CRANBROOK
SCHOOL

HSC Trial Examination

Mathematics (2 Unit)

Wednesday July 18, 2007

General Instructions

- Reading time – 5 minutes
- Writing time – 3 hours
- All questions should be attempted
- Total marks available - 120
- All questions are worth 12 marks
- Begin a new booklet for each question
- An approved calculator may be used
- A table of standard integrals can be found on page 13
- All relevant working should be shown for each question

Marks

Question 1 (12 marks) **START A NEW BOOKLET**

(a) Evaluate $\ln 27$ correct to two decimal places. **2**

(b) Solve $|x+4| = 1$ **2**

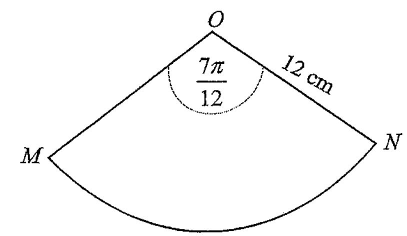
(c) Solve the simultaneous equations **2**

$$\begin{aligned} 3x - y &= -7 \\ 5x + 2y &= 3 \end{aligned}$$

(d) By rationalising the denominator, express $\dfrac{8}{3-\sqrt{5}}$ in the form $a + b\sqrt{5}$ **2**

(e) Find a primitive function of $2 + \dfrac{1}{x^2}$. **2**

(f) **2**



NOT TO SCALE

In the diagram, MN is an arc of a circle with centre O.

The angle MON is $\dfrac{7\pi}{12}$ radians. The radius ON is 12 cm.

Find the area of the sector MON.

Marks

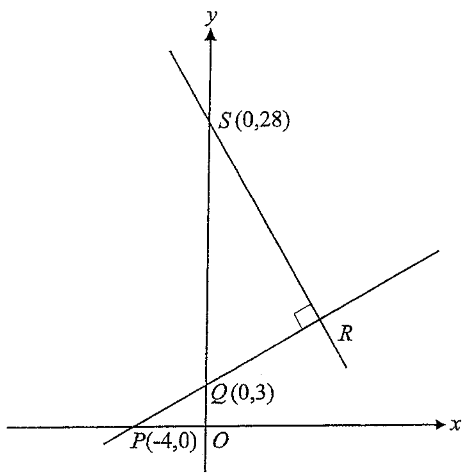
Question 2 (12 marks) **START A NEW BOOKLET**

(a) Find:

(i) $\displaystyle\int \frac{4x}{x^2-5}\,dx$ **2**

(ii) $\displaystyle\int_0^{\pi} \sin 2x\, dx$ **2**

(b)



NOT TO SCALE

The diagram shows the points $P(-4,0)$, $Q(0,3)$ and $S(0,28)$.
PQ intersects RS at right angles at the point R.

(i) Show that the distance PQ is 5 units. **1**

(ii) Show that the gradient of PQ is $\frac{3}{4}$. **1**

(iii) Find the equation of the line PR. **1**

(iv) Hence find the x-coordinate of R given that the y coordinate is 12. **1**

(v) Find the coordinates of the point T so that $OQRT$ is a parallelogram. **2**

(vi) Find the area of $OQRT$. **2**

Marks

Question 3 (12 marks) **START A NEW BOOKLET**

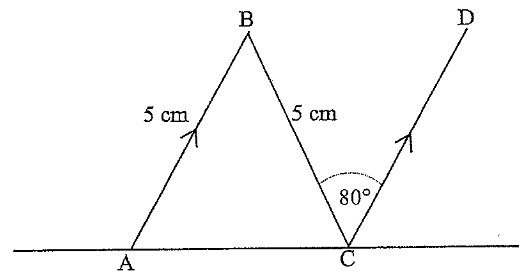
(a) Differentiate the following functions:

(i) $(e^x+2x)^5$ **2**

(ii) $\dfrac{\tan x}{x}$ **2**

(b) If $y = 2x^2\ln(5x)$ then find $\dfrac{d^2y}{dx^2}$. **4**

(c)



NOT TO SCALE

In the diagram, AB is parallel to CD, AB is 5 cm, BC is 5 cm and $\angle BCD$ is 80°.
Copy or trace the diagram onto your answer paper.

