Student Number: ______________

Teacher: CJL HRK SKB

CRANBROOK SCHOOL

MATHEMATICS EXTENSION 1

2004

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

General Instructions

- Reading time: five minutes
- Working time: two hours
- Calculators may be used.

- The examination consists of *7 questions* worth 12 marks each.

  Begin a new booklet for each question

  All questions should be attempted.

  All necessary working should be shown in every question.

  A table of standard integrals is provided at the back of this paper.

CRANBROOK SCHOOL

YEAR 12 MATHEMATICS – EXTENSION 1

Term 3 2004 **Time : 2 h / JJA, CJL, HRK and SKB**

All questions are of equal value.
All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved silent calculators may be used.
Submit your work in one 4 Page Booklet and three 8 Page Booklets.

1. (12marks) **(Begin a 4 page booklet.)** CJL

(a) Without using a calculator show that $\tan^{-1}\frac{2}{3}+\tan^{-1}\frac{1}{5}=\frac{\pi}{4}$ 3

(b) Show that $\int_0^{\frac{3}{2}}\frac{dt}{\sqrt{9-2t^2}}=\frac{\pi}{4\sqrt{2}}$ 3

(c) P $\left(2t,t^2\right)$ is a point on the parabola $x^2=4y$ with focus S. The point M divides the interval SP externally in the ratio 3:1.

(i) Find the coordinates of M

(ii) Show that the locus of M is $x^2=6y+3$
Hence find the coordinates of the focus and the equation of the directrix of M. 6

2. (12marks) **(Begin an 8 page booklet.)** HRK

(a) Sketch $y=(x+3)^3(2-x)^2$. 3

(b) Show that $x-2$ is a factor of the polynomial x^3-7x+6.
Hence, find the other factors and solve $x^3-7x+6>0$. 5

(c) Consider the curve $y = x^3 - 3x$

(i) Taking $x = 1$ as the first approximation use Newton's method to approximate the root that lies between zero and two.
Explain your result. **2**

(ii) Now use $x = \frac{1}{2}$ as the first approximation.
Explain your result. **2**

HRK

3. (12marks)

(a) Prove by mathematical induction that $(3n+1)7^n - 1$ is divisible by 9 for all integers $n \geq 1$. **4**

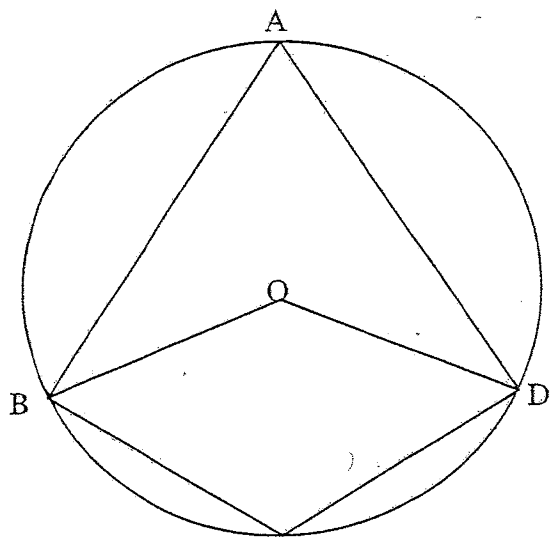
(b) Find the volume of the solid generated by revolving about the x-axis the region bounded by the x-axis, the curve $y = \cos 2x$ and the line $x = \frac{\pi}{4}$. **4**

(c) Find the exact value of $\int_{-1}^{2} \frac{t}{\sqrt{3-t}} dt$ using the substitution $t = 3 - u$. **4**

JJA

4. (12marks) (Begin an 8 page booklet.)

(a)

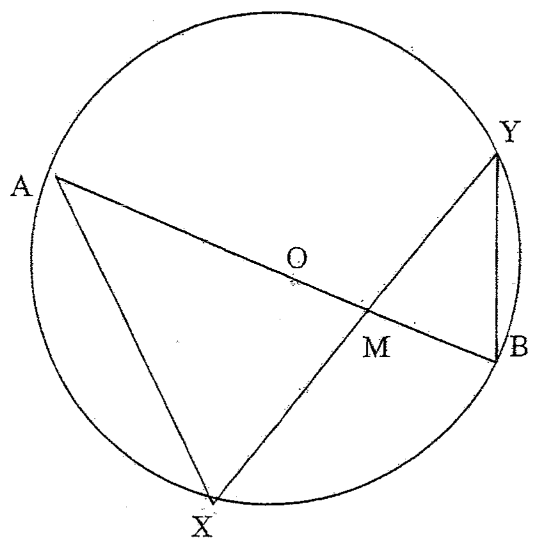


In the diagram A, B, C and D are points on a circle with centre O. $\angle BAD = x^o$ and $\angle BOD = \angle BCD$.

