Centre Number | 1 | 2 | 5 |

Student Number | | | | | | | | |

CRANBROOK SCHOOL

2015

Preliminary Examination

Assessment Task 3

Mathematics

Reading time	5 minutes
Writing time	2 hours
Total Marks	70
Task weighting	50%

General Instructions

- Write using blue or black pen
- Diagrams drawn using pencil
- A Board-approved calculator may be used
- A table of standard integrals can be found on page 10 of this paper
- All relevant working should be shown for each question

Additional Materials Needed

- Multiple Choice Answer Sheet
- 4 writing booklets

Structure & Suggested Time Spent

- **Section I** (Multiple Choice) 10 Marks
  Attempt all questions
  Allow about 15 minutes
- **Section II** (Extended Response) 70 Marks
  Attempt all questions
  Start a new booklet for each question
  Allow about 105 minutes

This paper must not be removed from the examination room

Section I

10 marks – Marked by KDJ
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is the value of $\dfrac{\sqrt{3.84}}{3.65+6.7}$ correct to two decimal places?

(A) 0.19
(B) 0.61
(C) 5.28
(D) 8.44

2 What is the solution to the equation $6x^2 = x+2$?

(A) $x=-\dfrac{2}{3}$ or $x=-\dfrac{1}{2}$

(B) $x=\dfrac{2}{3}$ or $x=-\dfrac{1}{2}$

(C) $x=-\dfrac{2}{3}$ or $x=\dfrac{1}{2}$

(D) $x=\dfrac{2}{3}$ or $x=\dfrac{1}{2}$

3 What is the midpoint of $(-2,5)$ and $(2,-3)$?

(A) $(0,1)$
(B) $(0,4)$
(C) $(2,1)$
(D) $(2,4)$

4 What is the simultaneous solution to the equations $2x-y=-8$ and $3x+2y=-5$?

(A) $x=-3$ and $y=-2$
(B) $x=-3$ and $y=2$
(C) $x=3$ and $y=-2$
(D) $x=3$ and $y=2$

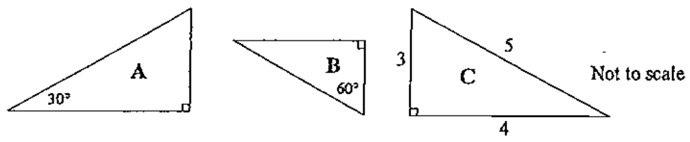
5 Which of the following is true for the function $f(x)=8x^3-7x$?

(A) Even function
(B) Odd function
(C) Neither odd or even
(D) Zero function

6 What is the value of $\lim_{x\to 3}\dfrac{x^2-2x-3}{x-3}$?

(A) Undefined
(B) 0
(C) 1
(D) 4

7



Which of the following statements is correct?

(A) Triangle A is similar to Triangle B
(B) Triangle A is similar to Triangle C
(C) Triangle C is similar to Triangle B
(D) Triangle A, B and C are all similar

8 What is the exact value of $\cos 135^\circ + \operatorname{cosec} 60^\circ$?

(A) $\dfrac{2\sqrt{2}-\sqrt{3}}{\sqrt{6}}$

(B) $\dfrac{2\sqrt{2}-1}{\sqrt{2}}$

(C) $\dfrac{2\sqrt{2}+\sqrt{3}}{\sqrt{6}}$

(D) $\dfrac{2\sqrt{2}+1}{\sqrt{2}}$

9 What values of k does the equation $x^2+(k+3)x+5=0$ have equal roots?

(A) $k=-3\pm\sqrt{5}$
(B) $k=-3\pm 2\sqrt{5}$
(C) $k=3\pm\sqrt{5}$
(D) $k=3\pm 2\sqrt{5}$

10 What is the solution to the equation $\cos\left(\dfrac{\theta}{2}+30^\circ\right)=\sin\theta$ for $0^\circ\le\theta\le 90^\circ$?