(i) Find $\angle BAC$, giving reasons for your answer. **2**

(ii) Hence find the length of AC using the sine rule. Express your answer correct to 2 decimal places. **2**

Marks

Question 4 (12 marks) **START A NEW BOOKLET**

(a) The gradient function of a particular curve is $2x^3 + 7x$. Given that this curve passes through the point $(1, 6)$, find its equation. **2**

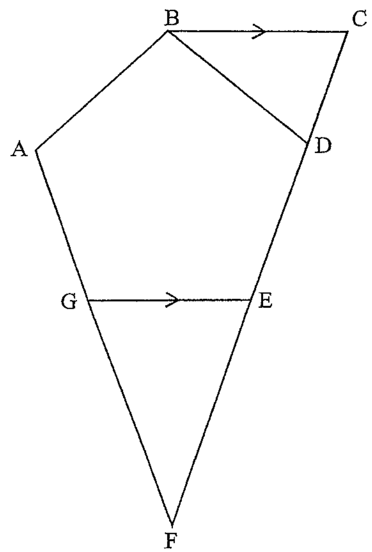
(b) A company produces 3 tonnes of cheese in its first year of production, 7 tonnes in its second year of production and 11 tonnes in its third year of production. The company continues to increase its cheese production by 4 tonnes each year thereafter.

(i) How many tonnes of cheese did the company produce in its 18th year of production? **1**

(ii) In which year of production did the amount of cheese produced for that year, first exceed 100 tonnes? **2**

(iii) What was the total amount of cheese produced by the company in its first 20 years of production? **2**

(c)



NOT TO SCALE

In the diagram, ABDEG is a regular pentagon. The points D and E lie on the line CF and BC is parallel to EG.
Copy or trace the diagram onto your answer paper.

(i) Explain why $\angle DEG = 108°$ **1**

(ii) Find $\angle CBD$. **2**

(iii) Show that ΔBCD is similar to ΔFGE. **2**

Marks

Question 5 (12 marks) **START A NEW BOOKLET**

(a) Calculate the limiting sum of the infinite geometric series given by **2**

$$2 - 1 + \frac{1}{2} \ldots$$

(b)

(i) Sketch the graph of $y = x^2 - 6$ and label all intercepts with the axes. **1**

(ii) On the same set of axes, neatly sketch the graph of $y = |x|$ **1**

(iii) Find the x coordinates of the two points where the graphs intersect. **2**

(iv) Hence solve the inequality $x^2 - 6 \le |x|$ **1**

(c) A particular isotope decomposes such that its mass M kg after t years is given by

$$M = Ae^{-kt}$$

where A and k are positive constants. Initially, there are 3 kg of the radioactive isotope and after 10 years the amount of the radioactive isotope remaining is 2 kg.

(i) Find the value of A. **1**

(ii) Find the value of k. Express your answer correct to 4 decimal places. **2**

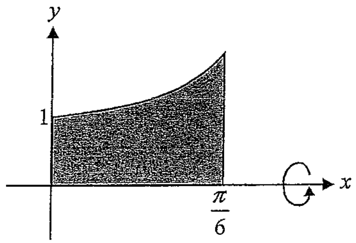
(iii) Find after how many years the amount of the radioactive isotope remaining would be 1 kg. Express your answer to the nearest whole year. **2**

Marks

Question 6 (12 marks) **START A NEW BOOKLET**

(a) Use the trapezoidal rule with three function values to find an approximate value of $\int_2^4 \frac{1}{x+1}\,dx$. **2**

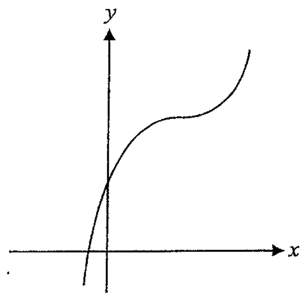
(b)



The diagram shows the graph of $y = \sec 2x$ between $x = 0$ and $x = \frac{\pi}{6}$.

The shaded region is rotated about the x-axis to form a solid of revolution. Find the exact volume of the solid formed. **3**

(c)



The graph of the function $f(x) = x^3 - 3x^2 + 3x + 1$ is shown in the diagram.

(i) Find $f'(x)$. **1**

(ii) Find the coordinates of the stationary point. **2**

(iii) Write down the values of x for which $f'(x) > 0$ **2**

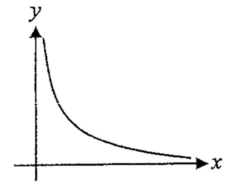
(iv) Without finding $f''(x)$, write down the values of x for which $f''(x) > 0$ **2**

Marks

Question 7 (12 marks) **START A NEW BOOKLET**

(a) Find all the values of x if $\tan x = \frac{1}{\sqrt{3}}$ and $0 \le x \le 2\pi$ **2**

(b)



The diagram shows the graph of $y = f'(x)$ for $x > 0$.

For this graph, we know that $f'(x) = \frac{1}{ax}$ where a is a positive constant,

$f(1) = 1$

and $f(e^4) = 3$.