(i) Accurately copy the above diagram into your workbook. 1

(ii) Find the value of x **giving reasons.** 3

(b) In the diagram below, AB is a diameter of a circle, whose centre is the point O. The chord XY passes through M, the mid-point of OB. AX and BY are joined.



(i) Copy the above diagram carefully, including all information, into your writing booklet. 1

(ii) Prove the two triangles formed (triangles AXM and MYB) are similar. 4

(iii) If $XM = 8cm$ and $YM = 6cm$, find the length of the radius of the circle. 3

5. **(12marks)** **JJA**

(a) Simplify **showing all working:**

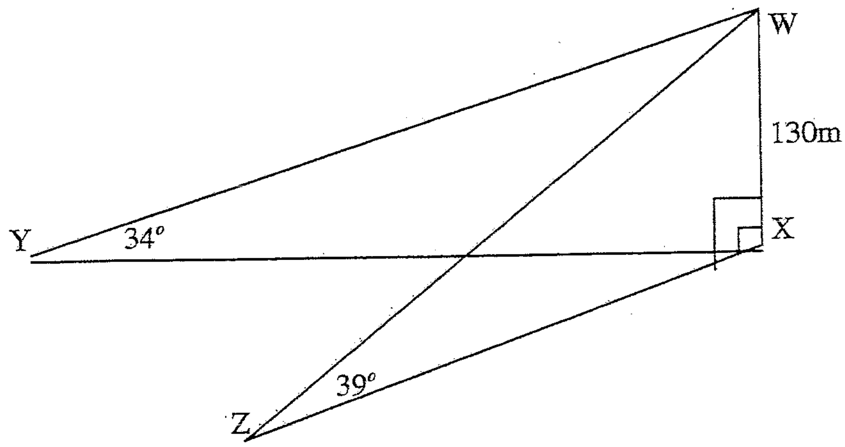
(i) $\sin(a+b)-\sin(a-b)$ 1

(ii) $\dfrac{\tan 64^o - \tan 19^o}{1+\tan 64^0 \tan 19^o}$ 2

(b) **Showing all working**, find the exact value of: $1-2\sin^2 75^o$ 2

(c) Find the general solution for: $\tan 2\theta = 3\tan\theta$. 3

(d) WX is a tower of height 130m. From points Y, Z on the same level as the foot of the tower (X), the angles of elevation to the top of the tower are 34^o and 39^o respectively.



(i) Find the distances XY and XZ to the nearest metre. 2

(ii) If $\angle YXZ = 44^0$, how far apart are Y and Z? 2

6. (12marks) (Begin an 8 page booklet.) SKB

(a) Sketch the curve $y=\dfrac{x^2+1}{x-1}$.
On your graph include any asymptotes, intercepts and coordinates of any turning points.
Note : the curve does not have any points of inflexion. 5

(b) When the interval joining the points (-5,6) and (-2,3) is trisected find the points of trisection. 3

(c) Solve for x: $\dfrac{|2x-1|}{x} \ge x+1$ 4

7. **(12marks)** **SKB**

(a) Two straight roads intersect at an angle of 120^0. A horse and cart starts from the intersection and travels along one road at 20 km/hr. One hour later a cyclist starts from the intersection and travels along the other road at 50 km/hr. At what rate is the distance between the horse and cart and the cyclist changing three hours after the cyclist starts? [Leave your answer to the nearest km/hr.] **6**

(b) A pendulum bob is oscillating in a straight line according to the equation $\ddot{x} = -4(x-8)$. Initially the bob is 3cm to the left of the origin O and is travelling at 4cm/s to the right.