(A) $\theta=20^\circ$
(B) $\theta=30^\circ$
(C) $\theta=40^\circ$
(D) $\theta=50^\circ$

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet.

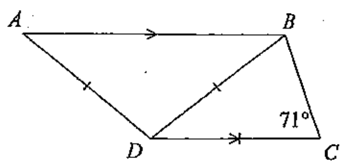
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marked by JJA** **Marks**

(a) Find the value of a and b if $\dfrac{4}{3-\sqrt{7}}=a+b\sqrt{7}$. **2**

(b) Expand and simplify $(5a+2b)(3a-4b)$. **1**

(c) In the diagram below, $AB \parallel CD$, $AD=BD=CD$ and $\angle BCD=71°$.



Not to scale

(i) Find the size of $\angle BDC$. Give reasons. **1**

(ii) Find the size of $\angle ADC$. Give reasons. **2**

(d) Factorise completely $x^2y-y-z+x^2z$. **2**

Question 11 continues on the next page

(e) Solve $|4-3x|<7$ **2**

(f) Solve

$$\frac{7x}{5}-\frac{x}{2}=9$$ **1**

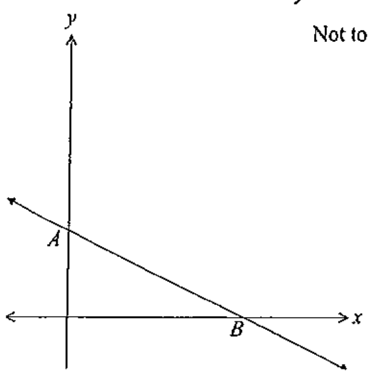
(g) State the domain and range of the function $f(x)=\sqrt{4-x^2}$. **2**

(h) Find the equation of the locus of a point that moves so that its distance from $A(2,2)$ is always the same as its distance from $B(-3,5)$. **2**

End of Question 11

Question 12 (15 marks) **Marked by RABS** **Marks**

(a) The line $x+2y-4=0$ cuts the x-axis at B and the y-axis at A.



(i) What are the coordinates of A and B? **2**

(ii) Find the exact perpendicular distance from $P(2,6)$ to $x+2y-4=0$. **1**

(iii) Find the gradient of AP. **1**

(iv) Hence or otherwise find the equation of AP. **1**

(v) What is the exact distance from A to B? **1**

(vi) Calculate the exact area of ΔAPB. **1**

(b) Prove $(\sec\theta-\cos\theta)^2=\tan^2\theta-\sin^2\theta$ **2**

(c) Draw neat sketches of the following equations on a separate set of axes.

(i) $(x-1)^2+y^2=36$ showing the centre and x and y intercepts. **2**

(ii) $y=x^2+2x+3$ showing the vertex and y intercept. **1**

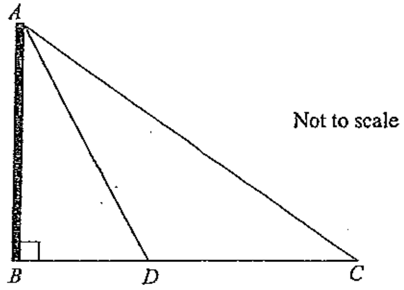
(iii) $y=|x+3|$ showing the x and y intercepts. **1**

(d) Find the value of $f'(-1)$ if $f(x)=4x^2-7x+5$. **2**

End of Question 12

Question 13 (15 marks) **Marked by KDJ** **Marks**

(a) A vertical tower AB with points B, C and D in a straight line on the ground is shown below. The distance from CD is 100 metres. The angle of elevation to the top of the tower from point C is 35° and from point D is 60°.



(i) Calculate $\angle CAD$ **1**

(ii) Hence show that $AD=\dfrac{100\sin 35^\circ}{\sin 25^\circ}$. **1**

(iii) Calculate the height of the tower. Answer to the nearest metre. **2**

(b) A parabola has the equation $x^2-6x-8y-7=0$.

