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CRANBROOK
SCHOOL

Monday September 4, 2006

Year 11 Mathematics

2006 Yearly Preliminary Examination

Time allowed: 3 hours

General Instructions:

- There are 10 questions, each question is worth 12 marks
- Attempt all questions
- Begin a new booklet at the beginning of each question.
- Show all necessary working to obtain maximum marks
- Marks will be deducted for poor and illegible work
- Only board approved calculators are permitted.

Question 1 [12 Marks] **Marked by CGH**

a) A clock is sold for \$112 making a 30%. What was the original cost price? **1**

b) Calculate S to 3 significant figures if **1**

$$S=\frac{a\left(1-R^n\right)}{1-R} \quad and \quad a=10, R=0.5, n=20$$

c) Simplify:

(i) $\frac{5x^3y^2}{(2xy)^3}\div\frac{10x^4}{6y^3}$ **2**

(ii) $\frac{5}{v+2}-\frac{4}{v^2-4}$ **2**

d) Write $1.3\dot{4}\dot{5}$ as a rational number. **2**

e) Factorise

(i) $3x^3-81$ **2**

(ii) $x^2-7x-4xy+28y$ **2**

Question 2 [12 Marks] **Begin a new booklet** **Marked by CGH**

a) Rationalise the denominator: $\frac{3+\sqrt{2}}{3-\sqrt{2}}$ **2**

b) Find a and b such that $\left(2\sqrt{3}-2\right)^2=a+b\sqrt{3}$ **2**

c) Solve:

(i) $5x^2-9x-2=0$ **2**

(ii) $2-\frac{x}{6}<4$ showing your solution on a number line **2**

(iii) $2^{x-2}=16$ **2**

d) Find the coordinates of the points of intersection of the parabola $y=x^2+5$ and the line $y=4x+50$. **2**

Question 3 [12 Marks] **Begin a new booklet** **Marked by BMM**

a) Consider the function: **2**

$$f(x)=\begin{cases}2x+3 & x>2\\ 1 & -2\le x\le 2\\ x^2 & x<-2\end{cases}$$

Find $\dfrac{f(3)+f(-4)}{f(0)}$

b) Sketch the function $y=\dfrac{1}{x-4}$ and hence state its domain and range. **3**

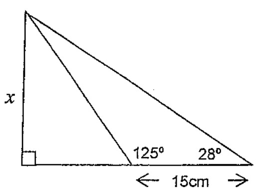
c) Is the function $f(x)=2x^2+4x+3$ odd, even or neither? **1**

d) Shade the region defined by $x^2+y^2\ge 25$, $y>3x-5$ **3**

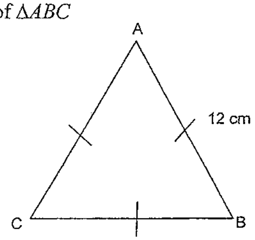
e) Sketch $y=2^x$ *and* $y=|x|+1$ on the same axes, and hence solve graphically $2^x=|x|+1$. **3**

Question 4 [12 Marks] **Begin a new booklet** **Marked by BMM**

a) Find x correct to 3 significant figures **2**



b) Find the exact area of ΔABC **2**



Question 4 contd.

c) Given that $\tan\alpha=\dfrac{8}{15}$, and $90°<\alpha<270°$, find the exact value of:

(i) $\sin\alpha$ **2**

(ii) $\sec\alpha$ **1**

d) Solve $\cos^2 x=\dfrac{1}{4}$ for $0°\le x\le 360°$ **3**

e) Simplify $\dfrac{\cos^2(90°-\theta)}{\sec^2\theta-1}$ **2**

Question 5 [12 Marks] **Begin a new booklet** **Marked by CAB**

a) Find the equation of the line perpendicular to the line $y=2x-3$ and passing through the point (1,1). **2**

b) $A(2, -2)$, $B(-2, -3)$ and $C(0, 2)$ are the vertices of a triangle ABC. **10**

(i) Draw a sketch diagram of the triangle in your answer booklet.

(ii) Find the length of the line AC and the gradient of AC.

(iii) Find the equation of the line AC in the general form.

(iv) Calculate the perpendicular distance of B from the side AC and hence find the area of ΔABC.

(v) Find the co-ordinates of D such that $ABCD$ is a parallelogram.