(i) Show that the velocity v, of the pendulum bob is given by $v = 2\sqrt{125-(x-8)^2}$. **2**

(ii) Determine the end points of its motion in irrational form. **1**

(iii) Determine an equation for the displacement, x, of the pendulum bob in terms of time, t, in exact form. **3**

Extension 1 Trial 2004

Question 1

a) let $\alpha = \tan^{-1}\frac{2}{3}$ $\quad \beta = \tan^{-1}\frac{1}{5}$

$\therefore \tan\alpha = \frac{2}{3}$ $\quad \tan\beta = \frac{1}{5}$ ✓

Prove $\tan\left(\tan^{-1}\frac{2}{3}+\tan^{-1}\frac{1}{5}\right) = \tan\frac{\pi}{4}$

$$\begin{aligned}
LHS &= \tan\left(\tan^{-1}\frac{2}{3}+\tan^{-1}\frac{1}{5}\right)\\
&= \tan(\alpha+\beta)\\
&= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\\
&= \frac{\frac{2}{3}+\frac{1}{5}}{1-\frac{2}{3}\cdot\frac{1}{5}} \quad ✓\\
&= \frac{\frac{13}{15}}{\frac{13}{15}}\\
&= 1
\end{aligned}$$

$RHS = \tan\frac{\pi}{4} = 1$ ✓

$\therefore \tan^{-1}\frac{2}{3}+\tan^{-1}\frac{1}{5} = \frac{\pi}{4}$

b) $$\int_0^{3/2}\frac{dt}{\sqrt{9-2t^2}} = \int_0^{3/2}\frac{dt}{\sqrt{2\left(\frac{9}{2}-t^2\right)}}$$

$$= \frac{1}{\sqrt{2}}\int_0^{3/2}\frac{dt}{\sqrt{\frac{9}{2}-t^2}} \quad ✓$$

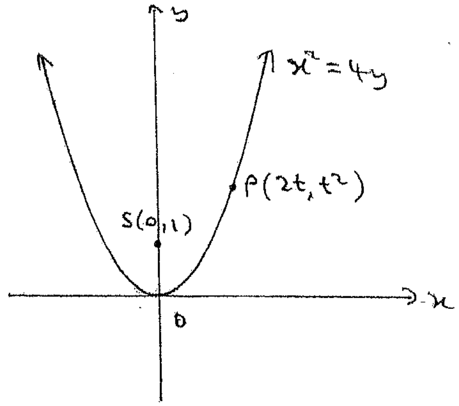
$$= \left[\frac{1}{\sqrt{2}}\sin^{-1}\frac{\sqrt{2}t}{3}\right]_0^{3/2} \quad ✓$$

$$= \frac{1}{\sqrt{2}}\left(\sin^{-1}\frac{\sqrt{2}}{2}-\sin^{-1}0\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4\sqrt{2}} \text{ as required.} \quad ✓$$

c)



✓

(i) $S(0,1)$ $\quad P(2t,t^2)$ $\quad 3:1$

$$M = \left(\frac{3(2t)-1(0)}{2}, \frac{3(t^2)-1(1)}{2}\right)$$

$\therefore x = 3t$ ✓ $\quad y = \frac{3t^2-1}{2}$ ✓

(ii) $t = \frac{x}{3}$ $\quad 2y = 3\left(\frac{x^2}{9}\right)-1$

$$2y = \frac{x^2}{3}-1$$
$$6y = x^2-3$$
$$\therefore x^2 = 6y+3$$
$$x^2 = 6\left(y+\frac{1}{2}\right)$$

vertex is $\left(0,-\frac{1}{2}\right)$

$4a = 6$

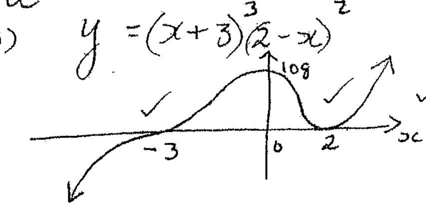
$a = \frac{3}{2}$

focus $(0,1)$ ✓

directrix $y=-2$ ✓

2a

(3) $y = (x+3)^3(2-x)^2$



✓

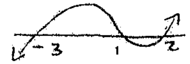
(b) (5) $P(x) = x^3-7x+6$

$$\begin{aligned}
P(2) &= 2^3-7(2)+6\\
&= 8-14+6 \quad ✓\\
&= 0
\end{aligned}$$

$\therefore x-2$ is a factor of x^3-7x+6

$$\begin{array}{r}
x^2+2x-3\\
x-2\,\overline{)\,x^3+0x^2-7x+6}\\
x^3-2x^2\\
\hline
2x^2-7x+6\\
2x^2-4x\\
\hline
-3x+6\\
-3x+6\\
\hline
\end{array} \quad ✓$$