(i) Find the vertex of the parabola. **2**

(ii) Find the focus of the parabola. **1**

(iii) What is the equation of the directrix of the parabola? **1**

(c) Differentiate with respect to x.

(i) $5-7x+13x^4$ **1**

(ii) $\dfrac{2x-3}{3x-2}$ simplifying your answer. **2**

Question 13 continues on the next page

(d) Find the value of x if $2^x \times 4^{x+1} = 0.5$ **2**

(e) Differentiate from first principles $f(x) = x^2 - 5x$ **2**

End of Question 13

Question 14 (15 marks) **Marked by HRK** **Marks**

(a) If α and β are roots of the quadratic equation $2x^2 - 7x + 8 = 0$, find

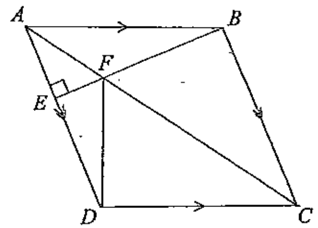
(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ **1**

(b) Find the equation of the normal to the curve $y = x^2 - 5x + 6$ at $(1, 2)$. **3**

(c) $ABCD$ is a rhombus, BE is perpendicular to AD and intersects AC at F.



Not to scale

(i) Explain why $\angle BCA = \angle DCA$. **1**

(ii) Prove that $\Delta BFC \equiv \Delta DFC$. **2**

(iii) Show that $\angle CBF$ is a right angle. **1**

(iv) Hence or otherwise find the size of $\angle FDC$ **1**

(d) Solve $\cot^2\theta = \operatorname{cosec}\theta + 1$ for $-180° < \theta \leq 180°$ **2**

(e) A line $y = mx$ is a tangent to the parabola $y = x^2 + 2$ so that their point of contact is in the first quadrant. **2**
Find the exact value of m.

End of paper

Multiple Choice

1. $\dfrac{\sqrt{3.84}}{3.65 + 6.7} = 0.19$ A

2. $6x^2 = x + 2$

$6x^2 - x - 2 = 0$

$(3x - 2)(2x + 1) = 0$

$3x - 2 = 0 \quad x = 2/3$

$2x + 1 = 0 \quad x = -1/2$ B

3 $\left(\dfrac{-2+2}{2}, \dfrac{5+-3}{2}\right) = (0, 1)$ A

4 $2x - y = -8 \xrightarrow{\times 2} 4x - 2y = -16$

$3x + 2y = -5 \qquad 3x + 2y = -5$

$7x = -21$

$x = -3$

$\therefore -6 - y = -8$

$2 = y$ B

5 $f(x) = 8x^3 - 7x$

$f(-x) = 8(-x)^3 - 7(-x)$

$= -8x^3 + 7x$

$= -(8x^3 - 7x)$

$= -f(x) \therefore$ odd B

6. $\lim\limits_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$

$= \lim\limits_{x \to 3} \dfrac{(x-3)(x+1)}{x-3}$

$= \lim\limits_{x \to 3} x + 1$

$= 3 + 1 = 4$ D.

7. A: angle 30, 60, 90

B: " " " "

