DEB$ is 180°)

✓ $\angle EAB = 90 - \alpha$ ($\angle$ btn tangent + chord = $\angle$ in alternate segment)

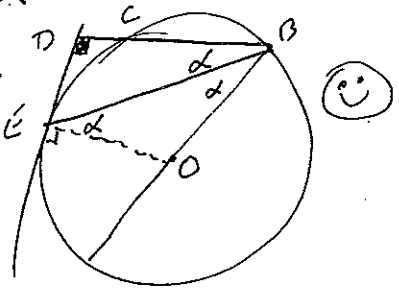
✓ $\angle AEB = 90^\circ$ ($\angle$ in a semi-circle is a rt $\angle$)

✓ $\angle EBA = \alpha$ ($\angle$ sum of $\triangle EBA = 180^\circ$)

✓ Thus $\angle CBE = \angle EBA = \alpha$

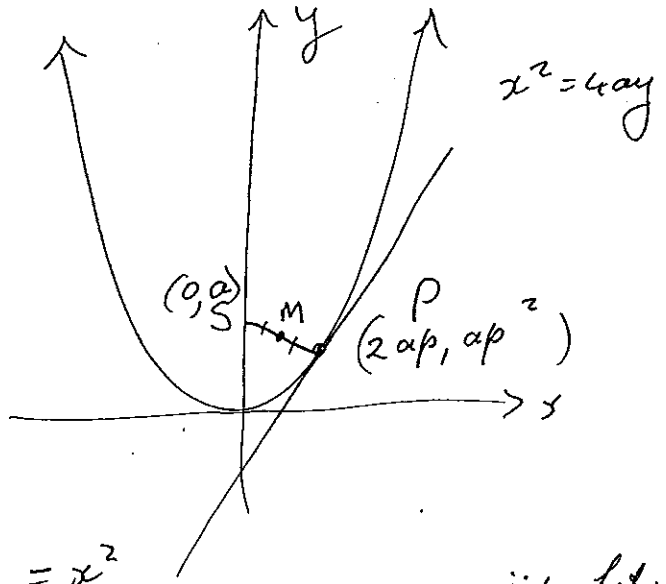
$\therefore$ BE bisects $\angle ABC$.

OR



Always begin with a large clear diagram

d)



(i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

$= \frac{x}{2a}$

at $x = 2ap$

$m_T = \frac{2ap}{2a}$

$= p$ ✓

Eqn of tangent

$y - y_1 = m(x - x_1)$ $\begin{cases} x_1 = 2ap \\ y_1 = ap^2 \\ m = p \end{cases}$

$y - ap^2 = p(x - 2ap)$ ✓

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$

ii) Let $y = 0$

$px = ap^2$

$x = ap$ ✓

(iii) $M = \left(\frac{0 + 2ap}{2}, \frac{a + ap^2}{2}\right)$

$= \left(ap, \frac{a + ap^2}{2}\right)$

iv) $x = ap$ $y = \frac{a + ap^2}{2}$ ✓

$p = \frac{x}{a}$

$y = \frac{1}{2}\left(a + \frac{ax^2}{a^2}\right)$

$y = \frac{x^2}{2a} + \frac{a}{2}$

$= \frac{x^2 + a}{2a}$ ✓½

again care needed with algebraic working

Question 8

a) i) $2\sin x - \cos x \qquad A\sin(x - \alpha)$

$A\sin(x - \alpha) = A\sin x\cos\alpha - A\sin\alpha\cos x$

$\therefore \quad A\cos\alpha = \frac{1}{\text{½}}2$

$A\sin\alpha = 1 \qquad A = \sqrt{5}$

$\tan\alpha = \frac{1}{2}$

$\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

$= 26°34'$ /2

ii) $2\sin x - \cos x = 1$

$\therefore \quad \sqrt{5}\sin\left(x - \left(\tan^{-1}\frac{1}{2}\right)\right) = 1$

$\sin\left(x - \tan^{-1}\left(\frac{1}{2}\right)\right) = \frac{1}{\sqrt{5}}$

$x - \tan^{-1}\left(\frac{1}{2}\right) = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ & $180 - \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$x = 53°8'$ & $180°$ /2

b) i) The chord, AB, subtends equal angles @ D & E. So they must all lie so the same circle. /1

ii) $\angle CEF + \angle CDF = 180°$

∴ This is a cyclic quadrilateral as opposite angles are supplementary. /1

iii) $\angle BAD = \angle BED$ (Angles subtended by the same chord are equal)