Question 6 [12 Marks] **Begin a new booklet** **Marked by JJA**

a) In each of the following, differentiate with respect to x:

(i) $5x^2-5x+3$ **1**

(ii) $(2x+8)^5$ **1**

(iii) $\dfrac{x^2+3x-5}{x}$ **2**

(iv) $\dfrac{1}{x^4}$ **2**

(v) $x^4(x^2-8)^5$ **3**

(vi) $\dfrac{x^2}{x-3}$ **3**

Question 7 [12 Marks] **Begin a new booklet** **Marked by JJA**

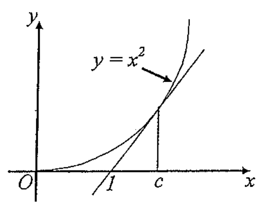
a) Differentiate $f(x)=x^2+3$ from first principles. **4**

ie: using $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

b) Find the equation of the normal to the curve $x^2=8y$ at the point where $x=2$

4

c) The diagram shows a graph of the parabola $y=x^2$ and the tangent to the parabola at $x=c$. **4**



(i) Find the gradient of the tangent at $x=c$.

(ii) Find the equation of the tangent at $x=c$.

(iii) Find the value of c if the tangent intersects the x-axis at $x=1$.

Question 8 [12 Marks] **Begin a new booklet** **Marked by HRK**

a)

(i) Determine whether the roots of the quadratic equation $9x^2-6x+1=0$ are real or unreal, rational or irrational, equal or unequal. **2**

(ii) For what values of k will the equation $x^2+kx-9=0$ have real and distinct roots? **2**

b) For the equation $x^2+(2r-3)x+(3-4r)=0$

(i) Find the values of r for which the equation has no real roots. **2**

(ii) Find the value of r for which the equation has one root equal to 0 **2**

c) Without sketching, show that the quadratic function $f(x)=2x^2-3x+7$ lies entirely above the x-axis. **2**

d) Find the value of r for which the roots of the quadratic equation $3x^2-4x+r=0$ are the reciprocals of one another **2**

Question 9 [12 Marks] **Begin a new booklet** **Marked by HRK**

a) Let α and β be the roots of the equation $5x^2+4x-6=0$. Without solving, find the value of: **5**

(i) $\alpha+\beta$

(ii) $\alpha\beta$

(iii) $\alpha^2+\beta^2$

(iv) $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}$

b) Rewrite the expression x^2-4x-3 in the form $A(x-1)^2+B(x+2)$ **2**

c) Solve for x:

(i) $x^4-6x^2+5=0$ **2**

(ii) $25^x-26(5^x)+25=0$ **3**

Question 10 [12 Marks] **Begin a new booklet** **Marked by BMM**

a) The point $P(x, y)$ moves such that PA is perpendicular to PB. Find the locus of P, where A is the point $(-3, 1)$ and B is the point $(4, -2)$. Describe the locus of P geometrically. **3**

b) Find the equation of the parabola with vertex $(0, 1)$ and focus $\left(\frac{1}{4}, 1\right)$ **3**

c) A parabola has equation $x^2 + 6x - 33 = 12y$. **6**

(i) Show that the coordinates of its vertex are $(-3, -3{\cdot}5)$

(ii) Show that its focal length is 3

(iii) Find the focus and the equation of its directrix.

(iv) Sketch the parabola labelling all essential features clearly.

End of Examination

2U SOLUTIONS ①

YR 11 EXAM 2006 (SEPT)

Question 1 (CGH)

a) $\$112 = 130\%$

$100\% = 112 \div 130 \times 100$

$= \$86.15$ ✓

Not asked to round up so don't!

b) $S = \dfrac{10(1-0.5^{20})}{1-0.5}$

$= 20.0$

too many mistakes in calculator work

Note $\sqrt{16+9} \neq 13$. i.e. use parentheses

c) (i) $\dfrac{5x^3y^2}{(2xy)^3} \times \dfrac{6y^3}{10x^4}$

$= \dfrac{5x^3y^2}{8x^3y^3} \times \dfrac{6y^3}{10x^4}$

$= \dfrac{3y^3}{8x^4}$

(ii) $\dfrac{5}{v+2} - \dfrac{4}{(v+2)(v-2)}$

$= \dfrac{5(v-2)-4}{(v+2)(v-2)}$ LCD please

$= \dfrac{5v-10-4}{v^2-4}$

$= \dfrac{5v-14}{v^2-4}$

d) $1.3\dot{4}\dot{5}$

$n = 1.34545 45\ldots$

$10n = 13.454545\ldots$

$1000n = 1345.4545\ldots$

$990n = 1332$

$n = \dfrac{74}{55}$

learn $a^3 - b^3$ please

e) (i) $3x^3 - 81$

$= 3(x^3 - 27)$

$= 3(x-3)(x^2+3x+9)$

(ii) $x^2 - 7x - 4xy + 28y$

$= x(x-7) - 4y(x-7)$

$= (x-7)(x-4y)$

Question 2 (CGH)