$\therefore x^3 = (x-2)(x^2+2x-3)$
$\quad = (x-2)(x+3)(x-1)$ ✓

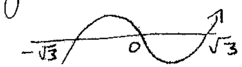


✓

$\therefore x^3-7x+6>0$ for

$-3<x<1,\ x>2$ ✓

(c) $y = x^3-3x = x(x^2-3)$



$y' = 3x^2-3$

$f'(1) = 0 \quad x=1 \quad f(x) = -2$

$x_1 = x - \frac{f(x)}{f'(x)}$ ✓

$\quad = 1 - \frac{-2}{0}$

No result as this is undefined due to horizontal tangent which does not cut x-axis ✓

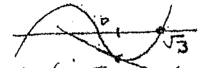
QZ+3 SOLNS HRK

$x_1 = \frac{1}{2} - \frac{-\frac{11}{8}}{-\frac{9}{4}}$ $\quad f\left(\frac{1}{2}\right) = -\frac{11}{8}$

$\quad f'\left(\frac{1}{2}\right) = -\frac{9}{4}$

✓ $\quad = -\frac{1}{9}$

Fails again



due to concavity & direction of curve ✓ this brings us closer to another root ie $x=0$. There is a MIN turning point between $x=\frac{1}{2}$ & $x=$

3 ① when $n=1$ $\quad (3n+1)7^n-1 = (3+1)7-1$
$\quad = 27 = 9\times 3$

is divisible by 9 $\therefore$ true for $n=$

② Assume true for $n=k$ ie assume $(3k+1)7^k$ ... where $Q\in J^+$

ie $(3k+1) = \frac{9Q+1}{7^k}$ ✓

③ Prove true for $n=k+1$ ie prove

$(3(k+1)+1)7^{k+1}-1 = 9P \quad (P\in J^+)$

$$\begin{aligned}
LHS &= (3k+3+1)7^{k+1}-1\\
&= \left(\boxed{3k+1}+3\right)7^{k+1}-1\\
&= \left(\frac{9Q+1}{7^k}+3\right)7^{k+1}-1\\
&= (9Q+1)7+3\times 7^{k+1}-1 \quad ✓\\
&= 9(7Q)+7+3\times 7\times 7^k-1\\
&= 9(7Q)+6+(21)7^k\\
&= 9(7Q)+3\left(2+7^{k+1}\right)
\end{aligned}$$

NOW PROVE BY INDUCTION THAT $2+7^{k+1}$ is divisible by 3

For $n=1$ $\quad 2+7^2 = 51 = 3\times 17$ $\therefore$ true for $n=1$

assume $2+7^{k+1} = 9R$ ie $7^{k+1} = 9R-2$ & prove $2+7^{k+2} =$ $(R, T\in J^+)$

$$\begin{aligned}
2+7^{k+2} &= 2+\left(7^{k+1}\right)7' \quad ✓\\
&= 2+(9R-2)7\\
&= 9\times 7R-12\\
&= 3(21R-4)
\end{aligned}$$

& Since $R\in J^+$, $21R-4\in$

$\therefore$ If true for $n=k$, true for $n=k+1$ but true for $n=1$ $\therefore$ true for $n=2$ & since true for $n=2$ true for $n=3$ & SO ON $\therefore$ Re is proven true by MI ✓

3(b) $V = \pi \int_0^{\frac{\pi}{4}} y^2\, dx$ $\qquad \cos^2 x = \frac{1}{2}(1+\cos 2x)$

$= \pi \int_0^{\frac{\pi}{4}} \cos^2 2x\, dx$ ✓ $\quad \therefore \cos^2 2x = \frac{1}{2}(1+\cos 4x)$

$\cancel{= \pi \int_0^{\frac{\pi}{4}} \cos^2 2x\, dx}$ ✓

$= \pi \int_0^{\frac{\pi}{4}} \frac{1}{2}(1+\cos 4x)\, dx$

$= \frac{\pi}{2} \int_0^{\frac{\pi}{4}} (1+\cos 4x)\, dx$

$= \frac{\pi}{2}\left[x + \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{4}}$ ✓