(i) Find the value of a. **3**

(ii) Hence find $f(x)$. **1**

(iii) Hence or otherwise, sketch the graph of $y = f(x)$. **1**

(iv) Explain why the equation $f'(x) = 0$ has no solutions. **1**

(c) Consider the function given by $y = \sin^2 x$

(i) Copy and complete the following table on your answer paper **2**

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y					

(ii) Apply Simpson's Rule with five function values to find an approximation to $\int_0^{\pi} \sin^2 x\,dx$. Give your answer correct to 2 decimal places. **2**

Marks

Question 8 (12 marks) **START A NEW BOOKLET**

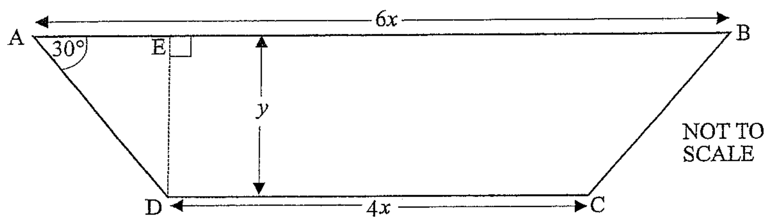
(a) Paula borrows \$15 000 at 4% per quarter reducible interest. She pays the loan off over 5 years by paying quarterly repayments of $\$R$.
Let $\$A_n$ be the amount of money Paula still owes after the n *th* repayment is made.

(i) Write an expression for A_1. **1**

(ii) Show that $A_n = 15000 \times 1.04^n - R(1.04^{n-1} + ... + 1.04 + 1)$ **2**

(iii) Hence find the value of R. **2**

(b)



Four lengths of tape in the shape of a trapezium which has $AD = BC$, are stuck to a stage floor as shown in the diagram. The length of AB is $6x$ metres, the length of CD is $4x$ metres, the length of DE is y metres and $\angle DAE = 30°$.

(i) If the total length of the tape used is 50 metres, show that $5x + 2y = 25$. **2**

(ii) Hence show that the area, A, of the trapezium is **2**

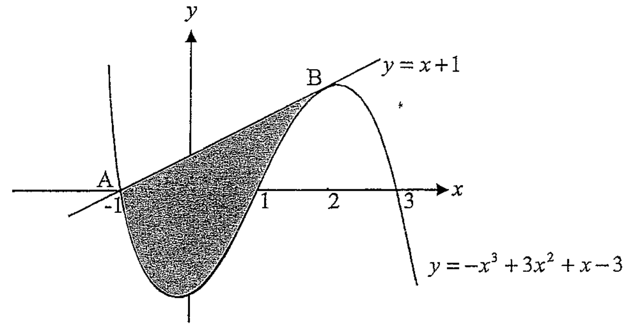
$$A = \frac{125x}{2} - \frac{25x^2}{2}.$$

(iii) Find the value of x for which the area of the trapezium is a maximum. Justify your answer. **3**

Marks

Question 9 (12 marks) **START A NEW BOOKLET**

(a)



The diagram shows the graph of $y = -x^3 + 3x^2 + x - 3$ and its tangent at the point B where $x = 2$. The point $A(-1, 0)$, is a point of intersection of the graph and the tangent.

(i) Use calculus to show that the equation of the tangent at point B is given by $y = x + 1$. **3**

(ii) Calculate the area of the shaded region. **3**

(b) Consider the function $y = 1 - 2\sin 2x$

(i) Find the value of y when $x = 0$. **1**

(ii) Find the first and second derivatives of the function. **2**

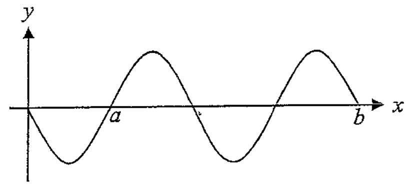
(iii) Find the value of $\frac{d^2y}{dx^2}$ when $x = \frac{\pi}{12}$ **1**

(iv) Sketch the curve for the domain $-\pi \le x \le \pi$ **2**

Marks

Question 10 (12 marks) **START A NEW BOOKLET**

(a)



The graph of $y = 2\cos(2x + \frac{\pi}{2})$ is shown over two complete cycles.

