



CRANBROOK
SCHOOL

Term 1, 2006

Year 11 Extension 1 Mathematics

Mini Examination

Wednesday April 5, 2006

All questions are of equal value.
All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved silent calculators may be used.
Submit your work in five 4 Page Booklets.

ESSE QUAM VIDERI

Question 1 [12 Marks]

Start a new booklet

Marked by BMM

- a) Express $1.5\bar{6}$ as a rational number 2
- b) i. Evaluate $\frac{1.015(1.015^{24}-1)}{1.015-1}$ correct to 3 decimal places 1
- ii. Find M given that $0 = 2000(1.015)^{24} - M\left(\frac{1.015(1.015^{24}-1)}{1.015-1}\right)$ correct to 2 decimal places. 2
- c) If the price of a mobile phone is reduced by 20% to \$480
- i. Find its price before the reduction. 1
- ii. If the reduced price is later increased by 20%, find this final price. 1
- d) If $x = -2$, evaluate $3x^3 + \frac{1}{(3x)^2}$ 1
- e) Write with negative indices:
- i. $\frac{4}{x^4}$ 1
- ii. $\frac{1}{4x^4}$ 1
- iii. $\frac{4}{7\sqrt[3]{(x+1)^3}}$ 1
- iv. $\frac{1}{x\sqrt{x}}$ 1

Question 2 [12 Marks]

Start a new booklet

Marked by JJA

- a) Fully factorise:
- i. $2a^3 + 10a^2 + 12a$ 2
 - ii. $-6x^2 + x + 35$ 2
- b) Simplify: $\frac{ax - ay + bx - by}{x^2 - y^2} \div \frac{ab^2 + a^2b}{x^3 + y^3}$ 3
- c) If $\frac{\sqrt{2}-4}{3+2\sqrt{2}} = a + b\sqrt{2}$, find a and b 3
- d) Prove that $\frac{\frac{1}{2\sqrt{x^2+1}} - 2(x-9)\sqrt{x^2+1}}{(x-9)^4}$ can be written as $\frac{-4x^3 + 36x^2 - 4x + 37}{2(x^2+1)^{\frac{1}{2}}(x-9)^4}$ 3

Question 3 [12 Marks]

Start a new booklet

Marked by HRK

- a) Solve for x
- i. $\frac{3x+2}{x-4} \geq 2$ 3
 - ii. $|x+1| = 1-4x$ 2
 - iii. $\frac{25^{2x}}{\sqrt{5}} = 125^{x-1}$ 2
- b) i. Solve simultaneously: $y = x^3$
 $y = x^2 + 6x$ 3
- ii. Represent these two curves on a neat sketch showing clearly all points of intersection. 2

Question 4 [12 Marks]

Start a new booklet

Marked by HRK

- a) i. Sketch $y = x^2 - 6x - 16$ showing the x and y -intercepts and vertex. 2
- ii. State the domain and range of $y = x^2 - 6x - 16$. 1
- b) If $f(x) = 10^x + 10^{-x}$ show that $f(2x) = (f(x))^2 - 2$. 2
- c) i. Find $\lim_{x \rightarrow 2^+} \frac{x^2}{x^2 + 9}$ 1
- ii. Sketch $y = \frac{x^2}{x^2 + 9}$ showing any intercepts and asymptotes. 2
- iii. Is $y = \frac{x^2}{x^2 + 9}$ odd, even or neither? 1
- d) Shade the region bounded by $y > 4^x - 3$, $y \leq 0$ and $x > 0$ 3

Question 5 [12 Marks]

Start a new booklet

Marked by HRK

- a) Given $y = 5x^2 - 2x$ find $\frac{dy}{dx}$ from first principles 3
- b) Differentiate
- i. $y = 3x^2(5x-1)^6$. Factorise your answer 3
 - ii. $y = \frac{3x+5}{2x+1}$. Simplify your answer. 2
- c) Find the equation, in general form, of the normal to the curve $y = 3x^2 - 5$ at the point $(2, 7)$. 4

Yr 11 EXT 1 Mini Solutions

Question 1 (BMM)

a) $n = 1.565656\dots$

$100n = 156.5656\dots$

$99n = 155$

$n = 155$

99

$n = \frac{155}{99}$

b) i) 29.063

ii) $M = 2000(1.015)^{24}$

29.063

$M = 98.37$

c) i) $80\% = \$480$

$100\% = \$600$

original price = $\$600$

ii) $480 \times 1.2 = \$576$

d) $3x^3 + \frac{1}{(3x)^4}$

$= 3(-2)^3 + \frac{1}{(3(-2))^4}$

$= -24 + \frac{1}{(3(-2))^2}$

$= -24 + \frac{1}{36}$

$= -23 \frac{35}{36}$

e) i) $4x^{-4}$

ii) $\frac{x^{-4}}{4}$

iii) $\frac{4(x+1)^{-3/4}}{7}$

iv) $x^{-3/2}$

Question 2 (JJA)

a) i) $2a^3 + 10a^2 + 12a$

$= 2a(a^2 + 5a + 6)$

$= 2a(a+3)(a+2)$

ii) $-6x^2 + x + 35$

$= -(6x^2 - x - 35)$

$= -(3x+7)(2x-5)$

b) $\frac{ax - ay + bx - by}{x^2 - y^2} \div \frac{ab^2 + a^2b}{x^3 + y^3}$

$= \frac{a(x-y) + b(x-y)}{(x-y)(x+y)} \times \frac{(x+y)(x^2 - xy + y^2)}{ab(b+a)}$