$= \frac{\pi}{2}\left[\frac{\pi}{4} + \frac{\sin 4\frac{\pi}{4}}{4}\right]$

$= \frac{\pi}{2}\left[\frac{\pi}{4} + 0\right]$ ✓

$\checkmark$ Volume $= \frac{\pi^2}{8} u^3$ //

(c) $\int_{-1}^{2} \frac{t}{\sqrt{3-t}}\, dt$ $\qquad t = 3-u$, $\quad \frac{dt}{du} = -1$, $\quad \therefore dt = -du$ $\qquad$ When $t = 2$, $u = 1$; $\quad t = -1$, $u = 4$

$= \int_4^1 \frac{(3-u)(-du)}{\sqrt{u}}$

$= \int_4^1 \frac{u-3}{\sqrt{u}}\, du$

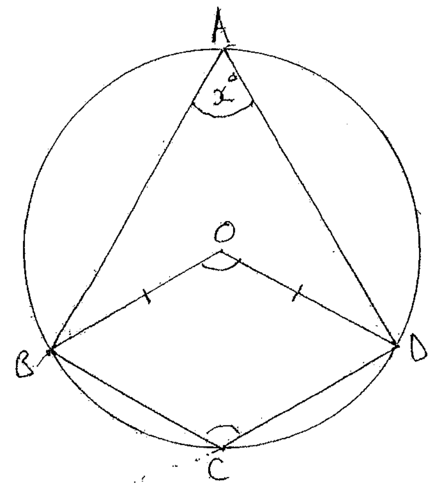
$= \int_4^1 \sqrt{u} - 3u^{-\frac{1}{2}}\, du$

$= \left[\frac{2u^{\frac{3}{2}}}{3} - 6u^{\frac{1}{2}}\right]_4^1$

$= \left(\frac{2}{3} - 6\right) - \left(\frac{16}{3} - 12\right)$ $= \frac{4}{3}$ //

4.

(a)



1 MARK

(1 MARK) $\angle BOD = 2x$ (Angle at centre is equal to twice angle at circumference)

$\angle BOD = \angle BCD$ (given)

(1 MARK) $\angle BCD = 180° - x$ (Opposite $\angle$s in cyc quad. are supp.)

(1 MARK) $\therefore 2x = 180° - x$

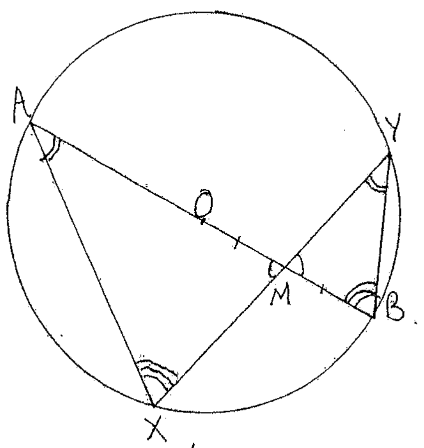
$3x = 180°$

$\therefore x = 60°$

(b)

(i)

(1 MARK)



(ii) In Δs AXM and ΔYBM

ONLY NEED 2!

-(1 MARK FOR EACH REASON)

- (1 MARK FOR LAYOUT)

$\angle AMX = \angle YMB$ (Vert. opp. angles)

$\angle XAM = \angle BYM$ (Angles subtended by same arc (XB) are equal)

$\angle AXM = \angle YBM$ (Angles subtended by same arc (AY) are equal)

(1 MARK) $\therefore \Delta AXM \,|||\, \Delta YBM$ (Equiangular)

(iii) (1 MARK) $\frac{XM}{BM} = \frac{AM}{YM}$ (corresponding sides in similar triangles)

(1 MARK) $AM = 3 \times BM$ (M is midpoint of OB and AB is diameter)