(i) Find the value of b. **1**

(ii) Given that $\int_0^a 2\cos(2x + \frac{\pi}{2})dx = -8$, find $\int_0^b 2\cos(2x + \frac{\pi}{2})dx$ without using calculus. **2**

(iii) One solution to the equation $2\cos(2x + \frac{\pi}{2}) = \sqrt{3}$, is $x = \frac{2\pi}{3}$. Using the graph or otherwise, find the other three solutions to the equation between $x = 0$ and $x = b$. **3**

Marks

(b) At a beach house, the only water supply comes from bore water which has to be pumped into a tank using a hand pump. Each day, the owners of the house, Bill and Gwen, pump for a total of 60 minutes. Gwen always pumps first and then, with no delay in between, Bill takes over until the 60 minutes is up.

The rate at which Gwen can pump the water is $\frac{100}{t+5}$ litres per minute, where t represents the number of minutes after Gwen began pumping.

The rate at which Bill can pump the water is $\frac{-t_B}{6} + 10$ litres per minute, where t_B represents the number of minutes after Bill began pumping.

(i) Explain why, according to this model, the longest time for which Bill could pump is 60 minutes. **1**

(ii) Given that V is the total volume of water pumped by the pair and T represents the time, in minutes, that Gwen pumped for, show that **3**

$$V = 100\ln(\frac{T+5}{5}) + 300 - \frac{T^2}{12}$$

(iii) Hence find how long Gwen should pump for if the volume of water they get into their tank is to be a maximum. Express your answer to the nearest minute. **2**

END OF TRIAL EXAM

Question 1

a) $\ln 27 = 3.2958368\ldots$

$= 3.30$

b) $|x+4| = 1$

$x+4=1$ $\qquad$ $-x-4=1$

$x=-3$ $\qquad$ $-x=5$

$x=-5$

Both verify (no need to show checking)

c) $3x-y=-7$ $\times 2$

$5x+2y=3$

$6x-2y=-14$

$5x+2y=3$ $+$

$11x=-11$

$x=-1$

$\therefore -3-y=-7$

$-y=-4$

$y=4$

d) $\frac{8}{3-\sqrt5} \times \frac{3+\sqrt5}{3+\sqrt5} = \frac{24+8\sqrt5}{9-5}$

$= \frac{24+8\sqrt5}{4}$

$= 6+2\sqrt5$

$\therefore a=6 \quad b=2$

e) $\int 2+\frac{1}{x^2}\,dx = \int 2+x^{-2}\,dx$

$= 2x+\frac{x^{-1}}{-1}+c$

$= 2x-\frac{1}{x}+c$

(do NOT deduct mark if 'c' omitted)

f) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 12^2\times\frac{7\pi}{12}$

$= 42\pi \text{ cm}^2$

$= 131.95$ (to 2d.p)

(give 2nd mark for any sensible rounding. Exact value is best)

Question 2

a) (i) $\int \frac{4x}{x^2-5}\,dx$

$= 2\int\frac{2x}{x^2-5}\,dx$

$= 2\ln(x^2-5)+c$

(ii) $\int_0^{\pi}\sin 2x\,dx$

$= \left[-\frac{1}{2}\cos 2x\right]_0^{\pi}$

$= \left(-\frac{1}{2}\cos 2\pi\right)-\left(-\frac{1}{2}\cos 0\right)$

$= -\frac{1}{2}+\frac{1}{2}$

$= 0$

b) (i) $PQ = \sqrt{(0+4)^2+(3-0)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

(ii) $m_{PQ} = \frac{3-0}{0+4}$

$= \frac{3}{4}$

(iii) $y = mx+b$

$y = \frac{3}{4}x+3$

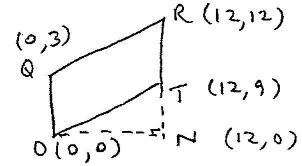
(iv) sub $y=12$

$\therefore 12 = \frac{3}{4}x+3$

$9 = \frac{3}{4}x$

$x = 12$

(v) $R(12,12)$



T is $(12,9)$

(vi) N is $(12,0)$ from diagram

Area OQRT = area OQRN − area OTN

$= \frac{1}{2}\times 12(3+12) - \frac{1}{2}\times 12\times 9$

$= 90-54$

$= 36\text{ u}^2$

Question 3

a) (i) $y' = 5(e^x+2x)^4\cdot(e^x+2)$

(ii) $y' = \frac{x\cdot\sec^2 x - \tan x\cdot 1}{x^2}$

b) $y = 2x^2\ln(5x)$

$y' = 2x^2\cdot\frac{5}{5x}+\ln 5x\cdot 4x$

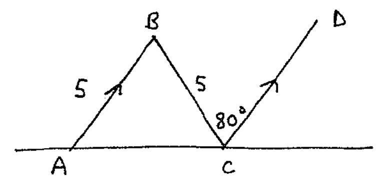
$= 2x+4x\ln 5x$

$y'' = 2+4x\cdot\frac{5}{5x}+\ln 5x\cdot 4$

$= 2+4+4\ln 5x$

$= 6+4\ln 5x$

c)