C: none of the above

$\therefore$ A + B similar (AAA) A

8. $\cos 135^\circ + \operatorname{cosec} 60$

$= -\cos 45 + \dfrac{1}{\sin 60}$

$= -\dfrac{1}{\sqrt{2}} + \dfrac{2}{\sqrt{3}}$

$= \dfrac{2\sqrt{2} - \sqrt{3}}{\sqrt{6}}$ A

9. $x^2 + (k+3)x + 5 = 0$

$b^2 - 4ac = 0$

$(k+3)^2 - 4 \times 1 \times 5 = 0$

$k^2 + 6k + 9 - 20 = 0$

$k^2 + 6k - 11 = 0$

$k = \dfrac{-6 \pm \sqrt{36 - 4 \times 1 \times -11}}{2}$

$= \dfrac{-6 \pm \sqrt{80}}{2}$

$= \dfrac{-6 \pm 4\sqrt{5}}{2}$

$= -3 \pm 2\sqrt{5}$ B

10. $\cos\left(\dfrac{\theta}{2} + 30^\circ\right) = \sin\theta$

$\cos\left(\dfrac{\theta}{2} + 30\right) = \cos(90 - \theta)$

$\therefore \dfrac{\theta}{2} + 30 = 90 - \theta$

$\dfrac{3}{2}\theta = 60 \quad \therefore \theta = 40^\circ$ C.

11a $\frac{4}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$

$\frac{12+4\sqrt{7}}{9-7}$

① Correctly multiplying through by conjugate

$\frac{12+4\sqrt{7}}{2}$

② Correctly solving for a and b.

$a = 6$

$b = 2$

) $(5a+2b)(3a-4b)$

$15a^2 + 6ab - 20ab - 8b^2$

$= 15a^2 - 14ab - 8b^2$ ✓

c) $\angle DBC = 71°$ (Base angles isosceles Δ are equal

$\angle BDC = 180 - 2(71°)$ (Angles in a Δ add to 1

$= 38°$

ii) $\angle ABC = 180 - 71°$ (Cointerior angles on parallel lines are ~~equal~~ supplementary

$= 109°$

$\angle DAB = \angle ABD = 109° - 71°$ (Base angles isosceles Δ are equal)

$= 38°$

$\therefore \angle ADB = 180 - 2(38°)$ (Angles in Δ add to 180°)

$= 104°$

$\therefore \angle ADC = 104 + 38$

$= 142°$

To receive full marks, all reasons must be given. Students should write words in full ie alternate, cointerior, parallel. Many students found $\angle ADB$ and didn't add on $\angle BDC$ for $\angle ADC$.

d) $x^2y - y - z + x^2z$

$y(x^2-1) + z(x^2-1)$

$(y+z)(x^2-1)$ 1 Mark

$(y+z)(x+1)(x-1)$ 2 Marks

e) $|4-3x| < 7$ $\qquad -4+3x < 7$

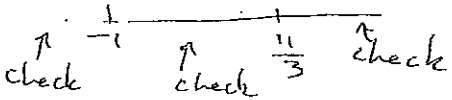
$4-3x < 7$ $\qquad 3x < 11$

$-3x < 3$ $\qquad x < \frac{11}{3}$ ✓

$x > -1$ ✓

Best answers $-1 < x < \frac{11}{3}$

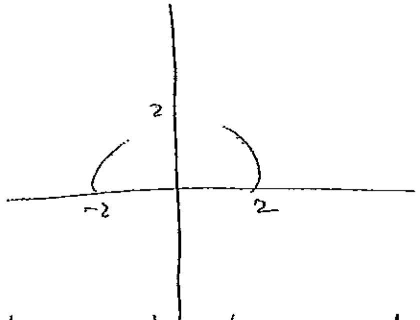
Note: You doa not need to check this. If you are going to check should use a numberline and check points in region.



g) $f(x) = \sqrt{4-x^2}$

D: $-2 \le x \le 2$ ✓

R: $0 \le y \le 2$ ✓



* Need to state which is domain, which is range.

* 1 Mark for $-4 \le x \le 4$, $0 \le y \le 4$

h) $P(x,y)$ $\quad A(2,2)$ $\quad B(-3,5)$

$PA = PB$

$\sqrt{(x-2)^2+(y-2)^2} = \sqrt{(x+3)^2+(y-5)^2}$

$x^2-4x+4+y^2-4y+4 = x^2+6x+9+y^2-10y+25$

$-10x+6y-26=0$

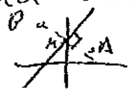
$-5x+3y-13=0$

$5x-3y+13=0$ ✓

1 Mark use of distance formula correctly ie PA = PB

2 Correct equation of line.

Note: Could also find midpoint + gradient ⊥ to AB and find eqn of line this way



QUESTION 12

a.