$\underline{8}$ $\quad 3BM$

$48 = 3.BM^2$

$16 = BM^2$

$\therefore BM = 4$

$\therefore$ radius $= 2 \times MB$

$= 2 \times 4$

(1 MARK) $= 8cm$

5.A).i) $\sin(a+b) - \sin(a-b)$

$= \sin a\cos b + \cos a\sin b - [\sin a\cos b - \cos a\sin b]$

$= 2\cos a\sin b$

$= \underline{\underline{2\sin b\cos a}}$

A.2) $\dfrac{\tan 64^\circ - \tan 19^\circ}{1+\tan 64^\circ \tan 19^\circ}$

$= \tan(64^\circ - 19^\circ)$

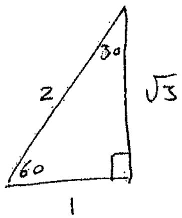
$= \tan 45^\circ$

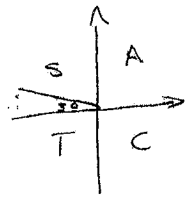
$= \underline{\underline{1}}$

B) $1 - 2\sin^2 75^\circ$

$= \cos 150^\circ$

$= \underline{\underline{-\dfrac{\sqrt{3}}{2}}}$





c) $\tan 2\theta = 3\tan\theta$

Let $t = \tan\theta$

$\therefore \tan 2\theta = \dfrac{2t}{1-t^2}$

$\dfrac{2t}{1-t^2} = 3t$

$2t = 3t(1-t^2)$

$2t = 3t - 3t^3$

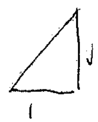
$3t^3 - t = 0$

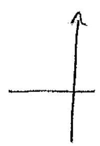
$t(3t^2 - 1) = 0$

$t = 0,$

$t^2 = \dfrac{1}{3}$

$t = \pm\dfrac{1}{\sqrt{3}}$





$\tan\theta = 0,$

$\theta = 0, \pi \ldots$

$= n\pi$

$\tan\theta = \pm\dfrac{1}{\sqrt{3}}$

$= \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6} \ldots$

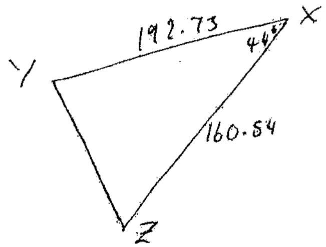
D).i) $\tan 34^\circ = \dfrac{130}{XY}$

$\underline{\underline{XY = 192.73m}}$

$\tan 39^\circ = \dfrac{130}{XZ}$

$\underline{\underline{XZ = 160.54m}}$

.ii)



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 192.73^2 + 160.54^2 - 2\times 192.73\times 160.54\cos 44^\circ$

$a^2 = 18403.93996\ldots$

$= 135.66$

$= \underline{\underline{136m}}$

6(a) $y=\dfrac{x^2+1}{x-1}$

• y is undefined when $x-1=0$
$\therefore x=1$ is a vertical asymptote

• $$\begin{array}{r} x+1 \\ x-1\,\overline{)\,x^2+1} \\ -(x^2-x) \\ x+1 \\ -(x-1) \\ 2 \end{array}$$

$\therefore y = x+1+\dfrac{2}{x-1}$

As $x\to\pm\infty$ $y\to x+1$
$\therefore$ oblique asymptote at $y=x+1$.

• When $x=0$ $y=-1$ $\therefore$ y-intercept at $(0,-1)$
when $y=0$ $x^2+1=0$ $\therefore$ No x-intercepts

• $y=\dfrac{x^2+1}{x-1}$

$\therefore y' = \dfrac{(x-1).2x-(x^2+1).1}{(x-1)^2}$

$= \dfrac{2x^2-2x-x^2-1}{(x-1)^2}$

$= \dfrac{x^2-2x-1}{(x-1)^2}$

For stat. pts $y'=0$ $\therefore x^2-2x-1=0$
$\therefore (x-1)^2-2=0$
$\therefore x = 1\pm\sqrt{2}$

At $x=1-\sqrt{2}$

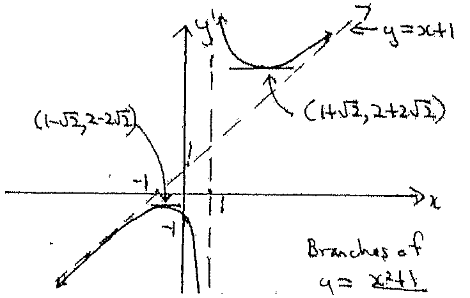
x	$(1-\sqrt{2})^-$	$1-\sqrt{2}$	$(1-\sqrt{2})^+$
y'	$+$	0	$-$