a) $\dfrac{3+\sqrt{2}}{3-\sqrt{2}} \times \dfrac{(3+\sqrt{2})}{(3+\sqrt{2})}$ Think Conjugates

$= \dfrac{9+6\sqrt{2}+2}{9-2}$

$= \dfrac{11+6\sqrt{2}}{7}$

$(\sqrt{2} \times \sqrt{2}) = \sqrt{4}$ Not 4

b) $(2\sqrt{3}-2)^2 = 4(3) - 8\sqrt{3} + 4$

$= 16 - 8\sqrt{3}$

$\therefore a = 16$

$b = -8$

②

c) (i) $5x^2 - 9x - 2 = 0$

99% of HSC quadratics factor

$(x-2)(5x+1) = 0$

$x = 2, -\frac{1}{5}$

So try factoring before going to $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ & avoid mechanical errors

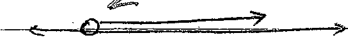
(ii) $2 - \dfrac{x}{6} < 4$

$12 - x < 24$

$-x < 12$

$x > -12$

Make hollow Clear



-12

(iii) $2^{x-2} = 16$

$2^{x-2} = 2^4$

$x - 2 = 4$

$x = 6$

too many errors here! like $x = 2$???

d) $y = x^2 + 5$ — ①

$y = 4x + 50$ — (2)

$x^2 + 5 = 4x + 50$

$x^2 - 4x - 45 = 0$

$(x-9)(x+5) = 0$

$x = 9, -5$ too many stopped here & failed to answer the Question

At $x = 9$, $y = 86$ $\therefore (9, 86)$

At $x = -5$, $y = 30$ $\therefore (-5, 30)$

Q3 - BMM.

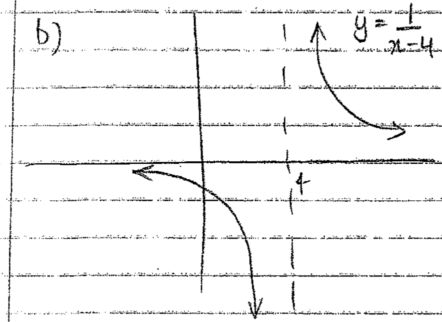
a) $\dfrac{f(3)+f(-4)}{f(0)} = \dfrac{(2(3)+3)+(-4)^2}{1}$

$= 9+16$

$= 27$

1 mk - correct substitution
1mk - answer
1mk - correct answer following incorrect substitution.

b)



Domain: All real x, $x \neq 4$
Range: All real y, $y \neq 0$

1mk - graph with correct asymptote
1mk - Domain (must state 'All real x')
1mk - Range (must state 'All real y')
1mk - If Domain + Range did not state 'All real'
1mk - If Domain + Range followed graph sketched.

c) $f(x) = 2x^2 - 4x + 3$

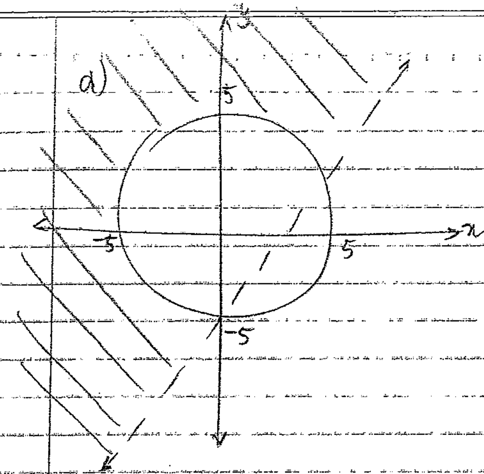
$f(-x) = 2(-x)^2 - 4(-x) + 3$

$= 2x^2 + 4x + 3$

$\therefore$ neither

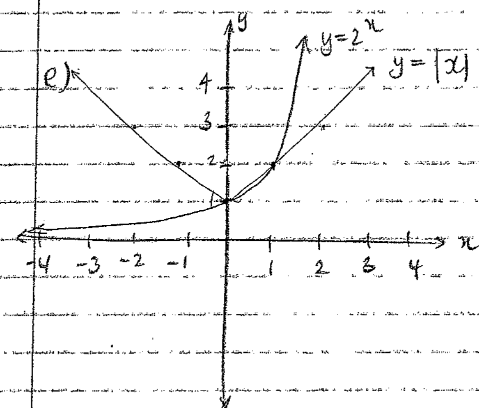
1mk - neither as correct answer.