i. $x + 2y - 4 = 0$

(A): IF $x = 0$, $y = 2$ $(0,2)$ ✓

(2)

(B): IF $y = 0$, $x = 4$ $(4,0)$ ✓

· A WORRYING NO. OF STUDENTS DIDN'T READ THE QUESTION & DIDN'T PROVIDE CO-ORDINATES.

ii. $P(2,6)$ $x + 2y - 4 = 0$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

$$= \frac{|(1\times2) + (2\times6) - 4|}{\sqrt{1^2+2^2}}$$

$$= \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$ ✓ (1)

· PROBLEMS: NOT REMEMBERING FORMULA

iii. $A: (0,2)$ $P: (2,6)$

$$\therefore m_{AP} = \frac{6-2}{2-0} = \frac{4}{2} = 2$$ ✓ (1)

· NO PROBLEMS.

iv. $P(2,6)$ $m = 2$

$$\therefore y - 6 = 2(x - 2)$$
$$y - 6 = 2x - 4$$
$$0 = 2x - y + 2$$ ✓ (1)

OR

$$y = 2x + 2$$

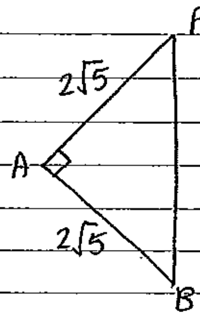
· SOME STUDENTS FAILED TO PUT IN CORRECT GENERAL FORM.

v. $\triangle AOB$ (O = ORIGIN) IS RIGHT-ANGLED
$\therefore$ USE PYTHAG.

$AB^2 = 2^2 + 4^2$ → FROM $(0,2)$ & $(4,0)$
$= 4 + 16$
$AB^2 = 20$
$\therefore AB = \sqrt{20} = 2\sqrt{5}$ ✓ (1)

· AGAIN, NO PROBLEMS.

vi.



USING ii. & v.

$A = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 2\sqrt{5} \times 2\sqrt{5}$
$= \frac{1}{2} \times 20$
$= 10$ UNITS^2 ✓ (1)

· ONLY SOURCES OF ERRORS WERE FROM PREVIOUS WORKING.

b. $(\sec\theta - \cos\theta)^2 = \tan^2\theta - \sin^2\theta$

$$\begin{aligned}
LHS &= \sec^2\theta - 2\sec\theta\cos\theta + \cos^2\theta \\
&= \sec^2\theta - \frac{2\cos\theta}{\cos\theta} + \cos^2\theta \\
&= \sec^2\theta - 2 + \cos^2\theta \quad ✓ \\
&= (\sec^2\theta - 1) + (\cos^2\theta - 1) \quad \cdot \text{BREAK UP } -2 \\
&= \tan^2\theta - \sin^2\theta \quad ✓ \\
&= RHS
\end{aligned}$$

(2)

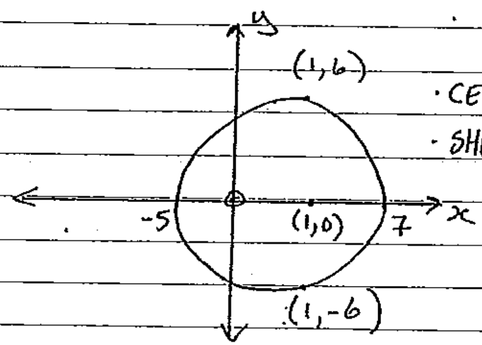
$\boxed{\begin{array}{l} 1 + \tan^2\theta = \sec^2\theta \\ \therefore \sec^2\theta - 1 = \tan^2\theta \end{array}}$

$\boxed{\begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ \cos^2\theta - 1 = \sin^2\theta \end{array}}$

- TOUGH QUESTION FOR MOST
- MANY STRUGGLED WITH TRIG. ALGEBRA
- MANY DID NOT THINK TO USE THE 2 TRIG IDENTITIES
- SOME MADE TOO BIG A "JUMP" IN THEIR WORKING FROM ONE STEP TO ANOTHER.

c.

i.