$\Rightarrow$ max. turning point at $(1-\sqrt{2},\ 2-2\sqrt{2})$

At $x=1+\sqrt{2}$

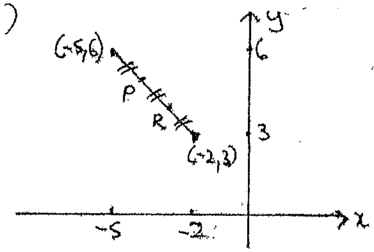
x	$(1+\sqrt{2})^-$	$1+\sqrt{2}$	$(1+\sqrt{2})^+$
y'	$-$	0	$+$


$\Rightarrow$ min. turning point at $(1+\sqrt{2},\ 2+2\sqrt{2})$



Branches of $y=\dfrac{x^2+1}{\ldots}$

(b)



Let $(-5,6)=(x_1,y_1)$, $(-2,3)=(x_2,y_2)$

For point R $m:n=2:1$

$\therefore R=\left(\dfrac{2\times-2+1\times-5}{3},\dfrac{2\times3+1\times6}{3}\right)=(-3,4)$

P = midpoint of (x_1,y_1) and R

$\therefore P=\left(\dfrac{-5-3}{2},\dfrac{6+4}{2}\right)=(-4,5)$

$\therefore$ points of trisection are $(-4,5)$ and $(-3,4)$

(c) $\dfrac{|2x-1|}{x}\geq x+1$, $x\neq0$

By definition: $|2x-1| = 2x-1$ if $2x-1\geq0$ i.e. $x\geq\frac12$
$= -(2x-1)$ if $x<\frac12$

Case 1: $x\geq\frac12$ $\therefore \dfrac{2x-1}{x}\geq x+1$

$\therefore x^2\left(\dfrac{2x-1}{x}\right)\geq x^2(x+1)$

$\therefore x(2x-1)\geq x^2(x+1)$

$\therefore x\,[(2x-1)-x(x+1)]\geq0$

$\therefore x\,[-x^2+x-1]\geq0$

$\therefore x(x^2-x+1)\leq0$

But x^2-x+1 is positive definite

$\therefore x\leq0$. But $x\geq\frac12$; contradiction

$\therefore$ No sol'ns for Case 1.

Case 2: $x<\frac12$ $\therefore \dfrac{-(2x-1)}{x}\geq x+1$

$\therefore x^2\left[\dfrac{-2x+1}{x}\right]\geq x^2(x+1)$

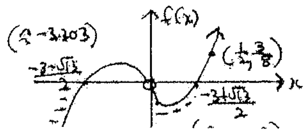
$\therefore x\,[(-2x+1)-x(x+1)]\geq0$

$\therefore x\,[-x^2-3x+1]\geq0$

$\therefore x\,[x^2+3x-1]\leq0$

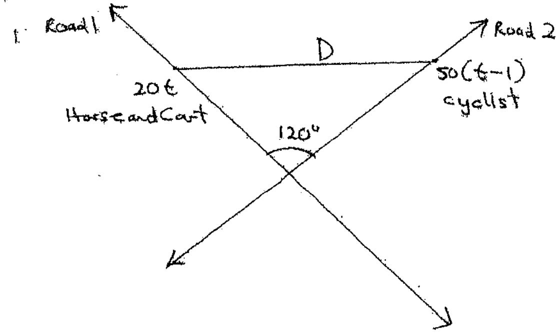
$\therefore x\left[\left(x+\tfrac32\right)^2-\tfrac{13}{4}\right]\leq0$

$\therefore x\left[x+\dfrac{3+\sqrt{13}}{2}\right]\left[x+\dfrac{3-\sqrt{13}}{2}\right]\leq0$



$\therefore x\leq\dfrac{-3-\sqrt{13}}{2}$ or $0<x\leq\dfrac{-3+\sqrt{13}}{2}$.

7(a)



Consider the relative positions of the Horse and Cart and the Cyclist after t hours.

$\therefore D^2=(20t)^2+(50(t-1))^2-2(20t)(50(t-1))\cos120^\circ$

$= 400t^2+2500(t-1)^2+1000(t^2-t)$

$\therefore 2D\dfrac{dD}{dt} = 800t+5000(t-1)+1000(2t-1)$

$\therefore \dfrac{dD}{dt} = \dfrac{400t+2500(t-1)+500(2t-1)}{D}$

when $t=4$ $D^2=80^2+150^2+12000$

$\therefore D=\sqrt{40900}$

$\therefore \dfrac{dD}{dt}=\dfrac{1600+7500+3500}{\sqrt{40900}}$

$= 62.30299023\ldots$

$\therefore$ Distance is changing at the rate of 62 km/hr (to nearest km/hr) between the horse and cart and the cyclist after 4 hours.