d)



1 mk - circle correctly sketched with unbroken line
1mk - straight line correctly sketched with broken line
1mk - correct area shaded.

e)

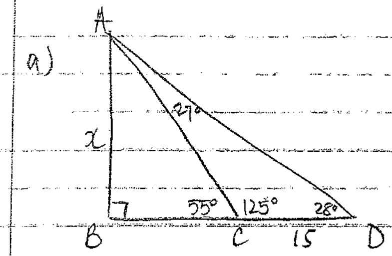


Solution: (0,1) and (1,2)

1 mk - correct sketch of $y=2^x$
1mk - correct sketch of $y=|x|+1$
1mk - BOTH pts of intersection.

Q4 - BMM

a)



Find AC first

$$\frac{AC}{\sin 28^\circ} = \frac{15}{\sin 27^\circ}$$

AC = 15.51149957

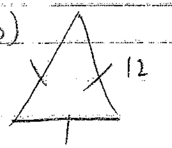
$$\sin 55^\circ = \frac{x}{15.51149957}$$

$x = 12.70627650$

$x = 12.7$ cm (3 sig fig)

1mk - correctly finding AC
1mk - correctly finding x to 3 sig fig
1mk - if incorrect AC was used to find x.

b)



Each angle = 60°

$$A = \frac{1}{2} \times a \times b \times \sin C$$

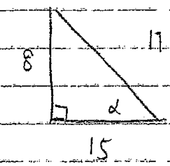
$$= \frac{1}{2} \times 12 \times 12 \times \sin 60^\circ$$

$$= 72 \times \frac{\sqrt{3}}{2}$$

$$= 36\sqrt{3} \text{ cm}^2$$

1mk - correct use of formula
1mk - correct simplified answer.

c) $\tan\alpha = \frac{8}{15}$



$90 < \alpha < 270^\circ$ and tan +ve

$\therefore \alpha$ in Q3

(i) sin -ve in Q3

$\therefore \sin\alpha = \frac{-8}{17}$

(ii) sec α -ve in Q3

$\therefore \sec\alpha = \frac{-17}{15}$

1mk - correct finding of hypotenuse
1mk - each answer, must be -ve.

d) $\cos^2 x = \frac{1}{4}$

$$\cos x = \pm\sqrt{\frac{1}{4}}$$

$$\cos x = \pm\frac{1}{2}$$

$\therefore$ All 4 quads.

In Q1, $x = 60^\circ$

in Q2, Q3, Q4 : 120°, 240°, 300°

1mk - correct manipulation of equation
1mk - finding x in 1st quadrant
1mk - for Q2, Q3, Q4 answers.
2mks - if equation was wrongly manipulated but correct answers of x followed.

e)

$$\frac{\cos^2(90-\theta)}{\sec^2\theta - 1}$$

$$= \frac{\sin^2\theta}{\tan^2\theta}$$

$$= \sin^2\theta \div \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \sin^2\theta \times \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cos^2\theta$$

1mk - change of numerator + denominator

1mk - simplifying

7

SOLUTIONS

QUESTION 5

a) $y = 2x - 3$

$m = 2$

$\therefore$ line perpendicular will have $m = -\frac{1}{2}$

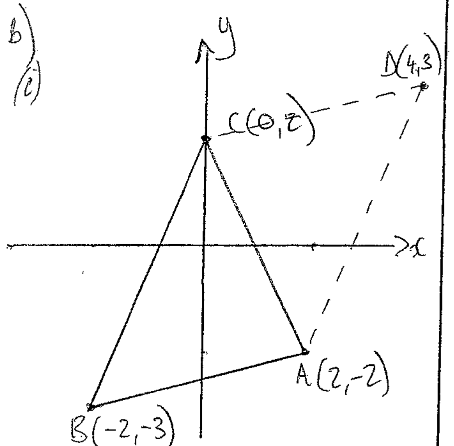
$y - 1 = -\frac{1}{2}(x-1)$

$2(y-1) = -(x-1)$

$2y - 2 = -x + 1$

$x + 2y - 3 = 0$

b) (i)