- CENTRE ✓
- SHOWING RADIUS = 6 ✓

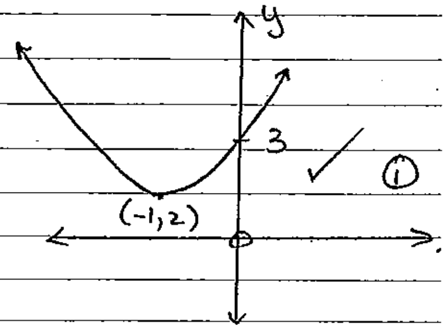
(2)

- THE USUAL PROBLEMS SURFACED... MESSINESS, NOT ENOUGH CRITICAL INFO. ON GRAPH, VERY NON-CIRCULAR SHAPES.

ii. VERTEX : $x = \frac{-b}{2a} = \frac{-2}{2} = -1$

$y = (-1)^2 - 2 + 3$
$= 2$

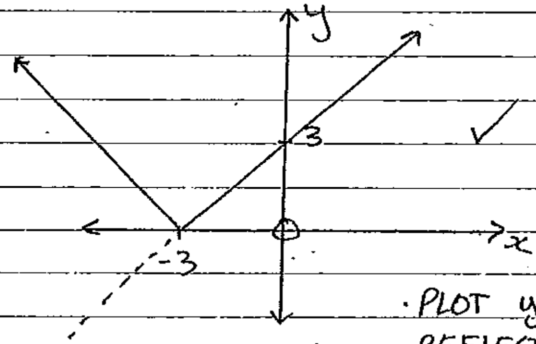
$\therefore$ VERTEX $= (-1, 2)$
y-INT $= 3$.



✓ (1)

- SOME HAD THE WRONG CONCAVITY
- SOME PUT THE VERTEX ON THE y-AXIS.

iii.



✓ (1)

- PLOT $y = x + 3$
- REFLECT THE PART BELOW y-AXIS.

- MAIN PROBLEM WAS NOT RECOGNISING / RECALLING THE SHAPE OF AN ABS. VALUE GRAPH.

d. $f(x) = 4x^2 - 7x + 5$

$f'(x) = 8x - 7$ ✓

$f'(-1) = (8 \times -1) - 7$
$= -15$ ✓

- MANY STUDENTS SUBBED IN $x = 1$ FOR SOME REASON.
- NO PROBS WITH DERIVATIVE.

Question 13

a) i) $\angle CAD = 180 - 120 - 35 = 25°$ ①

ii) $\dfrac{AD}{\sin 35} = \dfrac{100}{\sin 25}$ ①

$\therefore AD = \dfrac{100 \sin 35}{\sin 25}$

iii) $\sin 60 = \dfrac{AB}{AD}$

$\therefore AB = AD \sin 60$ ①

$= \dfrac{100 \sin 35}{\sin 25} \times \sin 60$

$= 117.5$ ①

$= 118M.$

b) $x^2 - 6x - 8y - 7 = 0$

$(x-3)^2 - 9 - 8y - 7 = 0$

$(x-3)^2 = 8y + 16$

$(x-3)^2 = 8(y+2)$ ① $\therefore a = 2$

i) vertex $= (3, -2)$ ①

ii) focus $= (3, 0)$ ①

iii) directrix: $y = -4.$ ①

c) i) $5 - 7x + 13x^4$

$\therefore \dfrac{d}{dx} = -7 + 52x^3$ ①

ii) $\dfrac{2x-3}{3x-2}$

$\therefore \dfrac{d}{dx} = \dfrac{(3x-2)(2) - (2x-3)(3)}{(3x-2)^2}$ ①

$\dfrac{d}{dx} = \dfrac{5}{(3x-2)^2}$ ①

d) $2^x \times 4^{x+1} = 0.5$

$2^x \times 2^{2(x+1)} = 2^{-1}$ ①

$\therefore x + 2(x+1) = -1$

$x + 2x + 2 = -1$

$3x = -3$

$x = -1$ ①

e) $f(x) = x^2 - 5x$

$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{[(x+h)^2 - 5(x+h)] - [x^2 - 5x]}{h}$ ①