(i) $\angle ABC = 80°$ (alt ∠'s equal as AB || CD)

ΔABC is isosceles (AB = BC)

$\therefore \angle BAC = \angle BCA$ (base ∠'s equal)

$2x+80=180$ (∠ sum of Δ)

$2x = 100$

$x = 50$

$\therefore \angle BAC = 50°$

(ii) $\frac{AC}{\sin 80°} = \frac{5}{\sin 50°}$

$AC = \frac{5\sin 80°}{\sin 50°}$

$= 6.43$

Question 4

a) $\frac{dy}{dx} = 2x^3+7x$

$y = \frac{1}{2}x^4+\frac{7}{2}x^2+c$

sub $(1,6)$

$6 = \frac{1}{2}+\frac{7}{2}+c$

$2 = c$

$\therefore y = \frac{1}{2}x^4+\frac{7}{2}x^2+2$

b) $3, 7, 11, \ldots$

A.P. $a=3 \quad d=4$

(i) $T_n = a+(n-1)d$

$T_{18} = 3+17\times 4$

$= 71$

$\therefore$ 71 tonnes are produced

(ii) $T_n > 100$

$3 + (n-1)4 > 100$

$3 + 4n - 4 > 100$

$4n > 101$

$n > 25.25$ ✓

$\therefore n = 26$

$\therefore$ In the 26th year ✓

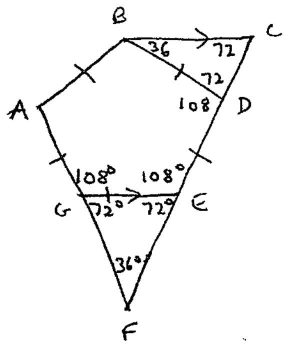
(iii) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = 10(6 + 19 \times 4)$ ✓

$= 820$ ✓

$\therefore$ 820 tonnes were produced

c)



(i) $\angle$ sum pentagon $= 3 \times 180°$

$= 540°$

each $\angle = 540° \div 5$

$= 108°$ ✓

$\therefore \angle DEG = 108°$

(ii) $\angle BCD = 180° - 108°$ ✓

$= 72°$ (cointerior $\angle$'s are suppl. as $BC \parallel GE$)

$\angle EDB = 108°$ (interior $\angle$ of pentagon)

$\therefore \angle BDC = 180° - 108°$

$= 72°$ (straight $\angle$)

$\therefore \angle CBD = 180 - 72 - 72$ ✓

$= 36°$ ($\angle$ sum of Δ is $180°$)

(iii) $\angle AGE = 108°$ (int. $\angle$ of pent.)

$\angle EGF = 72°$ (straight $\angle$)

$\angle GEF = 72°$ (straight $\angle$)

In ΔBCD and ΔGEF

$\angle BCD = \angle EGF = 72°$ ✓

$\angle BDC = \angle GEF = 72°$

$\therefore \Delta BCD \,|||\, \Delta GEF$ equiangular

Question 5

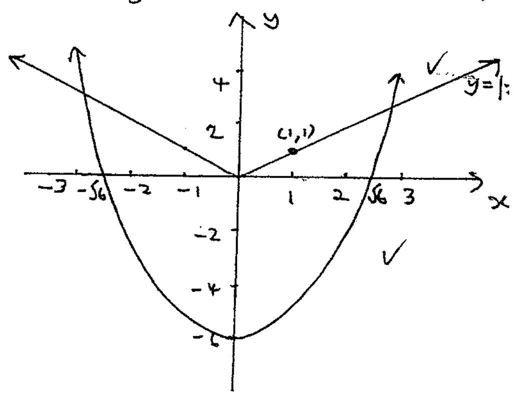
a) $a = 2 \quad r = -\frac{1}{2}$ ✓

$S_\infty = \frac{a}{1-r} = \frac{2}{1+\frac{1}{2}}$

$= 1\frac{1}{3}$ ✓

b) (i) $y = x^2 - 6$

$y = (x - \sqrt{6})(x + \sqrt{6})$



(iii) $x^2 - 6 = |x|$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, \ x = -2$ ✓