(ii) $d = \sqrt{(2-0)^2 + (-2-2)^2}$

$= \sqrt{2^2 + (-4)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$ or 4.5

$= 2\sqrt{5}$

$m = \frac{-2-2}{2-0} = \frac{-4}{2} = -2$

(iii) $y - 2 = -2(x - 0)$

$y - 2 = -2x$

$2x + y - 2 = 0$

(iv) $\perp d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$

$= \left|\frac{2\times(-2) + 1\times(-3) - 2}{\sqrt{2^2+1^2}}\right|$

$= \left|\frac{-4-3-2}{\sqrt{4+1}}\right|$

$= \frac{9}{\sqrt{5}}$

AREA OF $\triangle ABC = \frac{1}{2} \times 2\sqrt{5} \times \frac{9}{\sqrt{5}}$

$= 9$ units^2

(v) $D(4,3)$

By inspection

2 unit Yr 11 Exam 2006 8

Q6

a i) $\frac{d}{dx}(5x^2 - 5x + 3)$

$= 10x - 5$

ii) $\frac{d}{dx}(2x+8)^5$

$5(2x+8)^4 \times 2$

$10(2x+8)^4$

iii) $\frac{d}{dx}\left(\frac{x^2 + 3x - 5}{x}\right)$

Only one term on denominator

$\Rightarrow$ Divide through!!

Not Quotient rule

$= \frac{d}{dx}(x + 3 - 5x^{-1})$

$= 1 + 5x^{-2}$

$= 1 + \frac{5}{x^2}$

or, by quotient rule:

$\frac{x^2+5}{x^2}$

iv) $\frac{d}{dx}\left(\frac{1}{x^4}\right)$

$= \frac{d}{dx}(x^{-4})$

$= -4x^{-5}$

$= \frac{-4}{x^5}$

Can also do by quotient rule but no need!

v) $\frac{d}{dx}[x^4(x^2-8)^5]$

2 functions $\therefore$ Product rule!

$uv' + vu'$

$= x^4 \times 5(x^2-8)^4 \times \cancel{2x} + 4x^3(x^2-8)^5$

$= 10x^5(x^2-8)^4 + 4x^3(x^2-8)^5$

vi) $\frac{d}{dx}\frac{x^2}{x-3}$ 2 function $\therefore$ Quotient rule

$\frac{vu' - uv'}{v^2}$

$\frac{2x(x-3) - x^2}{(x-3)^2}$

$\frac{2x^2 - 6x - x^2}{(x-3)^2} \Rightarrow \frac{x^2 - 6x}{(x-3)^2}$

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7. a) $f(x) = x^2+3$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

Now $f(x+h) = (x+h)^2 + 3$

$= x^2 + 2xh + h^2 + 3$ ✓

Remember brackets! Lots forgot this

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{(x^2+2xh+h^2+3)-(x^2+3)}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2+\cancel{3}-\cancel{x^2}-\cancel{3}}{h}$ ✓

Anything without h should cancel.

$= \lim_{h\to 0} \frac{2xh+h^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$ ✓ Cancel h's.

$= \lim_{h\to 0} 2x+h$ ✓ Let $h \to 0$.

$= 2x$

Check answer to be sure

$\frac{d}{dx}\, x^2+3$

$= 2x$ ✓

B) $x^2 = 8y$

$\Rightarrow y = \frac{x^2}{8}$ (*) Make y subject

$\frac{dy}{dx} = \frac{x}{4}$ Must be able to differentiate.

let $x = 2$

$m = \frac{1}{2}$ ✓ tangent

Normal: $m_n = -2$

Generally Poorly done.