Alternative 7(b)(iii) $t=-\frac12\cos^{-1}\left(\dfrac{x-8}{5\sqrt5}\right)+c_3$ (**)

when $t=0, x=-3$ $\therefore 0=-\frac12\cos^{-1}\left(\dfrac{-11}{5\sqrt5}\right)+c_3$

$\therefore c_3=\frac12\cos^{-1}\left(\dfrac{-11}{5\sqrt5}\right)$

$\therefore t=-\frac12\cos^{-1}\left(\dfrac{x-8}{5\sqrt5}\right)+\frac12\cos^{-1}\left(\dfrac{-11}{5\sqrt5}\right)$

$\therefore -2t+\cos^{-1}\left(\dfrac{-11}{5\sqrt5}\right)=\cos^{-1}\left(\dfrac{x-8}{5\sqrt5}\right)$

$\therefore x=5\sqrt5\cos\left[-2t+\cos^{-1}\left(\dfrac{-11}{5\sqrt5}\right)\right]+8$

(b) $\ddot{x}=-4(x-8)$

(i) $\ddot{x}=\dfrac{d}{dx}\left(\tfrac12v^2\right)=-4(x-8)$

$\therefore \tfrac12v^2=-4\int(x-8)\,dx$

$= -4.\dfrac{(x-8)^2}{2}+c$

when $x=-3$, $v=4$

$\therefore 8=-2(-11)^2+c$

$\therefore c=250$

$\therefore \tfrac12v^2=250-2(x-8)^2$

$\therefore v^2=500-4(x-8)^2$

$\therefore v=\pm\sqrt{4(125-(x-8)^2)}$

$v=2\sqrt{125-(x-8)^2}$

[taking +ve square root as initial v was in positive direction.]

(ii) At end points of motion $v=0$

$\therefore 125=(x-8)^2$

$\therefore x-8=\pm5\sqrt5$

$\therefore x=8-5\sqrt5$ and $x=8+5\sqrt5$ are the end points of the mot

(iii) $\dfrac{dx}{dt}=2\sqrt{125-(x-8)^2}$

$\therefore \dfrac{dt}{dx}=\dfrac{1}{2\sqrt{125-(x-8)^2}}$

$t=\frac12\displaystyle\int\dfrac{dx}{\sqrt{(5\sqrt5)^2-(x-8)^2}}$

$\therefore t=\frac12\sin^{-1}\left(\dfrac{x-8}{5\sqrt5}\right)+c_1$ (*)

when $t=0, x=-3$

$\therefore 0=\frac12\sin^{-1}\left(\dfrac{-11}{5\sqrt5}\right)+c_1$

$\therefore c_1=\frac12\sin^{-1}\left(\dfrac{11}{5\sqrt5}\right)$

$\therefore t=\frac12\sin^{-1}\left(\dfrac{x-8}{5\sqrt5}\right)+\frac12\sin^{-1}\left(\dfrac{11}{5\sqrt5}\right)$

$x=5\sqrt5\sin\left[2t-\sin^{-1}\left(\dfrac{11}{5\sqrt5}\right)\right]+8$

7.(a) Alternative solution:

As before:

$$D^2 = 400t^2 + 2500(t-1)^2 + 1000(t^2-t)$$

$$\therefore D = \sqrt{400t^2 + 2500(t-1)^2 + 1000\cdot(t^2-t)}$$

$$\therefore \frac{dD}{dt} = \frac{1}{2}\left(400t^2 + 2500(t-1)^2 + 1000(t^2-t)\right)^{-\frac{1}{2}}$$

$$\cdot\left(800t + 5000(t-1) + 1000(2t-1)\right)$$

when $t=4$ $\quad D^2 = 80^2 + 150^2 + 12000$

$$\therefore D = \sqrt{40900}$$

$$\therefore \frac{dD}{dt} = \frac{3200 + 15000 + 7000}{2\sqrt{40900}}$$

$$= 62.3029023\ldots$$

$\therefore$ Distance is changing at the rate of 62 km/hr (to nearest km/hr) between the horse and cart and the cyclist after 4 hours.