CRANBROOK SCHOOL

# Year 11 Extension 1 Mathematics

## Preliminary Half yearly Assessment

Friday 28th of May, 2010

**Time Allowed:** 50 Minutes

**Total Marks:** 40

There are 2 questions.

Start a new booklet for each question.

All necessary working should be shown in every question.

Full marks may not be awarded if work is careless or badly arranged.

Approved calculators may be used.

---

Year 11 Ext 1 — Term 2, 2010

50 minutes — 40 Marks

NAME: _____ Teacher: RDS GHW BG1

**Question 1 (19 Marks)** **- Start a new booklet**

MARKS

(a) Show that $\frac{d}{dx}$ of $\frac{1}{x}$ is $-\frac{1}{x^2}$ using first principles 3

(b) Find:

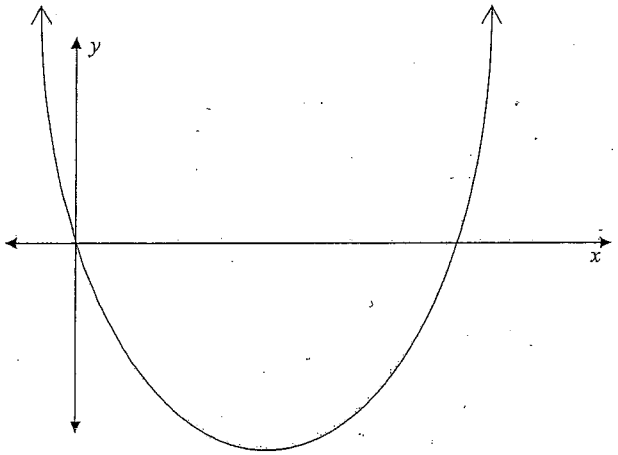
(i) the equation of the tangent to $y=\frac{x}{\sqrt{x}}$ at the point $(1,1)$ 3

(ii) $a$ and $b$ if the function $y=ax^2+b$ has a tangent at $(-4,10)$ that is parallel to $2y-8x-1=0$ 4

(c) Find $f'(x)$ in it's simplest form if

$f(x)=\frac{3-2x}{\sqrt{2-x}}$ 4

(e) The curve $y = x^2 - kx$ is drawn below.




(i) Show that the $x$ intercepts occur when $x = 0$ and $x = k$ 1

(ii) Find the gradient of the tangents at the $x$ intercepts 2

(iii) If the angle between the two tangents is 30° what is the value of $k$? 2

End of Question 1

**Question 2 (21 Marks)** **- Start a new booklet**

MARKS

a) What is the exact values of:

i) tan 330° 1

ii) cosec 300° 1

b) $A$ and $B$ are two lighthouses, $A$ being 20 km due north of $B$. The bearing of a ship is 145°T from $A$ and 055°T from $B$. Calculate the distance of each lighthouse from the ship. 4

c) The sides of a triangular field are 60 metres and 50 metres and the included angle is 140°. Calculate

i) the length of the third side by using the cosine rule, 2

ii) the area of the field. 2

d) Find all values of θ between 0° and 360° for which $\sin\theta = -\frac{\sqrt{3}}{2}$ 2

e) Prove the identity: 3

$$\frac{\cos\theta}{1-\sin\theta} - \tan\theta = \sec\theta$$

f) If $\tan\theta = \frac{7}{24}$ and $180° < \theta < 360°$, find the value of $\sin\theta$. 2

g) Simplify $\sin^3\theta + \sin\theta\cos^2\theta$ 1

h) Simplify $\frac{x^2}{\sqrt{x^2 - a^2}}$ when $x = a\sec\theta$ 3

End of Exam

Extension 1 – Term 3 Year 11

Question One

(a) $\frac{d}{dx}\frac{1}{x}$ 3

$f(x) = \frac{1}{x}$ $\quad f(x+h) = \frac{1}{x+h}$

$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x}$

$= \frac{x-(x+h)}{x(x+h)}$

$= \frac{-h}{x^2+xh}$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{-h}{x^2+xh} \times \frac{1}{h}$

$= \lim_{h\to 0} \frac{-1}{x^2+xh}$

$= \frac{-1}{x^2}$ 3

Lots of curious algebra moves were applied to this question

(b) (i) $y = \frac{x}{\sqrt{x}} = x \cdot x^{-\frac{1}{2}}$

$y' = \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

At $x=1$ $\quad y=1$

$y' = \frac{1}{2}$

$\therefore \frac{1}{2} = \frac{y-1}{x-1}$

$x-1 = 2y-2$

$2y-x-1=0$

(ii) $y = ax^2+b$ $\quad y' = 2ax$ $\quad 2y-8x-1=0$

$2y = 8x+1$

$y = 4x+\frac{1}{2}$

at $(-4,10)$, $y' = 4$

$\therefore 4 = 2a \cdot -4$

$-1 = 2a$

$a = -\frac{1}{2}$ (2)

$\therefore y = -\frac{1}{2}ax^2+b$ at $(-4,10)$

$10 = -\frac{1}{2}(-4)^2+b$

$10 = -8+b$

$b = 18$ (2)

Many students subbed m and pt into $y-y_1 = m(x-x_1)$ which did not help with the question.