$y - y_1 = m(x - x_1)$

Need y_1

When $x = 2$

$y = \frac{4}{8}$ (sub into $y = \frac{x^2}{8}$ (*))

$y = \frac{1}{2}$ ✓

$y - \frac{1}{2} = -2(x-2)$ ✓

$y = -2x + 4\frac{1}{2}$ ✓

Lots of Mistakes bringing ½ across.

c) i) $y = x^2$

$\frac{dy}{dx} = 2x$

let $x = c$

$m = 2c$ ✓

Don't be put off by letters. Exactly the same process

ii) At $x = c$

$y = c^2$ (ie $y = x^2$)

$y - c^2 = 2c(x - c)$

$y - c^2 = 2cx - 2c^2$

$y = 2cx - c^2$ ✓

iii) tangent intersects x axis at $x = 1$

ie $(1, 0)$

Let $x = 1,\ y = 0$

$0 = 2c - c^2$ ✓

$0 = c(2 - c)$

$c = 0$ or 2

c must $= 2$ from diagram and from tangent intersecting x-axis at one point

SEE NEXT PAGE FOR MORE NOTES ✳

✓ = 1 mark

Question 8 (HRK)

a) (i) $9x^2-6x+1=0$

$\Delta = b^2-4ac$
$= 36-4\times9\times1$
$= 0.$ ✓✓

∴ roots are real, rational, equal

(ii) $x^2+kx-9=0$

$\Delta > 0$

$\Delta = k^2-4\times1\times-9$ ✓

$k^2+36>0$ always

∴ for all values of k, equation has real + distinct roots ✓

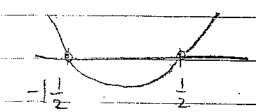
b) $x^2+(2r-3)x+(3-4r)=0$

$\Delta = b^2-4ac$
$=(2r-3)^2-4(3-4r)$
$= 4r^2-12r+9-12+16r$
$\Delta = 4r^2+4r-3$ ✓

(i) $\Delta<0$

$4r^2+4r-3<0$

$(2x-1)(2x+3)=0$



$r>-1\frac{1}{2}$ and

$r<\frac{1}{2}$

ie

$-1\frac{1}{2}<r<\frac{1}{2}$ ✓

(ii) JUST LET $x=0$ ✓✓

$0^2+0+3-4r=0$

$r=\frac{3}{4}$

OR Let 1 root be 0

the other be α

Product $\alpha\times0=\frac{c}{a}$

$0=\frac{3-4r}{1}$ ✓

$0=3-4r$

$4r=3$

$r=\frac{3}{4}$ ✓

(c) +ve definite

when $a>0$ (concave up)

and $\Delta<0$ (no real roots)

$f(x)=2x^2-3x+7$

$a=2$ $\quad \Delta=(-3)^2-4(2)\times7$

$a>0$ ✓ $\quad = 9-56$

$= -47$

<0 ✓

∴ $f(x)$ +ve definite ie lies entirely above x-axis

d) $3x^2+4x+r=0$

Let one root be α

the other $\frac{1}{\alpha}$ ✓

Product: $\alpha\times\frac{1}{\alpha}=\frac{c}{a}$

$1=\frac{r}{3}$ ✓

$r=3$

NOTES: ✳

8(a)(i) Consider the place of Δ in the quadratic formula. Here Δ is NOT negative ∴ roots are real

$\Delta=0$ means $x=\frac{-b\pm\sqrt{0}}{2a}$ ie 1 answer ie equal roots + rational

(ii) LOOK AT k^2+36 THIS IS ALWAYS POSITIVE ie square a number then ADD 36.

b) NOTE: $r>-1\frac{1}{2}$, $r<\frac{1}{2}$ gives whole number line must have intersection ie AND

Best to write 'between' ie $-1\frac{1}{2}<r<\frac{1}{2}$

(ii) Remember a root is an x-value that FITS the equation so do this in one line by letting $x=0$.

9 a(iii) $(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2$ (a basic expansion)

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ (rearranged ie $-2\alpha\beta$)

(iv) COMMON DENOMINATOR is $\alpha\beta$. Many need to revise fractions

b) Expand then collect like terms AND RTQ (Read the Question) + answer it !! ie "Rewrite the expression" as asked.

c(i) NOTE if $x^2=5$ $\quad$ likewise If $x^2=1$

$x=\pm\sqrt{5}$ $\quad$ $x=\pm1$

(ii) $25^x=(5^2)^x=5^{2x}=5^{x2}=(5^x)^2$

then in $(5^x)^2-26(5^x)+25=0$

Let $V=5^x$

$V^2-26V+25=0$

+ Solve etc ☺

* SEE PREVIOUS PAGE FOR NOTES

Question 9 (HRK)

a) $5x^2+4x-6=0$

(i) $\alpha+\beta = \frac{-4}{5}$ ✓

(ii) $\alpha\beta = \frac{-6}{5}$ ✓

(iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= \left(\frac{-4}{5}\right)^2 - 2\left(\frac{-6}{5}\right)$