$= \frac{(a/b)(x/y)}{(x/y)(x/y)} \times \frac{(x/y)(x^2 - xy + y^2)}{ab(b/a)}$

$= \frac{x^2 - xy + y^2}{ab}$

c) $\frac{1}{2\sqrt{x^2+1}} - \frac{2(x-a)\sqrt{x^2+1}}{(x-a)^4}$

OPERATOR: (Make a common denominator)

$\frac{1 - 2(x-a)\sqrt{x^2+1} \times (2\sqrt{x^2+1})}{2\sqrt{x^2+1}}$

$= \frac{1 - 4(x-a)(x^2+1)}{2\sqrt{x^2+1}}$

$= \frac{1 - 4(x^3 + x - 9x^2 - 9)}{2\sqrt{x^2+1}}$

$= \frac{1 - 4x^3 - 4x + 36x^2 + 36}{2\sqrt{x^2+1}}$

$= \frac{37 + 36x^2 - 4x - 4x^3}{2\sqrt{x^2+1}}$

with denominator now:

$\frac{37 + 36x^2 - 4x - 4x^3}{2\sqrt{x^2+1}} \times \frac{1}{(x-9)^4}$

$= \frac{-4x^3 + 36x^2 - 4x + 37}{3^2(x^2+1)^{1/2}(x-9)^4}$

c) $\frac{\sqrt{2-4}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{3\sqrt{2} - 12 + 4 + 8\sqrt{2}}{9 - 8}$

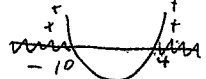
$= 11\sqrt{2} - 16$

$\therefore a = -16$

$b = 11$

3(a)
 i) $\frac{3x+2}{x-4} \times (x-4)^2 \geq 2(x-4)^2$

$(3x+2)(x-4) - 2(x-4)^2 \geq 0$
 $(x-4)[3x+2 - 2(x-4)] \geq 0$
 $(x-4)(x+10) \geq 0$



Note from question $x \neq 4$
 $\therefore x \leq -10, x > 4$

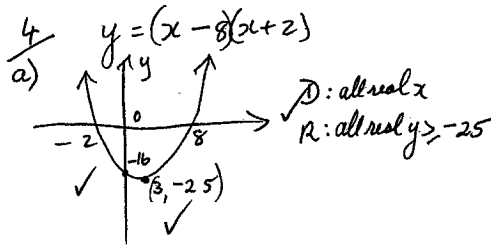
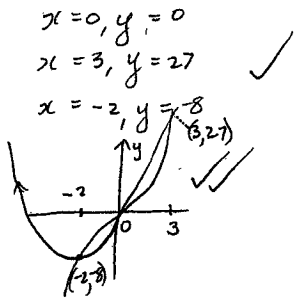
ii) $x+1 = 1-4x$
 $5x = 0$
 $x = 0$
 test in $|x+1| = 1-4x$
 $|| = 1$
 $1 = 1$

$\therefore x = 0$
 $-(x+1) = 1-4x$
 $-x-1 = 1-4x$
 $+3x = 2$
 $x = \frac{2}{3}$

test doesn't work
 $|\frac{2}{3}+1| \neq 1-\frac{4}{3} \therefore$ solution $x=0$

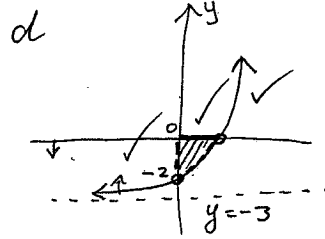
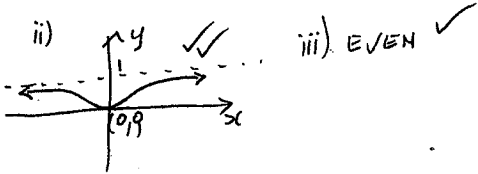
(iii) $5^{4x-\frac{1}{2}} = 5^{3x-3}$
 $4x-\frac{1}{2} = 3x-3$
 $x = -2\frac{1}{2}$

b) i) $x^3 = x^2 + 6x$
 $x(x^2 - x - 6) = 0$
 $x(x-3)(x+2) = 0$
 $x = 0, 3, -2$



b) $f(2x) = 10^{2x} + 10^{-2x}$
 $(f(x))^2 - 2 = (10^{2x} + 10^{-2x})^2 - 2$
 $= 10^{4x} + 10^{-4x} + 2 - 2$
 $= 10^{4x} + 10^{-4x}$
 $= f(2x)$

c) (i) $\lim_{x \rightarrow \pm \infty} \frac{x^2}{\frac{x^2}{x^2} + \frac{9}{x^2}}$
 $= \frac{1}{1+0} = 1$



5 $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{5(4+h)^2 - 2(4+h) - (5x^2 - 2x)}{h}$
 $= \lim_{h \rightarrow 0} \frac{5x^2 + 10xh + 5h^2 - 2x - 2h - 5x^2 + 2x}{h}$
 $= \lim_{h \rightarrow 0} \frac{10xh + 5h^2 - 2h}{h}$
 $= 10x - 2$

b) i) $y = 30x^2 6(5x-1)^5 + (5x-1)^6 6x$
 $= 90x^2(5x-1)^5 + 6x(5x-1)^6$
 $= 6x(5x-1)^5 [15x + 5x - 1]$
 $= 6x(5x-1)^5 (20x - 1)$

(ii) $y = \frac{3x+5}{2x+1}$ $y' = \frac{(2x+1)3 - (3x+5)2}{(2x+1)^2}$
 $= \frac{-7}{(2x+1)^2}$

c) $y' = 6x$
 at $x=2, M_T = 12 \therefore M_H = -\frac{1}{12}$
 $y - y_1 = m(x - x_1)$
 $y - 7 = -\frac{1}{12}(x - 2)$
 $12y - 84 = -x + 2$
 $x + 12y - 86 = 0$