From graph curves intersect at $x = \pm 3$. ✓

(iv) $x^2 - 6 \le |x|$

ie parabola is below absolute value graph

$-3 \le x \le 3$ ✓

c) $M = Ae^{-kt}$

(i) $A = 3$ ✓

(ii) $2 = 3e^{-10k}$ ✓

$\frac{2}{3} = e^{-10k}$

$-10k = \ln\frac{2}{3}$

$k = \frac{\ln\frac{2}{3}}{-10}$

$= 0.0405$ ✓

(iii) $1 = 3e^{-0.0405t}$ ✓

$\frac{1}{3} = e^{-0.0405t}$

$-0.0405t = \ln\frac{1}{3}$

$t = 27$ ✓

Question 6

a)

x	2	3	4
y	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
	y_0	y_1	y_n

$h = 1$ ✓

$\int_2^4 \frac{1}{x+1}\,dx \doteqdot \frac{1}{2}(1)\left(\frac{1}{3} + \frac{1}{5} + 2\left(\frac{1}{4}\right)\right)$

$= \frac{31}{60}$ ✓

b) $V = \pi \int y^2\,dx$

$= \pi \int_0^{\pi/6} \sec^2 2x\,dx$ ✓

$= \pi \left[\frac{1}{2}\tan 2x\right]_0^{\pi/6}$ ✓

$= \frac{\pi}{2}\left[\tan\frac{2\pi}{6} - \tan 0\right]$

c) $f(x) = x^3 - 3x^2 + 3x + 1$

(i) $f'(x) = 3x^2 - 6x + 3$ ✓

(ii) stat pts when $f'(x) = 0$

$\therefore x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$ ✓

$y = 2$ ✓

$\therefore$ stat pt is $(1, 2)$

(iii) $f'(x) > 0$

from graph for all x, ✓

except $x = 1$. ✓

(iv) $f''(x) > 0$ ie concave up ✓

$\therefore x > 1$ ✓

Question 7

a) $\tan x = \frac{1}{\sqrt{3}}$ ✓

B.A. $= 30°$

$x = 30°, 210°$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$ ✓✓

b) $f'(x) = \frac{1}{ax}$

$f(x) = \int \frac{1}{ax}\,dx$

$= \frac{1}{a}\int \frac{1}{x}\,dx$

$f(x) = \frac{1}{a}\ln x + c$ ✓

sub $f(1) = 1$

$1 = \frac{1}{a}\ln(1) + c$

$c = 1$ ✓

sub $f(e^4) = 3$

$\therefore 3 = \frac{1}{a} \ln e^4 + 1$

$2 = \frac{1}{a} \ln e^4$

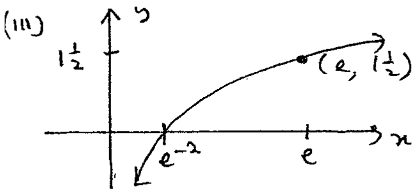
$2 = \frac{4}{a} \ln e$

$2 = \frac{4}{a}$

$a = 2$ ✓

(ii) $f(x) = \frac{1}{2} \ln x + 1$ ✓

(iii)



✓

iv) $f'(x) = 0$ has no solns as there are no stat pts on the graph. ✓

c) (i)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0
	y_0	y_1	y_2	y_3	y_n

$h = \frac{\pi}{4}$ ✓✓

(ii)

$\int_0^{\pi} \sin^2 x \, dx \doteqdot \frac{\frac{\pi}{4}}{3}\left(0 + 0 + 4\left(\frac{1}{2} + \frac{1}{2}\right) + 2(1)\right)$ ✓

$= \frac{\pi}{12}(6)$

$= \frac{\pi}{2}$

$= 1.57$ ✓

Question 8

a) (i) $A_1 = 15000(1.04) - R$ ✓

(ii) $A_2 = \left(15000(1.04) - R\right)1.04 -$

$= 15000(1.04)^2 - 1.04R - R$ ✓

$A_n = 15000(1.04)^n - 1.04^{n-1}R - \dots$

$= 15000(1.04)^n - R\left[1.04^{n-1} + \dots + 1\right]$ ✓

(iii) After 20 repays $0 is owed.

$0 = 15000(1.04)^{20} - R\left(1.04^{19} + \dots\right.$

$a = 1$
$r = 1.04$
$n = 20$

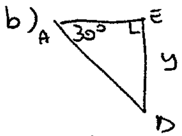
$S_{20} = \frac{1\left(1.04^{20} - 1\right)}{1.04 - 1}$ ✓

$= 29.778\dots$

$R = \frac{15000(1.04)^{20}}{29.778\dots}$

$= \$1103.73$ ✓

b)