$\angle BED (\angle FED) = \angle FCD$ /1

(Angles subtended by the same chord are equal)

∴ $\angle FCD = \angle DAB$.

c) i) $y = x^2$ $\quad y = (x-4)^2$

$x^2 = x^2 - 8x + 16$

$8(2-x) = 0$

$x = 2$ /1

∴ A (2, 4)

ii) $y = x^2$

$\frac{dy}{dx} = 2x$

∴ $m_1 = 4$

$y = (x-4)^2$

$\frac{dy}{dx} = 2(x-4)$

∴ $m_2 = -4$

$\tan\theta = \left| \frac{4+4}{1-16} \right|$

$\theta = 28°4'$ /2

∴ Obtuse $\angle = 151°56'$

d) i) $y = \dfrac{x}{2x^2 - x - 15}$

$x = 0, \quad y = 0$

$\therefore (0,0)$ (y intercept) ✓1

ii) $y = \dfrac{x}{(2x+5)(x-3)}$

$\therefore$ asymptotes $x = -\dfrac{5}{2}, 3$ ✓1

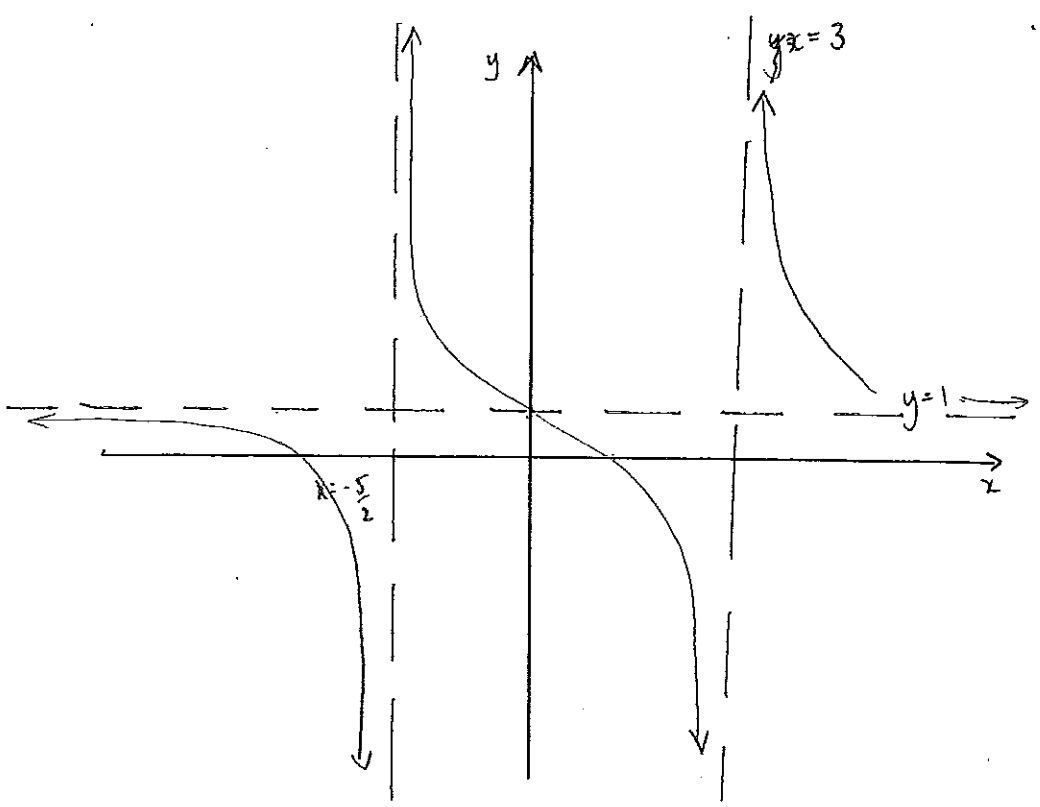
iii) $\lim\limits_{x \to \infty} \dfrac{x}{2x^2 - x - 15}$

$= \lim\limits_{x \to \infty} \dfrac{\frac{x}{x^2}}{\frac{2x^2}{x^2} - \frac{x}{x^2} - \frac{15}{x^2}}$

$= \lim\limits_{x \to \infty} \dfrac{\frac{1}{x}}{2 - \frac{1}{x} - \frac{15}{x^2}}$ As $\lim\limits_{x \to \infty} \frac{1}{x^2} = 0$

$= \dfrac{0}{2 - 0 - 0}$ ✓1

$= 0$



✓2