$= \lim_{h \to 0} \dfrac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 5h}{h}$$

$$= \lim_{h \to 0} 2x + h - 5$$

$$= 2x - 5 \quad (1)$$

Q14

MARKED BY HRK

✓ = 1 mark

(a) (i) $\alpha + \beta = -\frac{b}{a}$

$= -\frac{-7}{2}$

$= \frac{7}{2}$

(ii) $\alpha\beta = \frac{c}{a}$

$= \frac{8}{2}$

$= 4$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{\frac{7}{2}}{\frac{4}{1}}$

$= \frac{7}{2} \times \frac{1}{4}$

$= \frac{7}{8}$

This question highlights times when your calculator will NOT do your fractions for you !! If fractions are a problem WORK ON THEM NOW

(b) $y = x^2 - 5x + 6$

$\frac{dy}{dx} = 2x - 5$

$\therefore$ at $x = 1$ $M_T = 2(1) - 5$
$= -3$

$\therefore M_N = \frac{1}{3}$

$y - y_1 = m(x - x_1)$ $\begin{cases} x_1 = 1 \\ y_1 = 2 \\ m = \frac{1}{3} \end{cases}$

(RTQ NORMAL) ☺

$y - 2 = \frac{1}{3}(x - 1)$

$3y - 6 = x - 1$

$x - 3y + 5 = 0$

These questions appear often and should be easy marks.

(c) (i) Diagonals of a rhombus bisect the angles through which they pass.

(ii) In $\triangle$'s BFC, DFC

1/ FC is common

2/ $\angle BCF = \angle DCF$ (from (i))

3/ $BC = DC$ (sides of a rhombus are equa

$\therefore \triangle BFC \equiv \triangle DFC$ (SAS test)

(iii) $\angle CBF = \angle AEF = 90°$ (alternate angles $AD \parallel BC$ and given the right angle)

ie $\angle CBF$ is a right angle

(iv) $\angle FDC = \angle FBC = 90°$
(corresponding (matching) angles in congruent $\triangle$

d) $1 + \cot^2\theta = \csc^2\theta$ (PYTH. Identities)

$\therefore \cot^2\theta = \csc^2\theta - 1$ ①

$\cot^2\theta = \csc\theta + 1$ ②

SUBST ① → ② $\csc^2\theta - 1 = \csc\theta + 1$

(a quadratic !) $\csc^2\theta - \csc\theta - 2 = 0$

$(\csc\theta - 2)(\csc\theta + 1) = 0$

$\sin\theta = \frac{1}{2}, -1$

$\theta = 30°, 150°, -90°$

e) $y = mx$ (1)
$y = x^2+2$ (2)

GIVEN it is a tangent solution has 1 answer
ie $\Delta = 0$

(1) → (2)

$mx = x^2+2$

$x^2 - mx + 2 = 0$ ✓

$\Delta = m^2 - 8$

$m^2 - 8 = 0$

$m = \pm\sqrt{8}$ But $m > 0$ since line is in 1st quadrant.

$\therefore m = 2\sqrt{2}$ ✓

OR $y = mx$ has gradient m

$y = x^2+2$ has gradient $2x$

$\therefore m = 2x$ & then

subst this into

$x^2 - mx + 2 = 0$

$x^2 - 2x^2 + 2 = 0$

$x^2 = 2$ (1ST Q $x > 0$)

$x = \sqrt{2}$

$\therefore m = 2\sqrt{2}$