$\sin 30° = \frac{y}{AD}$

$AD = \frac{y}{\sin 30°}$

$= \frac{y}{\frac{1}{2}}$

$= 2y$ ✓

Perimeter $= 6x + 2y + 4x + 2y$

$= 10x + 4y$

$\therefore 10x + 4y = 50$

$5x + 2y = 25$ ✓

(ii) $A = \frac{1}{2} y (6x + 4x)$

$= 5xy$ ✓

From (i) $2y = 25 - 5x$

$y = \frac{25 - 5x}{2}$

$\therefore A = 5x\left(\frac{25 - 5x}{2}\right)$

$= \frac{125x}{2} - \frac{25x^2}{2}$ ✓

iii) $A' = \frac{125}{2} - 25x$ ✓

maximum when $A' = 0$ and $A'' < 0$

$\therefore \frac{125}{2} - 25x = 0$

$125 - 50x = 0$

$50x = 125$

$x = 2.5$ ✓

$A'' = -25$

$< 0 \therefore$ max as required ✓

Question 9

a) (i) $y = -x^3 + 3x^2 + x - 3$

$y' = -3x^2 + 6x + 1$

When $x = 2$ $y' = -3(2)^2 + 6(2) + 1$

$= 1$ ✓

$= m_T$

When $x = 2$ $y = -8 + 12 + 2 - 3$ ✓

$= 3$ $\therefore$ pt is $(2, 3)$

$y - y_1 = m(x - x_1)$

$y - 3 = 1(x - 2)$ ✓

$y - 3 = x - 2$

$y = x + 1$

(ii) $A = \int_{-1}^{2} (x+1) - (-x^3 + 3x^2 + x - 3)$

$= \int_{-1}^{2} x^3 - 3x^2 + 4$

$= \left[\frac{1}{4}x^4 - x^3 + 4x\right]_{-1}^{2}$ ✓

$= (4 - 8 + 8) - \left(\frac{1}{4} + 1 - 4\right)$

$= 6\frac{3}{4}$ ✓

b) $y = 1 - 2\sin 2x$

(i) $x = 0$ $y = 1$ ✓

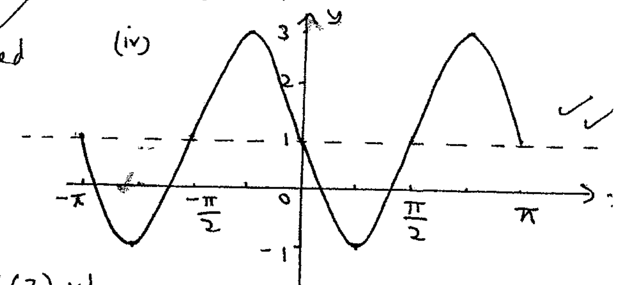
(ii) $y' = -4\cos 2x$ ✓

$y'' = 8\sin 2x$ ✓

(iii) $y'' = 8\sin\frac{2\pi}{12}$

$y'' = 4$ ✓

(iv)



✓✓

Question 10

a) (i) $b = 2\pi$ ✓

(ii) $\int_0^b 2\cos\left(2x + \frac{\pi}{2}\right) dx$

$= -8 + 8 - 8 + 8$ ✓

$= 0$ ✓

(iii) $2\cos\left(2x+\frac{\pi}{2}\right) = \sqrt{3}$

$\cos\left(2x+\frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$ B.A. $= \frac{\pi}{6}$

$2x+\frac{\pi}{2} = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}, \frac{25\pi}{6} \ldots$

$2x = -\frac{\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

$x = -\frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

not in domain

∴ other 3 values are $\frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

b) Bills rate is

i) $\frac{dV}{dt_B} = -\frac{t_B}{6} + 10$

If Bill stops pumping $\frac{dV}{dt_B} = 0$

$\therefore -\frac{t_B}{6} + 10 = 0$

$-t_B + 60 = 0$

$t_B = 60$

∴ longest time Bill pumps for is 60 minutes

ii) Bill starts pumping after Gwen

If Gwen pumps for T minutes, Bill starts pumping after $t - T$

$\therefore V = \int_0^T \frac{100}{t+5} + \int_T^{60} -\frac{(t-T)}{6} + 10$

$= \left[100\ln(t+5)\right]_0^T + \left[-\frac{(t-T)^2}{12} + 10t\right]_T^{60}$

$= 100\ln(T+5) - 100\ln 5 + \left(-\frac{(60-T)^2}{12} + 600\right) - \left(-\frac{(T-T)^2}{12} + 10T\right)$

$= 100\ln\left(\frac{T+5}{5}\right) + \left(-\frac{(3600-120T+T^2)}{12} + 600 - 10T\right)$