$= \frac{16}{25} + \frac{12}{5}$

$= 3\frac{1}{25}$ ✓

(iv) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$= \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= 3\frac{1}{25} \div \frac{-6}{5}$

$= -2\,{}^{8}/_{15}$ ✓

b) $A(x-1)^2 + B(x+2)$

x^2-4x-3

$A(x^2-2x+1)+Bx+2B$

$= Ax^2-2Ax+A+Bx+2B$

$= Ax^2-(2A-B)x+(A+2B)$ ✓

$A=1$

$2A-B=4$

$2(1)-B=4$

$-B=2$

$B=-2$

$\therefore$

$x^2-4x-3 \equiv (x-1)^2-2(x+2)$ ✓

c)(i) $x^4-6x^2+5=0$

let $u=x^2$

$u^2-6u+5=0$

$(u-5)(u-1)=0$

$u=5,\ u=1$ ✓

$x^2=5,\ x^2=1$

$x=\pm5,\ \pm1$ ✓

(ii) $25^x-26(5^x)+25=0$

let $u=5^x$ ✓

$u^2-26u+25=0$

$(u-25)(u-1)=0$ ✓

$u=25,\ 1$

$5^x=25,\ 5^x=1$

$x=5,\ 0$ ✓

Q10 - BMM

a) $A(-3,1)$ $B(4,-2)$

$P(x,y)$

$PA \perp PB$

$\therefore m_{PA} \times m_{PB} = -1$

$\frac{y-1}{x+3} \times \frac{y+2}{x-4} = -1$

$\frac{y^2-y-2}{x^2-x-12} = -1$

$x^2-x+y^2-y-14=0$

$\therefore$ locus is a circle.

1mk - correct substitution into gradient formula

1mk - simplifying and writing in general form

1mk - stating the geometric description of locus.

b) $(y-1)^2 = 4\left(\frac{1}{4}\right)(x-0)$

$y^2-2y+1=x$

$y^2-2y-x+1=0.$

1mk - correct formula of parab.

1mk - correct substitution of focal length + vertex

1mk - correct eqn of parabola

c)

(i) $x^2+6x = 12y+33$

$(x+3)^2 = 12y+33+9$

$(x+3)^2 = 12y+42$

$(x+3)^2 = 12(y+3.5)$

$\therefore$ vertex $= (-3, -3.5)$

1mk - complete the square and write in the form $(x-h)^2=4a(y-k)$

1mk - correct reading of vertex

1mk - if vertex given followed working before

(ii) $4a = 12$

$a = 3$

1mk - correct focal length.

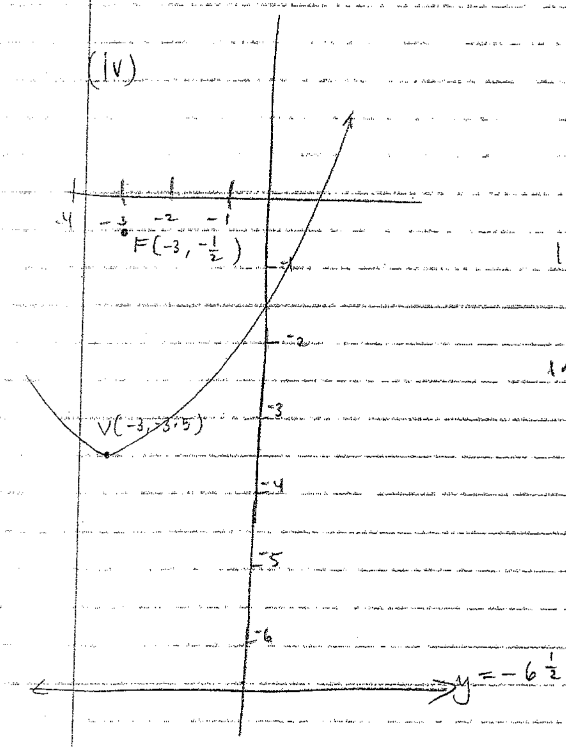
(iii) Focus $(-3, -\frac{1}{2})$

Directrix $y = -6\frac{1}{2}$

1mk each

1mk each if followed correctly from previous working.

(iv)



1mk - correct sketching + labelling of all points.

1mk - if sketch followed correctly from previous work