4

(c) $f(x) = \frac{3-2x}{\sqrt{2-x}}$ $\quad u$ $\quad v$ $\quad u' = -2$ $\quad v' = \frac{1}{2}\cdot(2-x)^{-\frac{1}{2}} \cdot -1$

$= \frac{-1}{2\sqrt{2-x}}$

$v^2 = 2-x$

$f'(x) = \frac{vu'-uv'}{v^2}$

$= \frac{-2\sqrt{2-x} - (3-2x)\frac{-1}{2\sqrt{2-x}}}{2-x}$

$= \frac{-2\sqrt{2-x} + \frac{3-2x}{2\sqrt{2-x}}}{2-x} \times \frac{1}{2-x}$

$= \frac{-4(2-x)+3-2x}{2\sqrt{2-x}} \times \frac{1}{(2-x)^1}$

4

Lots of errors on this question

$= \dfrac{-5+2x}{2\sqrt{(2-x)^3}}$

(ii) 1) $y = x^2 - kx$ | 1

$0 = x^2 - kx$

$= x(x-k)$

$\therefore x = 0 \quad , x = k$

ii) $y' = 2x - k$

At $x = 0 \quad y' = -k$

$x = k \quad y' = k$ 2

iii) $\tan\theta = \left| \dfrac{m_1 - m_2}{1 + m_1 m_2} \right|$ if $m_1 = k$, $m_2 = -k$

~~tan0~~

$\tan 30° = \dfrac{1}{\sqrt{3}}$ 2

$\dfrac{1}{\sqrt{3}} = \dfrac{k+k}{1-k^2}$

$1 - k^2 = 2\sqrt{3}k$

$k^2 + 2\sqrt{3}k - 1 = 0$

$k = \dfrac{-2\sqrt{3} \pm \sqrt{12 - 4.1.-1}}{2}$

$= \dfrac{-2\sqrt{3} \pm \sqrt{16}}{2}$

$= \dfrac{-2\sqrt{3} \pm 4}{2}$

$= -\sqrt{3} \pm 2$

question was very poorly done. Few people remembered the formula correctly, let alone used it.

## Question 2 4HL (Q)

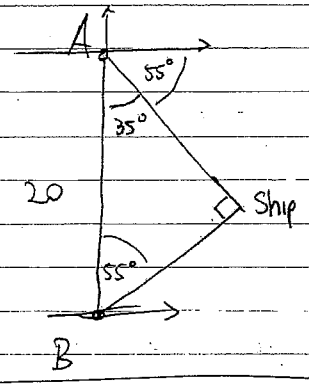
a) i) $\tan 330° = -\tan 30°$

$= \dfrac{-1}{\sqrt{3}}$ | 1

ii) $\operatorname{cosec} 300° = \dfrac{1}{\sin 300°} = \dfrac{1}{-\sin 60°}$

$= \dfrac{1}{-\frac{\sqrt{3}}{2}} = \dfrac{-2}{\sqrt{3}}$ | 1

b)




Few students saw right angle and used ~~via~~ SOHCAHTOA.

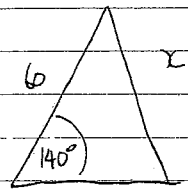
$\dfrac{AS}{20} = \sin 55°$

$AS = 16.38$ (2dp) 2

$\dfrac{BS}{20} = \sin 35°$

$BS = 11.47$ (2dp) 2

c)




i) $x^2 = 60^2 + 50^2 - 2 \times 50 \times 60 \cos 140°$

$x = 103.42$ m (2dp) 2

ii) $A = \frac{1}{2} \cdot 60 \cdot 50 \sin 140°$

$= 964.18$ m$^2$ (2dp) 2

Well done

d) $0 \leq \theta \leq 360°$

$\sin\theta = -\frac{\sqrt{3}}{2}$ $\quad \sin 60° = \frac{\sqrt{3}}{2}$

$\therefore \theta = (180+60), (360-60)$

$= 240, 300$ 2

e) $\frac{\cos\theta}{1-\sin\theta} \quad \tan\theta = \sec\theta$ 3

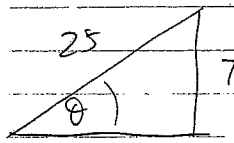
$LHS = \frac{\cos\theta}{1-\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta - \sin\theta + \sin^2\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{1-\sin\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{1}{\cos\theta} = \sec\theta = RHS.$

f) $\tan\theta = \frac{7}{24} \quad 180 \leq \theta \leq 360$ (3rd & 4th quadrants) tan is +ve ∴ only 3rd




$\therefore \sin\theta = -\frac{7}{25}$

in 3rd quadrant. 2

Many forgot negative

g) $\sin^3\theta + \sin\theta\cos^2\theta$ Good

$= \sin\theta(\sin^2\theta + \cos^2\theta)$ 1

$= \sin\theta$

h) $\frac{x^2}{\sqrt{x^2-a^2}} \quad x = a\sec\theta$

$x^2 = a^2\sec^2\theta$ 3

$\therefore = \frac{a^2\sec^2\theta}{\sqrt{a^2\sec^2\theta - a^2}}$ (1)

Very poorly done overall.

$= \frac{a^2\sec^2\theta}{\sqrt{a^2(\sec^2\theta - 1)}}$ (1)

$= \frac{a^2\sec^2\theta}{\sqrt{a^2\tan^2\theta}}$ (1) $= \frac{a^2\sec^2\theta}{a\tan\theta} = a$

$= \frac{a}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta}$

$= \frac{a}{\cos\theta\sin\theta}$ (1)