$= 100\ln\left(\frac{T+5}{5}\right) + 300 - \frac{T^2}{12}$

(iii) $\frac{dV}{dt} = 100 \times \frac{1}{T+5} - \frac{T}{6}$

max when $\frac{dV}{dt} = 0$ and $\frac{d^2V}{dt^2} < 0$

$\therefore \frac{100}{T+5} - \frac{T}{6} = 0$

$600 - T^2 - 5T = 0$

$T^2 + 5T - 600 = 0$

$T = \frac{-5 \pm \sqrt{25 - 4(1)(-600)}}{2}$

$T = \frac{-5 \pm \sqrt{2425}}{2}$

$T = 22.12, \ -27.12$ ignore

T	22	22.12	23
V'	/	—	\

∴ max as required

∴ Time is 22 minutes

2 UNIT COMMENTS

Q7: (a) Some students were confused with quadrants. Many students did not convert their answers to radians as required by the given domain.

(b) Many candidates performed poorly on this question. Operations with log functions need to be revised.

(c) Generally well done.

Q8: (a) Generally well done by most students. The development of A_2 in some cases was not shown fully which incurred a loss of marks.

(b) Some candidates could not obtain (i) but were able to use the result stated and successfully complete the question. This was a sensible examination technique undertaken.

Q9 (a) (i) Generally done well. Many students did not find coords of B and used A instead. To find B sub $x = 2$ into equation of y to get $(2, 3)$

(ii) Easiest way is to use
Area $= \int f(x) - g(x)$ or $\int$ top curve − bottom curve

(b) All parts done well except for (IV). Many students graphed $2\sin 2x$ instead of $-2\sin 2x$ so their graphs were upside down. Also others forgot to shift the graph 1 unit upwards

Q10(a)(i) Most students did not know to use Period $= \frac{2\pi}{n}$ and that the period is π. But there are 2 cycles, so b is 2π.

(II) Many did not know the difference between finding an area and evaluating an integral. An area will be positive BUT if evaluating an integral a negative stays negative.

(III) Most students did not realise that if domain is $0 \leq x \leq 2\pi$, then for $2x$ D: is $0 \leq 2x \leq 4\pi$ so did not give enough solutions

(b) Most students had NO CLUE f. this question. Answers were fudged for most parts!!

(i) Pumping stops when $\frac{dV}{dt_B} = 0$

(ii) Best way is to integrate Bill's rate and Gwen's rate separately

(III) Many students gave up instead of treating question as MAX/MIN prob and solving $\frac{dV}{dt} = 0$ using the $\frac{dV}{dt}$ given in the question. Also show you have a MAX using F.D.T and round off answer

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Q1. Mostly done well. A few transcription errors and lots of messy writing which caused 5's to become 3's half way through the working.

Q7 Well done.

4

Q3 COMMENTS (HRK)

a) i) WELL DONE
Mark lost if parentheses not maintained
ii) Generally well done but a number did not use their correctly quoted formula accurately!!.

b) Theory known by most but too many careless errors

c) i) Very well done including reasons
ii) V good.

4 a) A number of students tried to find a tangent !!
RTQ (Read the Question)

b) Formulae known & used well. But some interpreted $n > 25\frac{1}{2}$ as in the 25th year rather than the 26th

$\xleftarrow{\quad}$ 25TH YEAR $\xrightarrow{\quad}$ $\xleftarrow{\quad}$ 26TH YEAR $\xrightarrow{\quad}$

24 25 26

c) i) ii) Rules known and correctly applied
iii) NOT well presented
FORMAL PROOF LINKING part (ii) needs attention

Q5 (JSH)
Some students used $\frac{1}{2}$ rather than $r = -\frac{1}{2}$

Although $x^2 - 6 - x = 0$
gives $(x-3)(x+2)$
$x = \underline{3}, -2$
NB

MUST use the graph to answer THIS Question
ie $x = \pm 3$

Q5 c(ii) SOME CONFUSED for some students
$\ln\left(\frac{2}{3}\right)$ and $\frac{\ln 2}{\ln 3}$

LEARN
$\ln\left(\frac{2}{3}\right) = \ln 2 - \ln 3$
BUT
$\frac{\ln 2}{\ln 3} = \ln 2 \div \ln 3$

Q6 b TRIG
Notation issues.
led to mistakes in algebraic manipulation

NOTE
$\sec^2(2x) = (\sec 2x)^2$
$= \sec^2(\underline{2x})$

the angle STAYS AS $\underline{2x}$

again with
$\frac{1}{2}\tan\left(\frac{\pi}{3}\right)$
the angle STAYS as $\frac{\pi}{3}$ $= \frac{\tan\left(\frac{\pi}{3}\right)}{2}$